Controlling currency risk with options or forwards*

Nikolas Topaloglou, Hercules Vladimirou† and Stavros A. Zenios

HERMES European Center of Excellence on Computational Finance and Economics
School of Economics and Management
University of Cyprus
P.O.Box 20537, CY-1678 Nicosia, Cyprus

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Abstract

We consider alternative means for controlling currency risk exposure in actively-managed international portfolios. We extend multi-stage stochastic programming models to incorporate decisions for optimal selection of forward contracts or currency options for hedging purposes. We adapt a valuation procedure to price currency options consistently with discrete distributions of exchange rates that are used in the context of the stochastic programming model. We empirically assess the comparative effectiveness of alternative decision strategies through extensive numerical tests. Besides individual put options, we also consider trading strategies that involve combinations of options, and contrast them with optimal choices of forward contracts. We compare the alternative strategies both in static tests — in terms of their risk-return profiles — as well as in dynamic backtesting simulations using market data in a rolling horizon basis. We find that optimally selected currency forward contracts yield superior results in comparison to single protective puts per currency. However, option-trading strategies with suitable payoffs can improve performance in terms of higher portfolio returns. Moreover, we demonstrate that a multi-stage (dynamic) stochastic programming model consistently outperforms its single-stage (myopic) counterpart and yields incremental benefits.

1 Introduction

International investment portfolios are of particular interest to multinational firms, institutional investors, financial intermediaries and high net-worth individuals. Investments in financial assets denominated in multiple currencies provide a wider scope for diversification than investments localized in any market and mitigate the risk exposure to any specific market. However, internationally diversified portfolios are inevitably exposed to currency risk due to uncertain fluctuations of exchange rates.

Currency risk is an important aspect of international investments. With the abandonment of the Bretton Woods system in 1973, exchange rates were set free to float independently. Since then, exchange rates exhibited periods of high volatility; correlations between exchange rates, as well as between asset returns and exchange rates, have also changed substantially. Stochastic fluctuations of

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†Corresponding author. Fax: +357-22-892421, Email: hercules@ucy.ac.cy

exchange rates constitute an important source of risk that needs to be properly considered (see, e.g., Eun and Resnick [9]). Thus, it is important to investigate the relative effectiveness of alternative means for controlling currency risk.

Surprisingly, in practice, and usually in the literature as well, international portfolio management problems are addressed in a piecemeal manner. First, an aggregate allocation of funds across various markets is decided at the strategic level. These allocations are then managed pretty much independently, typically by market analysts who select investment securities in each market and manage their respective portfolio. Performance assessment is usually based on comparisons against preselected benchmarks. Currency hedging is often viewed as a subordinate decision; it is usually taken last so as to cover exposures of foreign investments that were decided previously. Changes in the overall portfolio composition are not always coordinated with corresponding adjustments to the currency hedging positions. Important and interrelated decisions are considered separately and sequentially. This approach neglects possible cross-hedging effects among portfolio positions and cannot produce a portfolio that jointly coordinates asset and currency holdings so as to yield an optimal risk-return profile. Jorion [15] criticized this overlay approach; he showed that it is suboptimal to a holistic view that considers all the interrelated decisions in a unified manner.

We consider models that jointly address the international diversification, asset selection, and the currency hedging decisions. An important part of this study is the comparison of alternative instruments and tactics for controlling currency risk in international financial portfolios. In dynamic portfolio management settings this is a challenging problem. We employ the stochastic programming paradigm to empirically assess the relative performance of alternative strategies that use either currency forward contracts or currency options as means of controlling currency risk.

A currency forward contract constitutes an obligation to sell (or buy) a certain amount of a foreign currency at a specific future date, at a predetermined exchange rate. A forward contract eliminates the downside risk for the amount of the transaction, but at the same time it forgoes the upside potential in the event of a favorable movement in the exchange rate. By contrast, a currency put option provides insurance against downside risk, while retaining upside potential as the option is simply not exercised if the exchange rate appreciates. So, currency forward contracts can be considered as more rigid hedge tools in comparison to currency options.

Few empirical studies on the use of currency options are reported in the literature. Eun and Resnick [10] examine the use of forward contracts and protective put options for handling currency risk. In ex ante tests, they find that forward contracts generally provide better performance in hedging currency risk than single protective put options. Albuquerque [3] analyzes hedging tactics and shows that forward contracts dominate the use of single put options as hedges of transaction exposures. The reason is that forward contracts pay more than single options on the downside, hence less currency needs to be sold forward to achieve the same degree of hedging; a smaller hedge ratio is required and the cost for hedging is less. Maurer and Valiani [21] compare the effectiveness of currency options versus forward contracts for hedging currency risk. They find that both ex-post, as well as in out-of-sample tests, forwards contracts dominate the use of single currency put options. Only put in-the-money options produce comparable results with optimally-hedged portfolios with forwards. Their results indicate more active use of put in-the-money options than at-the-money or out-of-the money put options which reveals the dependence of a hedging strategy based on put options on the level of the strike price.

Conover and Dubofsky [6] consider American options. They empirically examine portfolio insurance strategies employing currency spot and future options. They find that protective puts using future options are generally dominated by both protective puts that use options on spot currencies and by fiduciary calls on futures contracts. Lien and Tse [18] compare the hedging effectiveness of currency options versus futures on the basis of lower partial moments (LPM). They conclude that currency futures provide a better hedging instrument than currency options; the only situation in which options outperform futures occurs when the decision maker is optimistic (with a large target return) and not too concerned about large losses.

Steil [25] applies an expected utility analysis to determine optimal contingent claims for hedging foreign transaction exposure as well as optimal forward and option hedge alternatives. Using quadratic, negative exponential and positive exponential utility functions, Steil concludes that currency options play limited useful role in hedging contingent foreign exchange transaction exposures.

There is no consensus in the literature regarding a universally preferable strategy to hedge currency risk, although the majority of results indicate that currency forwards generally yield better performance than single protective put options. Earlier studies do not jointly consider the optimal selection of internationally diversified portfolios. Our study addresses this aspect of the portfolio management problem in connection with the associated problem of controlling currency risk, and contributes to the aforementioned debate. We empirically examine whether forward contracts are effective hedging instruments, or whether superior performance can be achieved by using currency options — either individual protective puts or combinations of options with appropriate payoffs.

To this end, we extend the multistage stochastic programming model for international portfolio management that was developed in Topaloglou et al. [27] by introducing positions in currency options to the decision set at each stage. The model accounts for the effects of portfolio (re)structuring decisions over multiple periods, including positions in currency options among its permissible decisions. The incorporation of currency options in a practical portfolio optimization model is a novel development. A number of issues are addressed in the adaptation of the model. Currency options are suitably priced at each decision stage of the stochastic program in a manner consistent with the scenario set of exchange rates. The scenario-contingent portfolio rebalancing decisions account for the discretionary exercise of expiring options at each decision point.

The dynamic nature of portfolio management problems motivated our development of flexible multi-stage stochastic programming models that capture in a holistic manner the interrelated decisions faced in international portfolio management. Multi-stage models help decision makers adopt more effective decisions; their decisions consider longer-term potential benefits and avoid myopic reactions to short-term movements that may lead to losses.

We use the stochastic programming model as a testbed to empirically assess the relative effectiveness of currency options and forward contracts to control the currency risk of international portfolios in a dynamic setting. We analyze the effect of alternative strategies on the performance of international portfolios of stock and bond indices in backtesting experiments over multiple time periods. Our empirical results confirm that portfolios with optimally-selected forward contracts outperform those that involve a single protective put option per currency. However, we find that trading strategies involving suitable combinations of currency options have the potential to produce better performance. Moreover, we demonstrate through extensive numerical tests the viability of a multi-stage stochastic programming model as a decision support tool for international portfolio management. We show that the dynamic (multi-stage) model consistently outperforms its single-stage (myopic) counterpart.

The paper is organized as follows. In section 2 we present the formulation of the optimization models for international portfolio selection. In section 3 we discuss the hedging strategies employed in the empirical tests. In section 4 we describe the computational tests and we discuss the empirical results. Section 5 concludes. Finally, in the Appendix we describe the procedure for pricing European currency options consistently with the discrete distribution of exchange rates on a scenario tree.

2 The International Portfolio Management Model

The international portfolio management model aims to determine the optimal portfolio that has the minimum shortfall risk at each level of expected return over the planning horizon. The problem is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. The portfolio is exposed to market and currency risk. To cope with the market risk, the portfolio is diversified across multiple markets. International diversification exposes the foreign investments to currency risk. To control the currency risk, the investor may enter into currency exchange contracts in the forward market, or buy currency options — either single protective puts, or combinations of options that form a particular trading strategy.

In this section we develop scenario-based stochastic programming models for managing investment portfolios of international stock and government bond indices. The models address the problems of optimal portfolio selection and currency risk management in an integrated manner. Their deterministic inputs are: the initial asset holdings, the current prices of the stock and bond indices, the current spot exchange rates, the forward exchange rates or the currency option prices — depending on which instruments are used to control currency risk — for a term equal to the decision interval. We also specify scenario-dependent data, together with associated probabilities, that represent the discrete process of the random variables at any decision stage in terms of a scenario tree. The prices of the indices and the exchange rates at any node of the scenario tree are generated with the moment-matching procedure of Høyland et al. [11]; these, in turn, uniquely determine the option payoffs at any node of the tree.

We explore single-stage, as well as multi-stage stochastic programming models to manage international portfolios of financial assets. The multi-stage model determines a sequence of buying and selling decisions at discrete points in time (monthly intervals). The portfolio manager starts with a given portfolio and with a set of postulated scenarios about future states of the economy represented in terms of a scenario tree, as well as corresponding forward exchange rates or currency option prices depended on the postulated scenarios. This information is incorporated into a portfolio restructuring decision. The composition of the portfolio at each decision point depends on the transactions that were decided at the previous stage. The portfolio value depends on the outcomes of asset returns and exchange rates realized in the interim period and, consequently, on the discretionary exercise of currency options whose purchase was decided at the previous decision point. Another portfolio restructuring decision is then made at that node of the scenario tree based on the available portfolio and taking into account the projected outcomes of the random variables in subsequent periods.

The models employ the conditional value-at-risk (`CVaR`) risk metric to minimize the excess losses, beyond a prespecified percentile of the portfolio return distribution, over the planing horizon. The decision variables reflect asset purchase and sale transactions that yield a revised portfolio. Additionally, the models determine the levels of forward exchange contracts or currency option purchases to mitigate currency risk. Positions in specific combinations of currency options — corresponding to certain trading strategies — are easily enforced with suitable linear constraints. The portfolio optimization models incorporate practical considerations (no short sales for assets, transaction costs) and minimize the tail risk of final portfolio value at the end of the planning horizon for a given target of expected return. The models determine jointly the portfolio compositions (not only the allocation of funds to different markets, but also the selection of assets within each market), and the levels of currency hedging in each market via forward contracts or currency options.

To ensure internal consistency of the models we price the currency options at each decision node on the basis of the postulated scenario sets. To this end, we adapt a suitable option valuation procedure that accounts for higher-order moments exhibited in historical data of exchange rates, as described in the Appendix. The option prices are used as inputs to the optimization models together with the postulated scenarios of asset returns and exchange rates. We confine our attention to European currency options that may be purchased at any decision node and have a maturity of one period. At any decision node of the scenario tree the selected options in the portfolio may be exercised and new option contracts may be purchased.

We use the following notation:

Sets:

$\boldsymbol{C_0}$	the set of currencies (synonymously, markets, countries),
$\ell \in \boldsymbol{C_0}$	the index of the base (reference) currency in the set of currencies,
$\boldsymbol{C} = \boldsymbol{C_0} \backslash \{\ell\}$	the set of foreign currencies,
$\boldsymbol{I_c}$	the set of assets denominated in currency $c \in \boldsymbol{C_0}$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term government bond index in each country),
$\boldsymbol{N}$	the set of nodes of the scenario tree,
$n \in \boldsymbol{N}$	a typical node of the scenario tree ($n = 0$ is the root node at $t = 0$),
$\boldsymbol{N_t} \subset \boldsymbol{N}$	the set of distinct nodes at time period $t = 0, 1, ..., T$,
$\boldsymbol{N_T} \subset \boldsymbol{N}$	the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios,
$\boldsymbol{S_n} \subset \boldsymbol{N}$	the set of immediate successor nodes of node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n.
$p(n) \in \boldsymbol{N}$	the unique predecessor node of node $n \in \boldsymbol{N} \setminus \{0\}$,
$\boldsymbol{J_c}$	the set of available currency options for foreign currency $c \in \boldsymbol{C}$ (differing in terms of exercise price).

Input Data:

(a). Deterministic parameters:

T	length of the time horizon (number of decision periods),
b_{ic}	initial position in asset $i \in \boldsymbol{I_c}$ of currency $c \in \boldsymbol{C_0}$ (in units of face value),
h_c^0	initially available cash in currency $c \in \boldsymbol{C_0}$ (surplus if +ve, shortage if -ve),
δ	proportional transaction cost for sales and purchases of assets,
d	proportional transaction cost for currency transactions in the spot market,
μ	prespecified target expected portfolio return over the planning horizon,
α	prespecified percentile for the `CVaR` risk measure,
π_{ic}^0	current market price (in units of the respective currency) per unit of face value of asset $i \in \boldsymbol{I_c}$ in currency $c \in \boldsymbol{C_0}$,
e_c^0	current spot exchange rate for foreign currency $c \in \boldsymbol{C}$,
f_c^0	currently quoted one-month forward exchange rate for foreign currency $c \in \boldsymbol{C}$,
K_j	the strike price of an option $j \in \boldsymbol{J_c}$, on the spot exchange rate of foreign currency $c \in \boldsymbol{C}$.

(b). Scenario-dependent parameters:

p_n	probability of occurrence of node $n \in \boldsymbol{N}$,
e_c^n	spot exchange rate of currency $c \in C$ at node $n \in \boldsymbol{N}$,
f_c^n	one-month forward exchange rate for foreign currency $c \in \boldsymbol{C}$ at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$,
π_{ic}^n	price of asset $i \in \boldsymbol{I_c},\ c \in \boldsymbol{C_0}$ on node $n \in \boldsymbol{N}$ (in units of local currency),
$cc^n(e_c^n, K_j)$	price of European call currency option $j \in \boldsymbol{J_c}$ on the exchange rate of currency $c \in \boldsymbol{C}$, at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity of one month,
$pc^n(e_c^n, K_j)$	price of European put currency option $j \in \boldsymbol{J_c}$ on the exchange rate of currency $c \in \boldsymbol{C}$, at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity of one month.

All exchange rates $(e_c^0, f_c^0, e_c^n, f_c^n)$ are expressed in units of the base currency per one unit of the foreign currency $c \in \boldsymbol{C}$. Of course, the exchange rate of the base currency to itself is trivially equal to one, $f_\ell^n = e_\ell^n \equiv 1,\ \forall n \in \boldsymbol{N}$. The prices cc and pc of currency call and put options, respectively, are expressed in units of the base currency ℓ.

Computed Parameters:

V_ℓ^0 total value (in units of the base currency) of the initial portfolio.

$$V_\ell^0 \;=\; h_\ell^0 + \sum_{i \in \boldsymbol{I_\ell}} b_{i\ell}\, \pi_{i\ell}^0 + \sum_{c \in \boldsymbol{C}} e_c^0 \left(h_c^0 + \sum_{i \in \boldsymbol{I_c}} b_{ic}\, \pi_{ic}^0 \right) \tag{1}$$

Decision Variables:

Portfolio (re)structuring decisions are made at all non-terminal nodes of the scenario tree, thus $\forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$.

(a). Asset purchase, sale, and hold quantities (in units of face value):

- x_{ic}^n units of asset $i \in \boldsymbol{I_c}$ of currency $c \in \boldsymbol{C_0}$ purchased,
- v_{ic}^n units of asset $i \in \boldsymbol{I_c}$ of currency $c \in \boldsymbol{C_0}$ sold,
- w_{ic}^n resulting units of asset $i \in \boldsymbol{I_c}$ of currency $c \in \boldsymbol{C_0}$ in the revised portfolio.

(b). Currency transfers in the spot market:

- $x_{c,e}^n$ amount of base currency exchanged in the spot market for foreign currency $c \in \boldsymbol{C}$,
- $v_{c,e}^n$ amount of the base currency collected from a spot sale of foreign currency $c \in \boldsymbol{C}$.

(c). Forward currency exchange contracts:

- $u_{c,f}^n$ amount of base currency collected from sale of currency $c \in \boldsymbol{C}$ in the forward market (i.e., amount of a forward contract, in units of the base currency). A negative value indicates a purchase of the foreign currency forward. This decision is taken at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, but the transaction is actually executed at the end of the respective period, i.e., at the successor nodes $\boldsymbol{S_n}$.

(d). Variables related to currency options transactions:

- $ncc_{c,j}^n$ purchases of European call currency option $j \in \boldsymbol{J_c}$ on the exchange rate of currency $c \in \boldsymbol{C}$, with exercise price K_j and maturity of one month,
- $npc_{c,j}^n$ purchases of European put currency option $j \in \boldsymbol{J_c}$ on the exchange rate of currency $c \in \boldsymbol{C}$, with exercise price K_j and maturity of one month.

When currency options are used in the portfolio management model, only long positions in the respective trading strategies of options are allowed.

Auxiliary variables:

- y_n auxiliary variables used to linearize the piecewise linear function in the definition of the `CVaR` risk metric; they measure the portfolio losses at leaf node $n \in \boldsymbol{N_T}$ in excess of `VaR`,
- z the value-at-risk (`VaR`) of portfolio losses over the planning horizon (i.e., the *alpha*-th percentile of the loss distribution),
- V_ℓ^n the total value of the portfolio at the end of the planning horizon at leaf node $n \in \boldsymbol{N_T}$ (in units of the base currency),
- R_n return of the international portfolio over the planning horizon at leaf node $n \in \boldsymbol{N_T}$.

2.1 International Portfolio Management Models

We consider either forward contracts or currency options in the optimization models, but not both, as means to mitigate the currency risk of international portfolios. Hence, we formulate two different variants of the international portfolio optimization model; the models differ in the cashflow balance constraints and the computation of the final portfolio value.

Portfolio Optimization Model with Currency Options

$$
\begin{aligned}
\texttt{min} \quad & z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N_T}} p_n y_n && \text{(2a)} \\
\texttt{s.t.} \quad & h_\ell^0 + \sum_{i \in \boldsymbol{I_\ell}} v_{i\ell}^0 \pi_{i\ell}^0 (1-\delta) + \sum_{c \in \boldsymbol{C}} v_{c,e}^0 (1-d) = \sum_{i \in \boldsymbol{I_\ell}} x_{i\ell}^0 \pi_{i\ell}^0 (1+\delta) + \sum_{c \in \boldsymbol{C}} x_{c,e}^0 (1+d) \\
& \quad + \sum_{c \in \boldsymbol{C}} \left\{ \sum_{j \in \boldsymbol{J_c}} \left[npc_{c,j}^0 * pc^0(e_c^0, K_j) \right] \right\} && \text{(2b)} \\
& h_c^0 + \sum_{i \in \boldsymbol{I_c}} v_{ic}^0 \pi_{ic}^0 (1-\delta) + \frac{1}{e_c^0} x_{c,e}^0 = \sum_{i \in \boldsymbol{I_c}} x_{ic}^0 \pi_{ic}^0 (1+\delta) + \frac{1}{e_c^0} v_{c,e}^0 , \quad \forall\, c \in \boldsymbol{C} && \text{(2c)} \\
& h_\ell^n + \sum_{i \in \boldsymbol{I_\ell}} v_{i\ell}^n \pi_{i\ell}^n (1-\delta) + \sum_{c \in \boldsymbol{C}} v_{c,e}^n (1-d) + \sum_{c \in \boldsymbol{C}} \left\{ \sum_{j \in \boldsymbol{J_c}} \left[npc_{c,j}^{p(n)} * \max(K_j - e_c^n, 0) \right] \right\} \\
& \quad = \sum_{i \in \boldsymbol{I_\ell}} x_{i\ell}^n \pi_{i\ell}^n (1+\delta) + \sum_{c \in \boldsymbol{C}} x_{c,e}^n (1+d) + \sum_{c \in \boldsymbol{C}} \left\{ \sum_{j \in \boldsymbol{J_c}} \left[npc_{c,j}^n * pc^n(e_c^n, K_j) \right] \right\}, \\
& \quad \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} && \text{(2d)} \\
& h_c^n + \sum_{i \in \boldsymbol{I_c}} v_{ic}^n \pi_{ic}^n (1-\delta) + \frac{1}{e_c^n} x_{c,e}^n = \sum_{i \in \boldsymbol{I_c}} x_{ic}^n \pi_{ic}^n (1+\delta) + \frac{1}{e_c^n} v_{c,e}^n , \\
& \quad \forall\, c \in \boldsymbol{C} ,\ \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} && \text{(2e)} \\
& V_\ell^n = \sum_{i \in \boldsymbol{I_\ell}} w_{i\ell}^{p(n)} \pi_{i\ell}^n + \sum_{c \in \boldsymbol{C}} \left\{ e_c^n \left[\sum_{i \in \boldsymbol{I_c}} w_{ic}^{p(n)} \pi_{ic}^n \right] + \sum_{j \in \boldsymbol{J_c}} \left[npc_{c,j}^{p(n)} * \max(K_j - e_c^n, 0) \right] \right\}, \\
& \quad \forall n \in \boldsymbol{N_T} && \text{(2f)} \\
& \sum_{j \in \boldsymbol{J_c}} npc_{c,j}^n \le \sum_{i \in \boldsymbol{I_c}} e_c^n \left(w_{ic}^n \pi_{ic}^n \right) , \quad \forall c \in \boldsymbol{C},\ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} && \text{(2g)} \\
& R_n = \frac{V_\ell^n}{V_\ell^0} - 1 , \quad \forall n \in \boldsymbol{N_T} && \text{(2h)} \\
& \sum_{n \in \boldsymbol{N_T}} p_n R_n \ge \mu , && \text{(2i)} \\
& y_n \ge L_n - z, \quad \forall n \in \boldsymbol{N_T} && \text{(2j)} \\
& y_n \ge 0 , \quad \forall n \in \boldsymbol{N_T} && \text{(2k)} \\
& L_n = -R_n , \quad \forall n \in \boldsymbol{N_T} && \text{(2l)} \\
& w_{ic}^0 = b_{ic} + x_{ic}^0 - v_{ic}^0 , \quad \forall\, i \in \boldsymbol{I_c} ,\ \forall\, c \in \boldsymbol{C_0} && \text{(2m)} \\
& w_{ic}^n = w_{ic}^{p(n)} + x_{ic}^n - v_{ic}^n, \quad \forall\, i \in \boldsymbol{I_c} ,\ \forall\, c \in \boldsymbol{C_0} ,\ \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} && \text{(2n)} \\
& x_{ic}^n \ge 0 , \quad w_{ic}^n \ge 0 , \quad \forall\, i \in \boldsymbol{I_c} ,\ \forall\, c \in \boldsymbol{C_0} ,\ \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} && \text{(2o)} \\
& 0 \le v_{ic}^0 \le b_{ic}, \quad \forall\, i \in \boldsymbol{I_c} ,\ \forall\, c \in \boldsymbol{C_0} && \text{(2p)} \\
& 0 \le v_{ic}^n \le w_{ic}^{p(n)}, \quad \forall\, i \in \boldsymbol{I_c} ,\ \forall\, c \in \boldsymbol{C_0} ,\ \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} && \text{(2q)}
\end{aligned}
$$

This model minimizes the Conditional Value-at-Risk (`CVaR`) of portfolio losses over the planning horizon, while also requiring that expected portfolio return meets a prespecified target, μ, (2i). Expectations are computed over the set of terminal states (leaf nodes). The objective value (2a) measures the `CVaR` of portfolio losses at the end of the horizon, while the corresponding `VaR` of portfolio losses (at percentile α) is captured by the variable z; see [22, 26].

We adopt the `CVaR` risk metric as it is suitable for asymmetric distributions. Asymmetry in the returns of the international portfolios arises not only because of the skewed and leptokurtic distributions of exchange rates, but mainly because of the highly asymmetric payoffs of options. The choice of the `CVaR` metric, that captures the tail risk, is entirely appropriate for the purposes of this study that aims to explore the effectiveness of currency options — or forward contracts — as means to mitigate and control currency risk so as to minimize the excess shortfall of portfolio returns over the planning horizon.

The purchase of an option entails a cost (price) that is payable at the time of purchase. The cost of option purchases is considered in the cash balance constraints of the base currency (2b) and (2d). Similarly, the conditional payoffs of the options are also accounted for in the cash balance conditions at the respective expiration dates. We consider only European options. Specifically, we use options with a single-period maturity (one month in our implementation). So, options purchased at some decision stage mature at exactly the next decision period, at which time they are either exercised, if they yield a positive payoff, or are simply left to expire.

The exercise prices of the options are specified exogenously as inputs to the model. By considering multiple options with different strike prices on the same currency, we can provide the model flexibility to chose the most appropriate options at each decision stage. The option prices at each node of the scenario tree are computed according to the valuation procedure summarized in the Appendix. The corresponding payoffs at the successor nodes on the tree are also computed and entered as inputs to the portfolio optimization program. The optimal portfolio rebalancing decisions, as well as the optimal positions in currency options, are considered in a unified manner at each decision node of the scenario tree. The model does not directly relate positions in options on different currencies, thus allowing selective hedging choices.

Model, (2a)–(2q) is a stochastic linear program with recourse. Equations (2b) and (2c) impose the cash balance conditions in every currency at the first decision stage (root node); the former for the base currency, ℓ, and the latter for the foreign currencies, $c \in \boldsymbol{C}$. Each constraint equates the sources and the uses of funds in the respective currency. The availability of funds stems from initially available cash reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of funds include the expenditures for the purchase of assets, the outgoing currency exchanges in the spot market, and the costs for the purchase of currency options; the latter for the cash equation of the base currency only. All currency exchanges are made through the base currency. Linear transaction costs (i.e., proportional to the amount of a transaction) are considered for purchases and sales of assets, as well as for currency exchanges in the spot market. Note that all available funds are placed in the available assets; that is, we don't have investments in money market accounts in any currency, nor do we have borrowing. These could be simple extensions of the model.

Similarly, equations (2d) and (2e) impose the cash balance conditions at subsequent decision states for the base currency, ℓ, and the foreign currencies, $c \in \boldsymbol{C}$, respectively. Now cash availability comes from exogenous inflows, if any, revenues from the sale of asset holdings in the portfolio at hand, incoming spot currency exchanges and potential payoffs from the exercise of currency option contracts purchased at the predecessor node. Again, the uses of funds include the purchase of assets, outgoing currency exchanges in the spot market and the purchase of currency options with maturity one period ahead. The cash flows associated with currency options (purchases and payoffs) enter only the cash balance equations of the base currency.

The final value of the portfolio at leaf node $n \in \boldsymbol{N_T}$ is computed in (2f). The total terminal value, in units of the base currency, reflects the proceeds from the liquidation of all final asset holdings at the corresponding market prices and the payoffs of currency put options expiring at the end of the horizon. Revenues in foreign currencies are converted to the base currency by applying the respective spot exchange rates at the end of the horizon.

Constraints (2g) limit the put options that can be purchased on each foreign currency. The total position in put options of each currency is bounded by the total value of assets that are held in the respective currency after the portfolio revision. So, currency puts are used only as protective hedges for investments in foreign currencies, and can cover up to the foreign exchange rate exposure of the portfolio held at the respective decision state.

Equation (2h) defines the return of the portfolio during the planning horizon at leaf node $n \in \boldsymbol{N_T}$. Constraint (2i) imposes a minimum target bound, μ, on the expected portfolio return over the planning horizon. Constraints (2j) and (2k) are the definitional constraints for `CVaR`, while equation (2l) defines portfolio losses as negative returns. Equations (2m) enforce the balance constraints for each asset, at the first decision stage, while equations (2n) similarly impose the balance constraint for each asset, at subsequent decision states. These equations determine the resulting composition of the revised portfolio after the purchase and sale transactions of assets at the respective decision nodes. Short positions in assets are not allowed, so, constraints (2o) ensure that asset purchases, as well as the resulting holdings in the rebalanced portfolio are nonegative. Finally, constraints (2p) and (2q) restrict the sales of each asset by the corresponding holdings in the portfolio at the time of a rebalancing decision.

Starting with an initial portfolio and using a representation of uncertainty for the asset prices and exchange rates by means of a scenario tree, as well as the prices and payoffs of the currency put options at each decision node, the multistage portfolio optimization model determines optimal decisions under the contingencies of the scenario tree. The portfolio rebalancing decisions at each node of the tree specify not only the allocation of funds across markets but also the positions in assets within each market. Moreover, positions in currency options are appropriately determined so as to mitigate the currency risk exposure of the foreign investments during the holding period (i.e., until the next portfolio rebalancing decision).

Portfolio Optimization Model with Currency Forward Contracts

$$\texttt{min} \quad z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N_T}} p_n y_n \tag{3a}$$

$$\texttt{s.t.} \quad h_\ell^0 + \sum_{i \in \boldsymbol{I_\ell}} v_{i\ell}^0 \pi_{i\ell}^0 (1-\delta) + \sum_{c \in \boldsymbol{C}} v_{c,e}^0 (1-d) \;=\; \sum_{i \in \boldsymbol{I_\ell}} x_{i\ell}^0 \pi_{i\ell}^0 (1+\delta) + \sum_{c \in \boldsymbol{C}} x_{c,e}^0 (1+d) \tag{3b}$$

$$h_c^0 + \sum_{i \in \boldsymbol{I_c}} v_{ic}^0 \pi_{ic}^0 (1-\delta) + \frac{1}{e_c^0} x_{c,e}^0 \;=\; \sum_{i \in \boldsymbol{I_c}} x_{ic}^0 \pi_{ic}^0 (1+\delta) + \frac{1}{e_c^0} v_{c,e}^0 \,, \quad \forall\, c \in \boldsymbol{C} \tag{3c}$$

$$\begin{aligned} & h_\ell^n + \sum_{i \in \boldsymbol{I_\ell}} v_{i\ell}^n \pi_{i\ell}^n (1-\delta) + \sum_{c \in \boldsymbol{C}} \left(v_{c,e}^n (1-d) + u_{c,f}^{p(n)} \right) \\ & \quad = \sum_{i \in \boldsymbol{I_\ell}} x_{i\ell}^n \pi_{i\ell}^n (1+\delta) + \sum_{c \in \boldsymbol{C}} x_{c,e}^n (1+d) \,, \qquad \forall\, n \in \boldsymbol{N_T} \setminus \{\boldsymbol{N_T} \cup 0\} \end{aligned} \tag{3d}$$

$$\begin{aligned} & h_c^n + \sum_{i \in \boldsymbol{I_c}} v_{ic}^n \pi_{ic}^n (1-\delta) + \frac{1}{e_c^n} x_{c,e}^n \;=\; \sum_{i \in \boldsymbol{I_c}} x_{ic}^n \pi_{ic}^n (1+\delta) + \frac{1}{e_c^n} v_{c,e}^n + \frac{1}{f_c^{p(n)}} u_{c,f}^{p(n)} \,, \\ & \quad \forall\, c \in \boldsymbol{C} \,,\ \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \end{aligned} \tag{3e}$$

$$V_\ell^n \;=\; \sum_{i \in \boldsymbol{I_\ell}} w_{i\ell}^{p(n)} \pi_{i\ell}^n + \sum_{c \in \boldsymbol{C}} \left[u_{c,f}^{p(n)} + e_c^n \Big[\sum_{i \in \boldsymbol{I_c}} w_{ic}^{p(n)} \pi_{ic}^n - \frac{1}{f_c^{p(n)}} u_{c,f}^{p(n)} \Big] \right], \quad \forall\, n \in \boldsymbol{N_T} \tag{3f}$$

$$0 \;\le\; u_{c,f}^n \;\le\; \sum_{m \in \boldsymbol{S_n}} \frac{p_m}{p_n} e_c^m \left(\sum_{i \in \boldsymbol{I_c}} w_{ic}^n \pi_{ic}^m \right), \quad \forall\, c \in \boldsymbol{C},\ \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{3g}$$

and also constraints (2h)–(2q).

Positions in currency forwards shell the value of foreign investments against potential depreciations of exchange rates. However, by fixing the exchange rate of forward transactions, these contracts forgo potential gains in the event of potential appreciations of exchange rates; this is the "penalty" for the protection against downside risk. We consider currency forward contracts with a single-period term (one month in our implementation). Hence, forward contracts decided in one period are executed in the next decision period. Positions in currency forward contracts are introduced as decision variables ($u^n_{c,f}$) at each decision state of the multistage portfolio optimization program. These decisions are determined jointly with the corresponding portfolio rebalancing decisions in an integrated manner. Forward exchange contracts in different currencies are not explicitly connected. The model can chose different coverage of the foreign exchange exposures in the different currencies (i.e., different hedge ratios across currencies), reflecting a flexible selective hedging approach.

This formulation differs from the previous model, that employed currency options, in the cash balance constraints and the valuation of the portfolio at the end of the planning horizon. Equations (3b) and (3c) impose the cash balance conditions in the first stage for the base currency, ℓ, and the foreign currencies, $c \in \boldsymbol{C}$, respectively. Equations (3d) and (3e) impose the cash balance conditions for every currency at subsequent decision states. These equations account for the cashflows associated with currency forward contracts that were decided at the predecessor state.

Equation (3f) computes the value of the portfolio, in units of the base currency, at leaf node $n \in \boldsymbol{N_T}$. The terminal value reflects the proceeds from the liquidation of the final asset holdings at the corresponding market prices and the proceeds of outstanding forward contracts in foreign currencies. The values in foreign currencies are converted to the base currency by applying the respective spot exchange rates at the end of the horizon, after settling the outstanding forward contracts.

Constraints (3g) limit the currency forward contracts. The amount of a forward contract in a foreign currency is restricted by the expected value of all asset holdings in the respective currency after the revision of the portfolio at that state. This ensures that forward contracts are used only for hedging, and not for speculative purposes. The right-hand side of (3g) reflects the expected value of the respective foreign positions at the end of the holding period. The conditional expectation is taken over the discrete outcomes at the successor nodes ($\boldsymbol{S_n}$) of the decision state $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$.

2.2 Scenario Generation

The scenario generation is a critical step of the modelling process. The set of scenarios must adequately depict the projected evolution of the underlying financial primitives (asset returns and exchange rates) and must be consistent with market observations and financial theory. We generate scenarios with the moment-matching method of Høyland et al. [11]. The outcomes of asset returns and exchange rates at each stage of the scenario tree are generated so that their first four marginal moments (mean, variance, skewness and kurtosis), as well as their correlations match their respective statistics estimated from market data. Thus, the outcomes on the scenario tree reflect the empirical distribution of the random variables as implied by historical observations.

We analyze the statistical characteristics of exchange rates over the period 05/1988–11/2001 that were used in the static and dynamic tests. As can be seen in Table 1, the monthly variations of spot exchange rates exhibit skewed distributions. They also exhibit considerable variance in comparison to their mean, as well as excess kurtosis, implying heavier tails than the normal distribution. Jarque-Berra tests [12] on these data indicate that normality hypotheses cannot be accepted.[1]

The skewed and leptokurtic distributions of the financial random variables (asset returns and exchange rates) motivated our choice of the moment-matching scenario generation procedure as this

[1]The Jarque-Berra statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.99 and 9.21, respectively. The normality hypothesis is rejected when the Jarque-Berra statistic has a higher value than the corresponding critical value at the respective confidence level.

Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Berra Statistic
UKtoUS	-0.116%	2.894%	-0.755	5.672	102.39
GRtoUS	-0.124%	3.091%	-0.215	3.399	5.69
JPtoUS	0.030%	3.615%	0.942	6.213	105.27

Table 1: Statistical characteristics and Jarque-Berra statistic of historical monthly changes of spot exchange rates over the period 05/1988–11/2001.

approach can capture the statistical characteristics implied by historical market data. An essential requirement in modelling stochastic financial variables is that they must satisfy the fundamental no-arbitrage condition. We exhaustively tested all the scenario sets used in the numerical experiments and empirically verified that the no-arbitrage requirement was always satisfied.

We note that the stochastic programming models presented above are not restricted to the moment-matching scenario generation procedure. A user may adopt an alternative approach that he finds preferable to project the evolution of the stochastic asset prices and exchange rates, as long as the scenarios effectively reflect the empirical distribution of the random variables and satisfy fundamental financial principles (i.e., must be arbitrage free). Dupačová et al. [8] review alternative scenario generation approaches.

In order to ensure internal consistency of the model we adapt a suitable valuation procedure to price the currency options consistently with the postulated scenarios of exchange rates. This option pricing approach — summarized in the Appendix — accounts for the higher moments of exchange rate fluctuations.

3 Currency Hedging Strategies

The pursuit of effective means to hedge currency risk in international financial portfolios has been a subject of active research. Over the years, there has been a considerable debate on this subject. The observation that, historically, changes in exchange rates had fairly low correlations with foreign stock and bond returns had raised doubts as to the potential benefit of currency hedging. This lack of a systematic relationship could, in principle, lower portfolio risk. Another argument is that, over a long time horizon, currency movements cancel out — the mean-reversion argument. In other words, exchange rates have expected return of zero in the long-run. On the other hand, for active portfolio managers who are concerned with shorter-term horizons, it is important to account for the impact of currency movements on the risk-return characteristics of international portfolios. Moreover, currency returns tend to be episodic; exchange rates can be volatile over short horizons, and the impact of this volatility needs to be controlled through appropriate means in the context of international portfolios. Currency movements also tend to exhibit some degree of persistence (volatility clamping). For these reasons, effective hedging strategies are actively sought by researchers and practitioners to improve the performance of international portfolios.

Currency hedging decisions typically concern the choice between foreign exchange forward contracts or currency options. A forward contract is an agreement between two parties (the investor and a bank) to buy (or sell) a certain amount of foreign currency at a future date at an exchange rate specified at the time of the agreement. Foreign exchange forward contracts are sold by major commercial banks and typically have fixed short-term maturities of one, six or nine months.

Currency forwards provide a simple and cost-effective way to alter the variability of revenues from foreign currency sources, but they may not be equally effective for all types of risk management problems. These contracts fix both the rate, as well as the amount, of a foreign exchange transaction, and thus protect the value of a certain amount in foreign currency against a potential reduction in the exchange rate. If the amount of foreign revenues is known with certainty, then an equivalent currency forward contract will completely eliminate the currency risk. However, in the case of foreign holdings of financial assets, their final value is stochastic due to their uncertain returns during the interim period; hence, full hedging is not attainable in this case. A limitation of currency forwards is that by fixing the exchange rate they forgo the opportunity for gains in the event that the exchange rate appreciates. One could argue that mitigating risk should be the primary consideration, while potential benefits from favorable exchange rate movements should be of a secondary concern. But it is the entire risk-return tradeoff that usually guides portfolio management decisions.

Options provide alternative means to control risks. An exporter could "shell" his future foreign exchange receipts by purchasing a currency put. A portfolio manager could protect his foreign asset holdings by buying currency options to mitigate currency risk exposure associated with foreign investments in the portfolio. Currency put options protect from losses in the event of a significant drop in the exchange rate, but with no sacrifice of potential benefits in the event of currency appreciation, as they would simply not be exercised in such a case. However, currency options entail a cost (purchase price).

In this study we experiment with two different trading strategies involving currency options that have different payoff profiles.

Using Protective Put Options

By buying a European put currency option, the investor acquires the discretionary right to sell a certain amount of foreign currency at a specified rate (exercise price) at the option's maturity date. In the numerical tests we consider protective put options for each foreign currency with three different strike prices, K_j, ("in-the-money, ITM, "at-the-money", ATM, and "out-of-the-money", OTM). These three options constitute the set of available options, $\boldsymbol{J_c}$, for each foreign currency $c \in \boldsymbol{C}$. The options have a term (maturity) of one-month that matches the duration of each decision stage in the model. The ATM options have strike prices equal to the respective spot exchange rates at the time of issue. As decisions are considered at non-leaf nodes of the scenario tree, the strike prices of the ATM options are equal to the scenario-dependent spot exchange rates specified for the corresponding node of the scenario tree. The ITM and OTM put options have strike prices that are 5% higher, respectively 5% lower, than the corresponding spot exchange rates at the respective decision state. These levels of option strike prices have been chosen fairly arbitrarily. Obviously, a larger set of options with different strike prices and cost can be easily included in the model.

The model is allowed to take only long positions in the protective put options. Thus, we add non-negativity constraints for the positions in options in model (2):

$$npc^n_{c,j} \;\geq\; 0\,, \qquad \forall\, j \in \boldsymbol{J_c}\,,\ \forall\, c \in \boldsymbol{C}\,,\ \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$$

Obviously, the OTM put option has a lower price than the ATM put which, in turn, is cheaper than the ITM put option. In the numerical tests we have observed that when the model selects options in the portfolios, these are OTM put options — ITM and ATM options are never selected in the solutions when they are considered together with OTM put options.

Using BearSpread Strategies

A BearSpread strategy is composed of two put options with the same expiration date. It involves a long position in an ITM put and a short position in an OTM put. The strike prices of the constituent options are set as described above.

Let $npc^n_{c,ITM}$ and $npc^n_{c,OTM}$ be the long position in the ITM and the short position in the OTM currency put option, respectively, constituting a BearSpread position in foreign currency $c \in \boldsymbol{C}$ at decision node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. To incorporate the BearSpeard strategy in the optimization model (2), we additionally impose the following constraints:

$$\begin{aligned} npc^n_{c,OTM} + npc^n_{c,ITM} &= 0\,, \quad \forall\, c \in \boldsymbol{C}\,,\ \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \\ npc^n_{c,ITM} &\geq 0\,, \quad \forall\, c \in \boldsymbol{C}\,,\ \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \end{aligned}$$

The first constraint ensures that the positions in the respective put options have the same magnitude, while the second constraint ensures that the long position is in the ITM option.

The payoff profile of the BearSpread is contrasted in Figure 1 to that of a long position in an OTM currency put option. We test both of these option tactics in the context of an international portfolio management problem. The numerical experiments aim to empirically assess the relative effectiveness of these tactics to control currency risk and enhance portfolio performance.

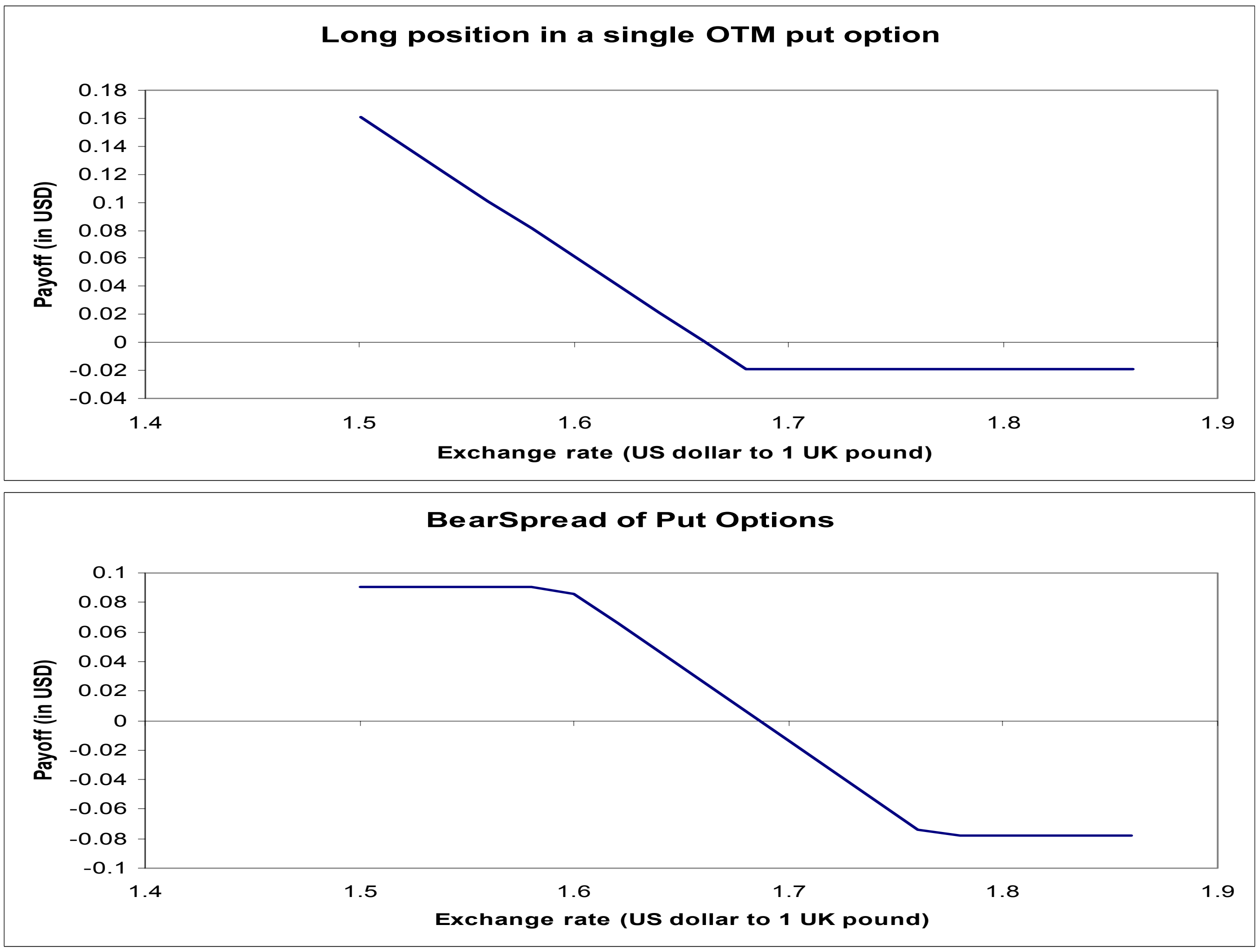


Figure 1: Payoff patterns of currency option strategies.

We have also tested alternative option trading strategies composed of long positions in put and call options with the same expiration date. The straddle, strip, and strap strategies are composed of ATM put and call options, i.e., with the same strike price; they differ only in the proportion of their long positions in the put and the call option. The proportions of positions in the put to the call option for the straddle, strip, and strap strategies are 1:1, 2:1, and 1:2, respectively. All three strategies yield positive payoffs in the event of sufficient movement in the underlying exchange rate, either downside or upside. Their proportional payoffs in the event of currency depreciation, or appreciation, differ depending on the proportion of the put option with respect to the corresponding call option. Although these strategies yield gains in the event of even moderate volatility in exchange rates, they have a higher cost as they are composed of ATM options. The strangle strategy involves long positions (equal in magnitude) in an OTM put and an OTM call option. It provides coverage against larger movements of the underlying exchange rate, in comparison to the previous three strategies, but at a lower cost.

We do not report results with these option trading strategies because in the numerical tests the straddle, strip and strap strategies were dominated both by the forward contracts, as well as by the use of a single protective OTM put option per currency. The performance of the strangle strategy was essentially indistinguishable from that of a single protective put per currency.

4 Empirical Results

As prior research suggests, currency risk is a main aspect of overall risk of international portfolios; controlling currency risk is important to enhance portfolio performance. We examine the effectiveness of alternative tactics to control the currency risk of international diversified portfolios of stock and bond indices. Alternative strategies, using either forward exchange contracts or currency options are evaluated and compared in terms of their performance in empirical tests using market data.

We solved single-stage and two-stage instances of the stochastic programming models described in section 2. The results of the numerical tests enable a comparative assessment of the following:

- Forward exchange contracts versus currency options,
- Alternative tactics with currency options,
- Single-stage vs two-stage stochastic programming models.

First, we compare the performance of a single-stage model with forward contracts against that of a corresponding model that uses currency options as means to mitigate currency risk. Second, we compare the performance of alternative tactics that provide coverage against unfavorable movements in exchange rates by means of currency options. Finally, we consider the performance of two-stage variants of the stochastic programming models. The two-stage models permit rebalancing at an intermediate decision stage, at which currency options held may be exercised, and new option contracts can be purchased. We investigate the incremental improvements in performance of the international portfolios that can be achieved with the two-stage models, over their single-stage counterparts.

As explained in section 2, the models select internationally diversified portfolios of stock and bond indices, and appropriate positions in currency hedging instruments (forward contracts or currency options), in order to minimize the excess downside risk while meeting a desirable target of expected return. Selective hedging is the norm followed in all tests.

Performance comparisons are made with static tests (in terms of risk-return efficient frontiers), as well as with dynamic tests. The dynamic tests involve backtesting experiments over a rolling horizon of 43 months: 04/1998–11/2001. At each month we use the historical data from the preceding ten years to calibrate the scenario generation procedure: we calculate the four marginal moments and correlations of the random variables and use these estimates as the target statistics in the moment-matching scenario generation procedure. We price the respective currency options on the nodes of the scenario tree using the method described in the Appendix. The scenario tree of asset prices and exchange rates, and the option prices, are used as inputs to the portfolio optimization model. Each month we solve one instance of the optimization model (single- or multi- stage) and record the optimal first-stage decisions. The clock is advanced one month and the market values of the random variables are revealed. Based on these we determine the actual return using the composition of the portfolio at hand, the observed market prices for the assets and the exchange rates, and the payoffs from the discretionary exercise of currency options that are held. We update the cash holdings accordingly.

Starting with the new initial portfolio composition we repeat the same procedure for the following month. The ex post realized returns are compounded and analyzed over the entire simulation period. These reflect the portfolio returns that would have been obtained had the recommendations of the respective model been adopted during the simulation period.

We ran backtesting experiments for each investment tactic that is studied, using the CVaR metric to minimize excess shortfall in all cases.

4.1 Efficient Frontiers

We now examine the potential performance of forwards and currency options, in comparison to unhedged portfolios, in terms of the risk-return profiles of their respective portfolios. Thus, we examine the potential effects of currency hedging through alternative means by comparing the efficient frontiers resulting from the alternative decision tactics. Single-stage CVaR models were used for all test reported in this section.

Figure 2 contrasts the efficient frontiers of CVaR-optimized international portfolios on August 2001 using optimal positions in currency options or forward contracts, versus totally unhedged portfolios. We consider two different strategies with currency options: (a) a single protective put option ("at-the-money", "in-the-money" or "out-of-the-money"), (b) a BearSpread strategy of put options.

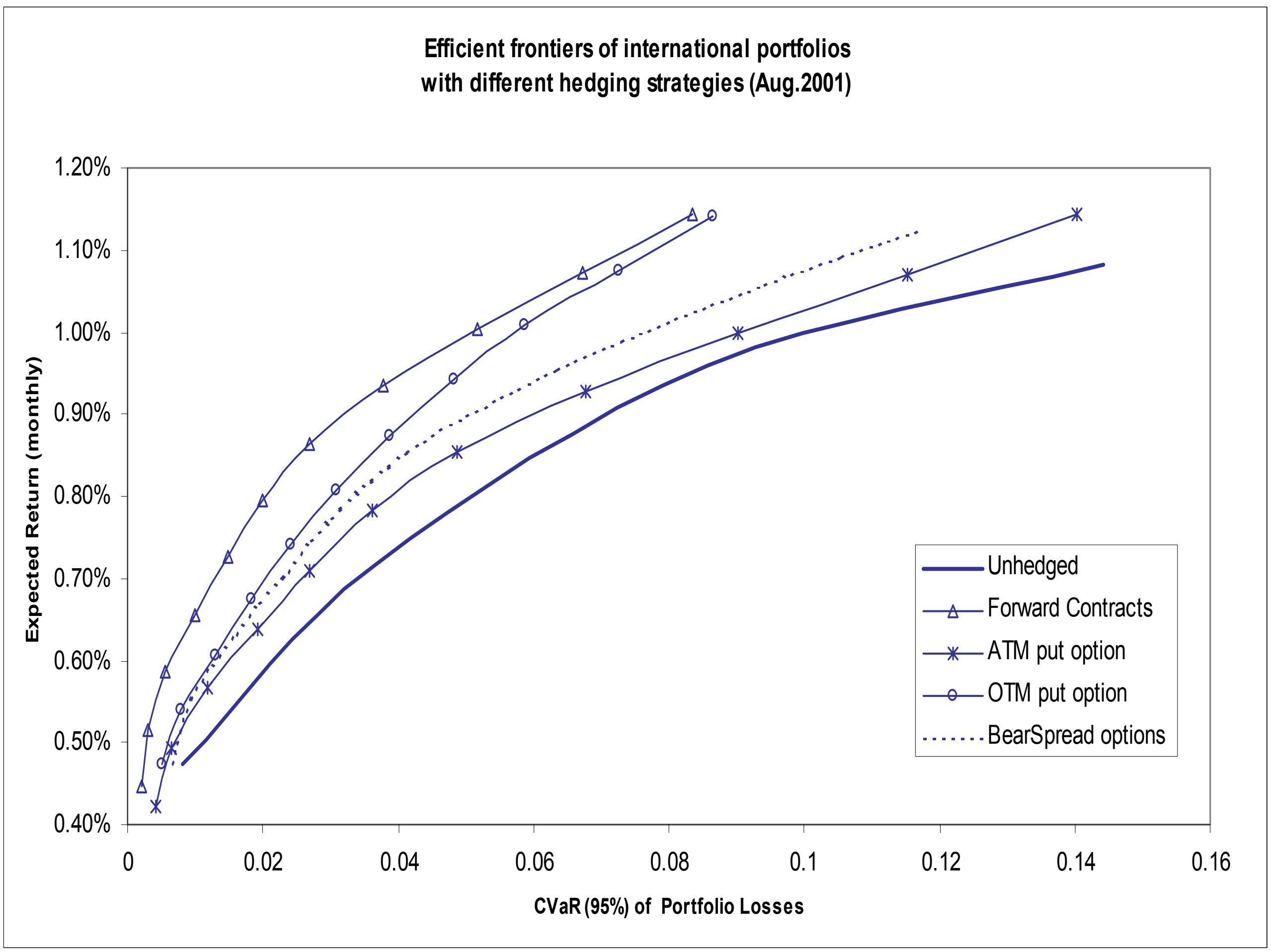


Figure 2: Efficient frontiers of CVaR-optimized international portfolios of stock and bond indices, and currency hedging instruments.

We observe that risk-return efficient frontiers of hedged portfolios (using either forwards or currency options) clearly dominate the efficient frontiers of unhedged portfolios. The efficient frontiers of unhedged portfolios extend into a range of higher risk levels. Clearly, the selectively hedged portfolios are preferable to unhedged portfolios; at any level of expected return, efficient hedged portfolios have a substantially lower level of risk — as measured by the `CVaR` metric — compared to efficient unhedged portfolios. We also observe that currency risk hedging (regardless of the strategy used) yields higher benefits, in terms of higher expected returns compared to the unhedged case, for the medium and high risk portfolios, rather than for the low risk portfolios. The potential gain from risk reduction is increasing for more aggressive targets of expected portfolio returns.

These ex ante results indicate that forward contracts exhibit superior potential as hedging instruments compared to currency options. The results in Figure 2 show that, while the various strategies of currency options produce efficient frontiers that dominate that of the unhedged portfolios, the most dominant efficient frontier is the one produced with the optimal selection of forward contracts. For any value of target expected return, the optimal hedged portfolios with forwards exhibit a lower level of risk than the efficient portfolios of any other strategy. The use of currency options improves the performance of international portfolios compared to unhedged portfolios, but forward contracts exhibit the most dominant performance in the static tests.

Among the trading strategies using currency options, we observe that the efficient frontier with "out-of-the-money" options is the closest to that obtained with forward contracts, especially in the highest levels of target expected return (i.e., most aggressive investment cases). The efficient frontier of the BearSpread strategy follows next, but the differences from the first two are increasing for more aggressive targets of expected portfolio returns. Portfolios with "in-the-money" or "at-the-money" options give almost indistinguishable risk-return efficient frontiers.

4.2 Dynamic Tests: Ex-post Comparative Performance of Portfolios with Currency Options

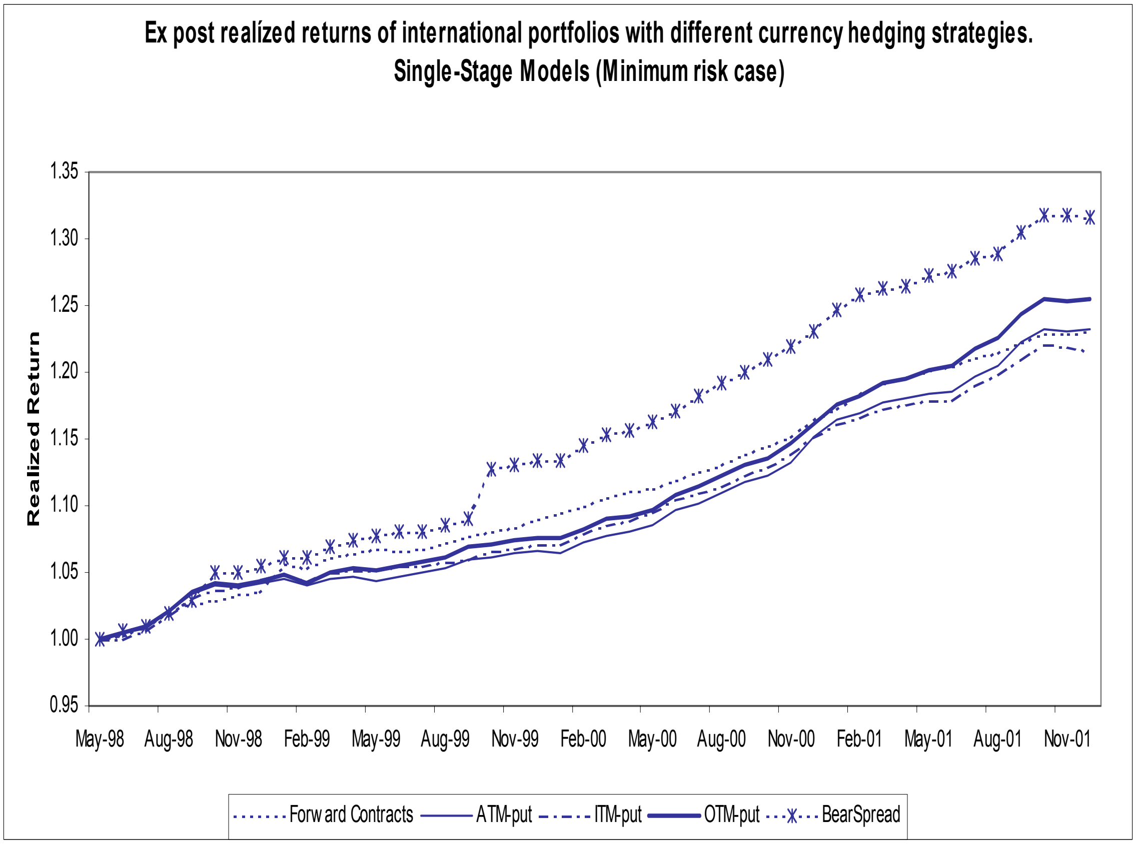
The results of the previous section indicate that, in static tests, forward contracts demonstrated better ex ante performance potential compared to currency options. We additionally ran a number of backtesting experiments on a rolling horizon basis for a more substantive empirical assessment of alternative currency hedging strategies.

Single-Stage Models

First, we compare the ex post realized performance of portfolios with various hedging strategies that are incorporated in single-stage stochastic programming models. The models have a holding period of one month, and consider portfolio restructuring decisions at a single point during the planning horizon. The joint distribution of the random variables (asset returns and exchange rates) during the one-month horizon of the models is represented by sets of 15,000 discrete scenarios.

Figure 3 contrasts the ex post performance of portfolios with optimal forward contracts with that of portfolios using different strategies of put options. The first graph compares performance in the minimum risk case — i.e., when the models simply minimize the `CVaR` risk measure at the end of the planning horizon, without imposing any minimal target on expected portfolio return; for the second graph the target expected return during the one-month planning horizon is $\mu = 1\%$.

We observe that in the minimum risk case of the dynamic tests, forward contracts and the use of a single protective put per currency resulted in very similar performance, regardless of the exercise price of the options (i.e., ITM, ATM or OTM). Forward contracts exhibited the most stable return path throughout the simulation period, indicating their effectiveness in hedging currency risk. In this minimum risk case, the models did not select a large number of currency options, resulting in low hedge ratios.



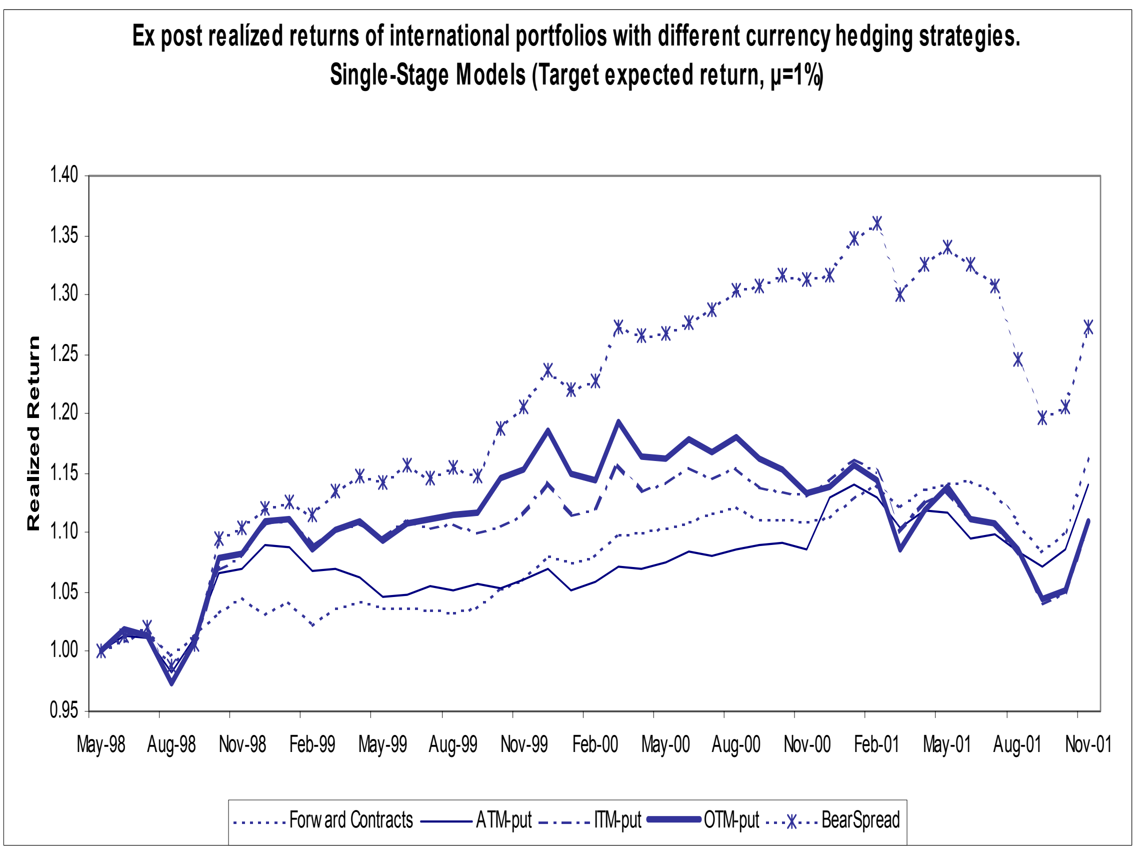


Figure 3: Ex-post realized returns of single-stage `CVaR`-optimized internationally diversified portfolios of stock and bond indices, and currency hedging instruments. Backtesting simulations over period 04/1998-11/2001.

Figure 3 also presents the ex post performance of portfolios that use a combination of currency put options comprising the BearSpread strategy. We observe a noticeable improvement in the performance of portfolios when the BearSpread strategy is employed. In the minimum risk case, the BearSpread strategy yields discernibly higher gains in the period of Sept.-Oct. 1999; this was due to its positions in Japanese bonds during this period, that allowed it to capitalize on the appreciation of the yen at that time. The remaining strategies had very limited positions in Japanese assets during that period.

In the minimum risk case, the optimal portfolios (regardless of the currency hedging strategy) were positioned almost exclusively in short-term government bonds in various currencies throughout the simulation period. These portfolios were able to weather the storm of the September 11, 2001 crisis unscathed, and actually generated profits during that period. That crisis had affected primarily the stock markets for a short period and had no material impact on the international bond markets.

The second graph in Figure 3 shows the performance of more aggressive portfolios — when a target expected return $\mu = 1\%$ is imposed over the models' one-month horizon. The differences in the performance of the currency hedging tactics are more pronounced in this case. Again, we observe that portfolios with forward contracts exhibit the most stable path of realized returns. We also observe that the BearSpread strategy of put options materially outperformed all the other tactics. In this case of an aggressive target on expected return, the models selected portfolios that involved sizable positions in the US stock index for most of the simulation period, and thus did not avoid the effects of the crisis in September 2001.

Multi-Stage Models

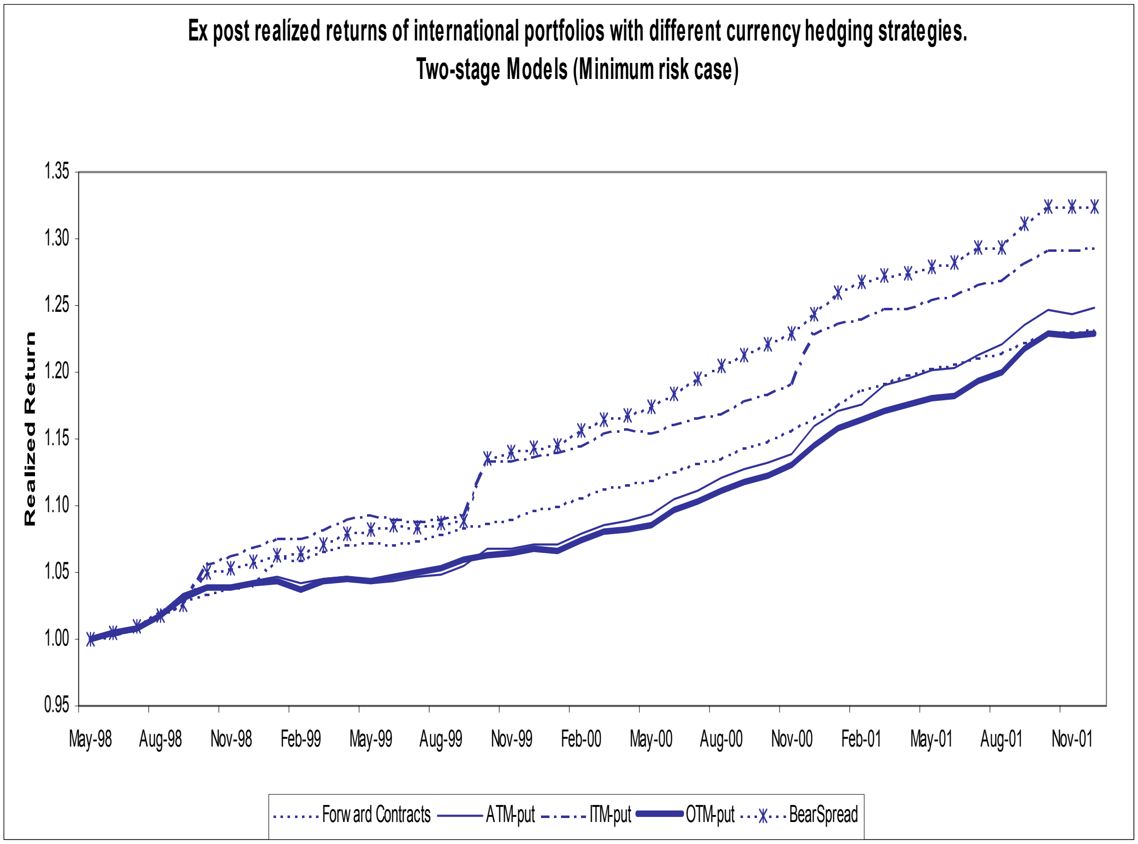
We also tested two-stage instances of the portfolio management models presented in section 2. Figure 4 contrasts the ex post performance of `CVaR`-optimized portfolios with alternative tactics for controlling currency risk. The first graph shows realized returns in the minimum risk case, and the second graph the realized returns for the aggressive investment case corresponding to a target expected return $\mu = 2\%$ during the two-month planning horizon of the models.

The comparative performance of the various currency hedging tactics remains similar, at least qualitatively, to that we had observed with the single-stage models. Again forward contacts yield the most stable path of realized returns and the BearSpread strategy of currency put options results in the best ex post performance. Portfolios with ITM currency options show a noticeable improvement in performance when the two-stage models are used; although these portfolios exhibit higher fluctuations in returns compared to the other strategies, they result in higher cumulative returns. In the multistage setting, the model with ITM options benefits the most from favorable exchange rate movements of the Japanese yen in Sept.-Oct. 1999 and the German mark in Nov.-Dec. 2000; the model maintained sizable positions in these currencies during the respective periods.

Overall, the results indicate that although forward contracts are generally more effective in hedging the currency risk compared to single protective put options per currency, appropriate combinations of put options lead to performance improvements.

Next, we turn to a comparative assessment of single- and two-stage models for international portfolio management in dynamic tests with real market data. The two-stage models use scenario trees composed of 150 joint outcomes of the random variables in the first month, each followed by a further 100 joint outcomes of the random variables in the subsequent month; thus, we have 15,000 scenarios over the two-month planning horizon of the models.

Figure 5 compares the performance of the models with currency options or forward contracts. The first graph presents the results of experiments minimizing the `CVaR` risk measure without any constraint on expected portfolio return. The second graph corresponds to more aggressive portfolios (target expected return $\mu = 1\%$ for single-stage models and $\mu = 2\%$ for two-stage models).



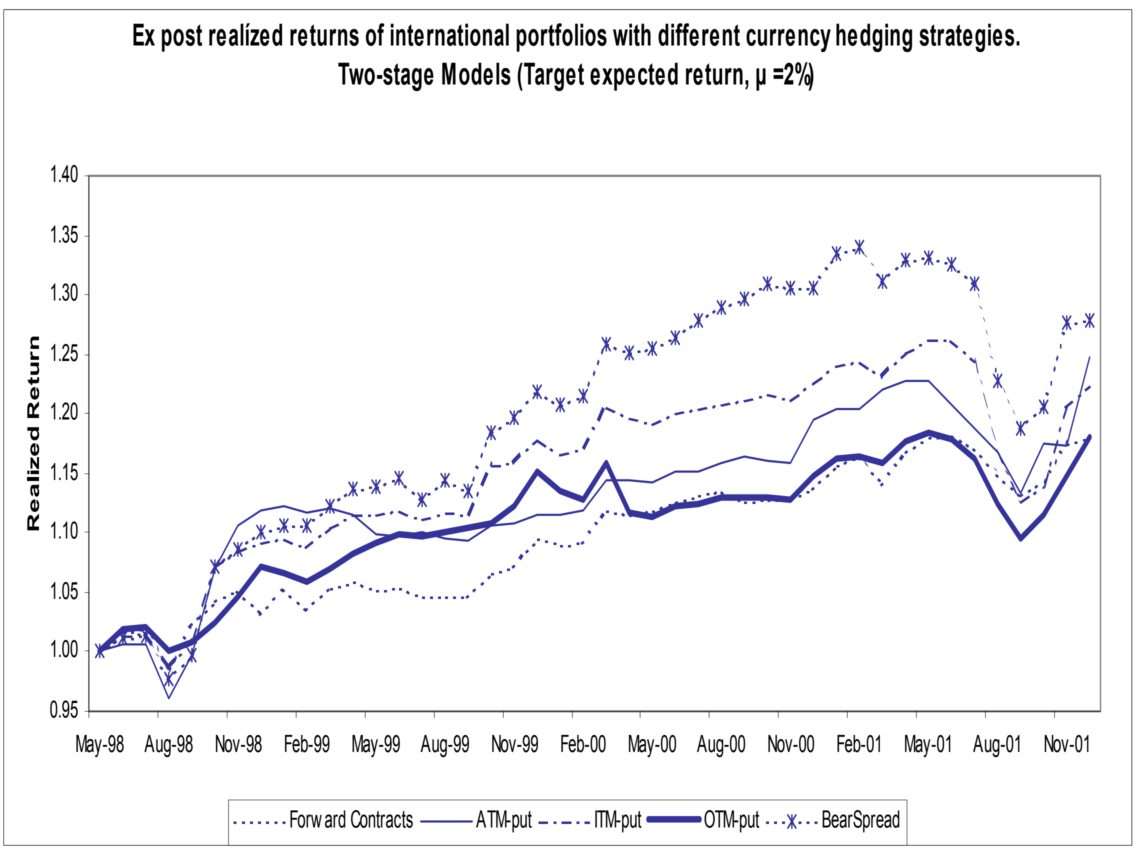
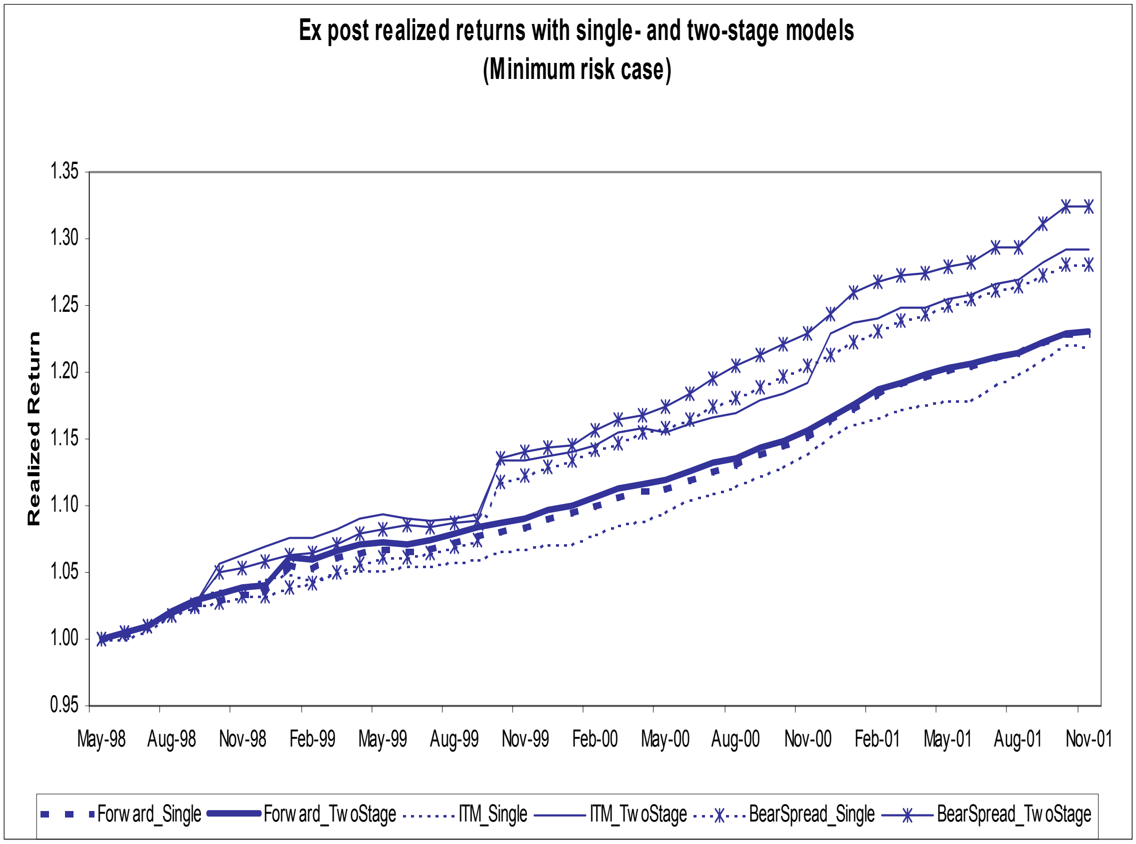


Figure 4: Ex-post realized returns of two-stage `CVaR`-optimized internationally diversified portfolios of stocks and bonds indices, and currency hedging tactics. Backtesting simulations over period 04/1998-11/2001.



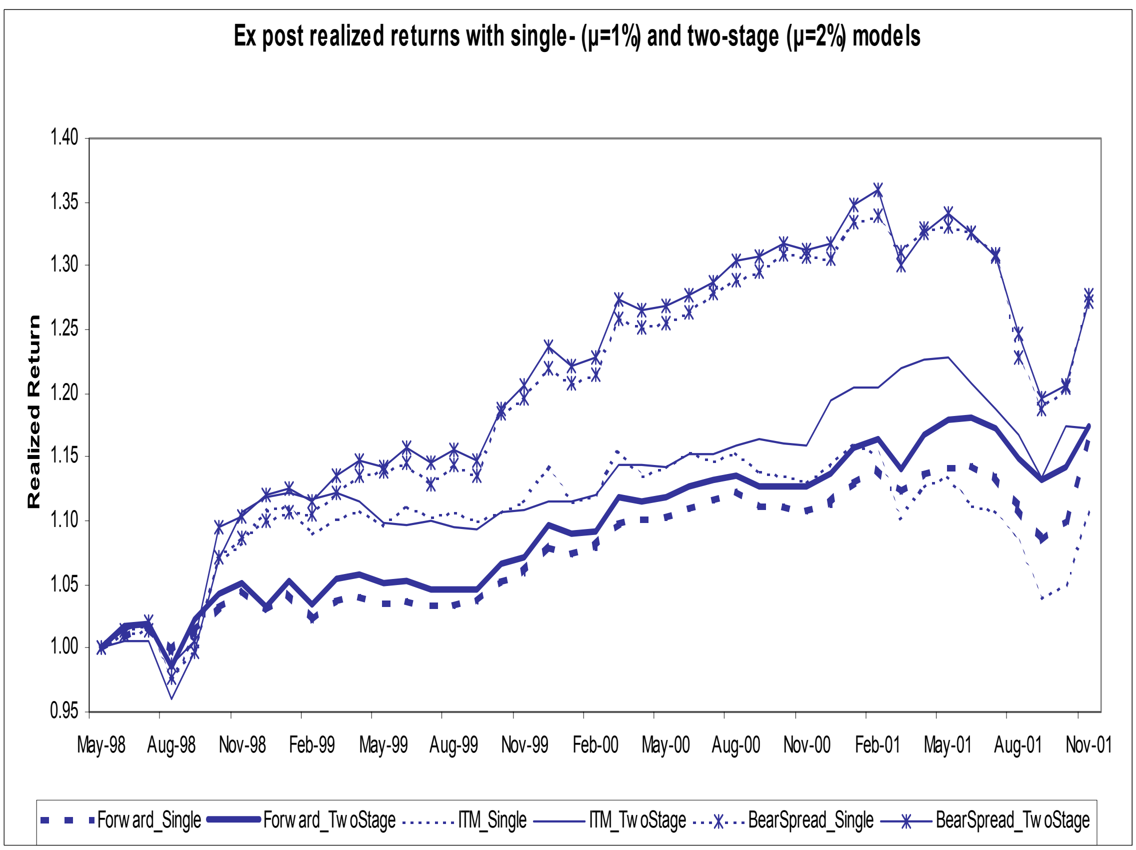


Figure 5: Ex-post realized returns of CVaR-optimized international portfolios of stocks and bond indices, and currency hedging tactics. Comparison of single- and two-stage models.

We observe that in the minimum risk case the models exhibit stable portfolio returns throughout the simulation period, with small losses in only very few periods. The more aggressive cases exhibit larger fluctuations in returns, reflecting riskier portfolios. In Figure 5 we observe that when currency risk is hedged with forward contracts, the two-stage model gives only slightly better results compared to the single-stage model. However, when currency options are used, the performance improvements of the two-stage models compared to the corresponding single-stage models are more evident, particularly for the cases that use ITM put options. Performance improvements with the two-stage model are also observed when the BearSpread strategy is employed. In all tests, and regardless of the trading strategy of options that is used, the two-stage models result in improved performance compared to the corresponding single-stage models.

In comparison to their single-stage counterparts, two-stage models incorporate the following advantages: (a) a longer planning horizon that permits to assess the sustained effects of investment choices, (b) increased information content as it accounts for the evolution of the random variables over the longer planning horizon, (c) the opportunity to account for the effect of portfolio rebalancing at an intermediate point during the planning horizon. The combined effects of these features lead to the selection of more effective portfolios with the two-stage models. The performance improvements are evident in higher and more stable portfolio returns that are achieved with the two-stage models in comparison to their single-stage counterparts.

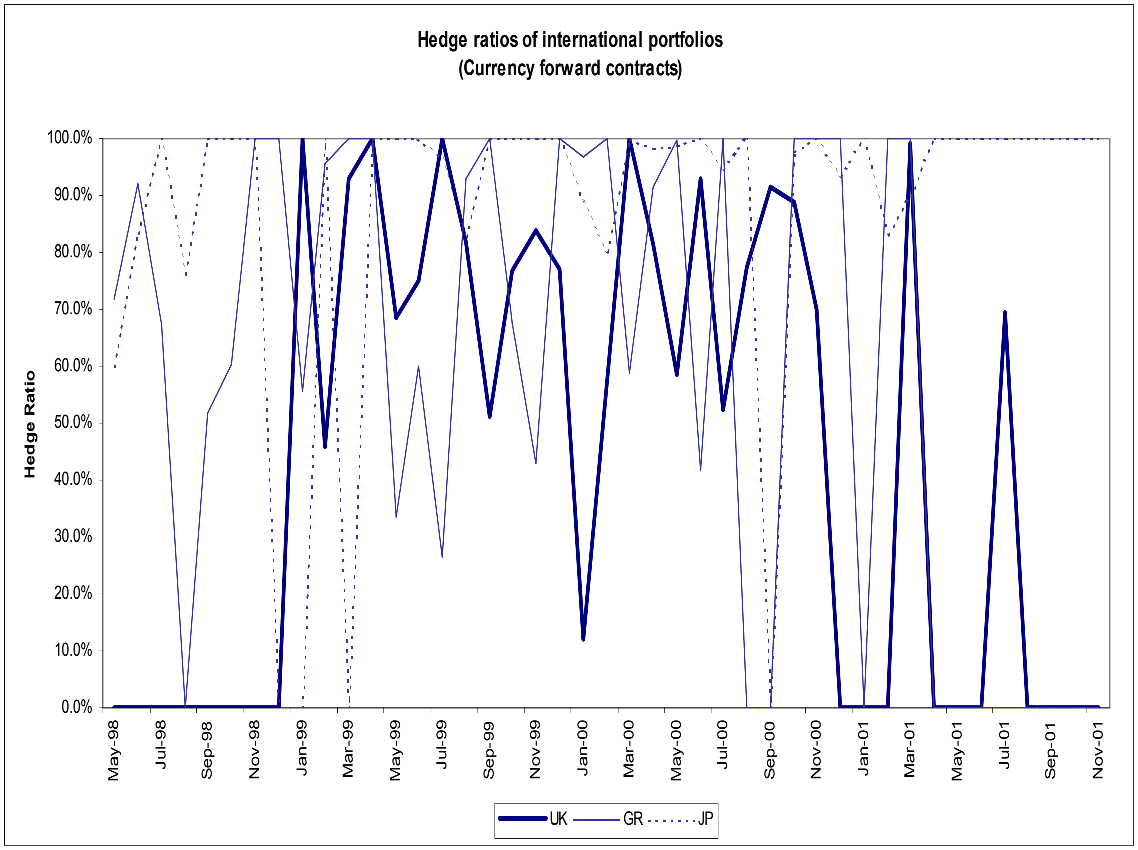
Empirical comparisons of multi-stage stochastic programming models and single-stage models are scantly found in the literature. The results of this study demonstrate the performance improvements that are achievable with the adoption of multi-stage stochastic programs — that account for information and decision dynamics — in comparison to single-stage (myopic) models.

Figure 6 shows the degree of currency hedging in each country (% of foreign investments hedged), when using forward contracts or currency options that form the BearSpread strategy to control currency risk; these results correspond to the first-stage decisions of the two-stage optimization models. The differences are evident. When using currency options, the model consistently chooses to hedge to a high degree — hedge ratios between 90% and 100% — foreign investments in the selected portfolios (see the second graph in Fig. 6). The hedge ratio is zero only when the selected portfolio does not include asset holdings in a particular foreign market. The degree of hedging is evidently different when forward contracts are used to control currency risk (first graph in Fig. 6). In this case the model makes much more use of the selective hedging flexibility; hedge ratios varying both in magnitude as well as across currencies are observed during the simulation period. Moreover, we have observed that, in comparison to the single-stage models, the two-stage models select more diversified portfolios throughout the simulation period and exhibit lower portfolio turnover.

Finally, we compute some measures to compare the overall performance of the models. Specifically, we consider the following measures of the exp-post realized monthly returns over the simulation period: geometric mean, standard deviation, Sharpe ratio and the upside potential and downside risk ratio (UP_{ratio}) proposed by Sortino and van der Meer [24]). This ratio contrasts the upside potential against a specific benchmark with the shortfall risk against the same benchmark. We use the risk-free rate of one-month T-bills as the benchmark. This ratio is computed as follows. Let r_t be the realized return of a portfolio in month $t = 1, \ldots, k$ of the simulation, where $k = 43$ is the number of months in the simulation period $04/1998 - 11/2001$. Let ρ_t be the return of the benchmark (riskless asset) at the same period. Then the UP_{ratio} is

$$UP_{ratio} = \frac{\frac{1}{k}\sum_{t=1}^{k} \max\,[0, r_t - \rho_t]}{[\frac{1}{k}\sum_{t=1}^{k}(\,\max\,[0, r_t - \rho_t]\,)^2]^{1/2}} \tag{4}$$

The numerator is the average excess return compared to the benchmark, reflecting the upside potential. The denominator is a measure of downside risk, as proposed in Sortino et al. [23], and can be thought of as the risk of failing to meet the benchmark.



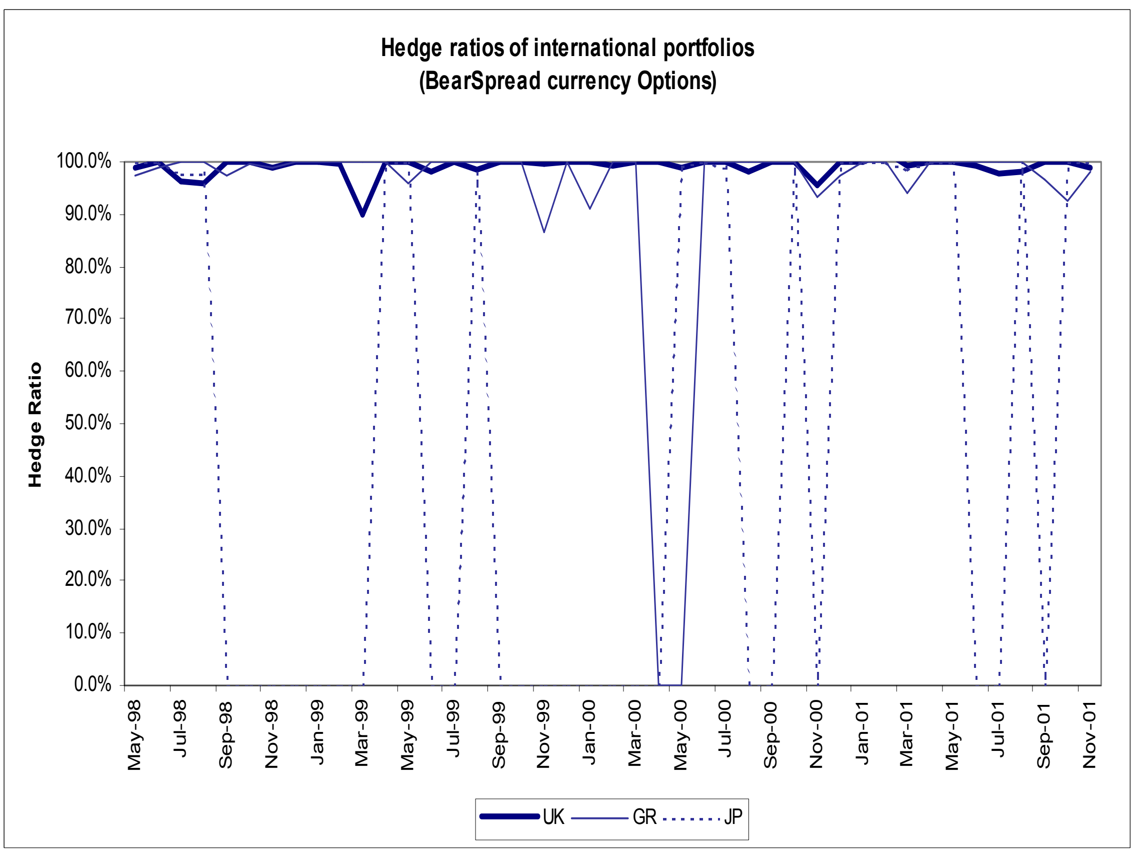


Figure 6: Comparison of optimal hedge ratios in each currency for different decision tactics. The first graph represents the use of forward contracts; the second graph shows the hedge ratios for the BearSpread strategy of currency put options.

Statistic	forwards contracts	ITM put	ATM put	OTM put	BearSpread strategy
Statistics of monthly realized returns, single-stage model ($\mu = 1\%$)					
Geometric Mean	0.0043	0.0035	0.0021	0.0022	0.0057
Stand. Dev.	0.0139	0.0148	0.0193	0.0222	0.0205
Sharpe Ratio	-0.0307	-0.0808	-0.1324	-0.1128	0.0468
UP_{ratio}	0.9500	0.8800	0.6910	0.7060	0.8300
Statistics of monthly realized returns, single-stage model (minimum risk)					
Geometric Mean	0.0057	0.0058	0.0054	0.0063	0.0076
Stand. Dev.	0.0027	0.0045	0.0036	0.0042	0.0061
Sharpe Ratio	0.3796	0.2350	0.1911	0.3641	0.4725
UP_{ratio}	11.1355	5.4853	6.7168	6.7518	25.1671
Statistics of monthly realized returns, two-stage model ($\mu = 2\%$)					
Geometric Mean	0.0046	0.0061	0.0056	0.0045	0.0065
Stand. Dev.	0.0124	0.0163	0.0193	0.0153	0.0208
Sharpe Ratio	-0.0120	0.0862	0.0458	-0.1023	0.0864
UP_{ratio}	0.9690	0.9470	0.7670	0.8380	0.833
Statistics of monthly realized returns, two-stage model (minimum risk)					
Geometric Mean	0.0058	0.0061	0.0071	0.0057	0.0078
Stand. Dev.	0.0025	0.0047	0.0078	0.0042	0.0075
Sharpe Ratio	0.4132	0.3006	0.3083	0.2302	0.4108
UP_{ratio}	12.4510	5.4270	10.1260	4.6344	21.134

Table 2: Statistics of realized monthly returns with alternative models and decision tactics.

The statistics of the realized monthly portfolio returns over the simulation period 04/1988 − −11/2001 are shown in Table 2. First, we observe that in the single-stage models, the optimal portfolios with forwards exhibit better statistics compared to portfolios with protective put options. When forward contracts are used, the Sharpe ratio is higher than that for the use of protective put options (ITM, ATM or OTM), and the standard deviation is lower. Also, the UP_{ratio} is substantially higher, indicating improved upside potential relative to downside risk. In some cases the portfolios with protective currency put options exhibit higher geometric mean than the portfolios with forward contracts; however, the rest of their statistics are worse in all simulation tests.

Next, we note that multi-stage models clearly outperform the corresponding single-stage models. All the statistics for the multi-stage models are improved compared to the results of their single-stage counterparts. These results clearly show that incremental benefits are gained over myopic models — in terms of improved performance (higher returns and lower risk) — with the adoption of two-stage (dynamic) portfolio optimization models.

Finally, we observe that a combination of currency put options that forms the BearSpread strategy outperforms the use of forward contracts. In our simulation experiments this was the case both for the two-stage and the single-stage models, and for both the minimum risk case as well as the more aggressive case (higher expected return targets). Hence, the judicious choice of option trading strategies with suitable payoff patterns can provide the means to improve performance in international portfolio management.

5 Conclusions

This paper investigated alternative strategies for controlling currency risk in international portfolios of financial assets. We carried out extensive numerical tests using market data to empirically assess the effectiveness of alternative means for controlling currency risk in international portfolios.

Empirical results indicate that the optimal choice of forward contracts outperforms the use of a single protective put option per currency. The results of both static, as well as dynamic, tests show that optimal portfolios with forward contacts achieve better performance than portfolios with protective put options. However, combinations of currency put options, like the BearSpread strategy, exhibit performance improvements. Yet, forward contracts consistently produced the more stable returns in all simulation experiments.

Finally, we point out the notable performance improvements of international portfolios that result from the adoption of dynamic (multi-stage) portfolio optimization models instead of the simpler, but more restricted myopic (single-stage) models. The two-stage models tested in this study consistently outperformed their single-stage counterparts in all cases (i.e., regardless of the decision tactics that were tested as means to control currency risk). These results strengthen the argument for the development, implementation and use of more sophisticated dynamic stochastic programming models for portfolio management. These models are more complex, have higher information demands (i.e., complete scenario trees), and are computationally more demanding because of their significantly larger size. However, as the results of this study demonstrate, dynamic models yield superior solutions, that is, more effective portfolios that attain higher returns and lower risk. Such improvements are important in an increasingly competitive financial environment.

The next step of this work is to investigate the use of appropriate instruments and decision tactics, in the context of dynamic stochastic programming models, so as to jointly control multiple risks that are encountered in international portfolio management. For example, we can incorporate in the models options on stocks as means to control market risk in addition to forward exchange contracts or currency options to control currency risk. We expect that such an approach to jointly manage all risk factors in the problem should yield additional benefits.

6 Appendix: Pricing Currency Options

In order to incorporate currency options in the stochastic programming model and maintain internal consistency, the options must be priced in accordance with the discrete distributions of the underlying as represented by the scenario tree. Conventional option pricing methods are not applicable as they rely on specific distributional assumptions on the underlying that are not satisfied in this case. In this study, the scenarios of asset returns and exchange rates are generated with a moment-matching method so as to closely reflect the empirical distributions of the random variables implied by historical market data.

We price the currency options based on empirical distributions of the exchange rates using a valuation procedure developed by Corrado and Su [7], based on an idea of approximating the density of the underlying by a series expansion that was introduced by Jarrow and Rudd [13]. Backus et al. [4] extended this approach to price currency options; we adapt their methodology.

We consider European currency options with maturity equaling a single period of the portfolio optimization model (i.e., one month). To price such a European option at a non-terminal node (state) $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ of the scenario tree, the essential inputs are: the price of the underlying (exchange rate) at the option's issue date (i.e., at node n) and the distribution of the underlying at the maturity date, conditional on the state at the issue date (i.e., conditional on state n). In the context of a scenario tree, this conditional distribution is represented by the discrete outcomes of the underlying associated with the immediate successor nodes (set $\boldsymbol{S_n}$) of node n. Hence, the option is priced on the basis of this discrete conditional distribution.

We use the following notation:

e_t	the underlying spot exchange rate at node n (deterministic for the pricing problem),
$\tilde{e}_{t+1}$	the random value of the underlying exchange rate at the maturity of the option,
r_t^d	the riskless rate in the base currency for the term of the option,
r_t^f	the riskless rate in the foreign currency for the term of the option,
K	the exercise price of the currency option (USD to one unit of the foreign currency).

The log of the underlying's appreciation during the term of the option, starting from node n, is

$$\tilde{x}_{t+1} \;=\; \ln(\tilde{e}_{t+1}) \,-\, \ln(e_t) \;=\; \ln(\frac{\tilde{e}_{t+1}}{e_t}) \,. \tag{5}$$

Then

$$\tilde{e}_{t+1} \;=\; e_t \,\exp(\tilde{x}_{t+1}) \tag{6}$$

and the conditional distribution of $\tilde{e}_{t+1}$ depends on that of $\tilde{x}_{t+1}$.

In the risk-neutral setting, the price at node n of a European call option (in units of the base currency) on the exchange rate $\tilde{e}_{t+1}$ with strike price K is computed as:

$$\begin{aligned} cc_t(e_t, K) \;&=\; \exp\left(-r_t^d\right)\, E_t\left[\,(\tilde{e}_{t+1} - K)^+\,\right] \\ &=\; \exp\left(-r_t^d\right) \int_{\ln(K/e_t)}^{\infty} (\,e_t\,\exp\left(x\right) - K\,)\,f(x)\,dx \end{aligned} \tag{7}$$

where $f(.)$ is the conditional density of $\tilde{x}_{t+1}$. Typically, the conditional density is not analytically available, and this is the case in this study where the uncertainty in exchange rates is represented in terms of an empirical distribution.

Corrado and Su [7] applied a Gram-Charlier series expansion to approximate the empirical distribution of the underlying's log-returns in order to derive the option price. The series expansion approximates the underlying distribution with an alternate (more tractable) distribution, specifically, with the log-normal. Hence, the normal density is augmented with additional terms capturing the effects of skewness and kurtosis in the distribution of the underlying random variable. The resulting truncated series may be viewed as the normal probability density function multiplied by a polynomial that accounts for the effects of departure from normality. The coefficients in the expansion are functions of the moments of the original and the approximating distribution. The underlying theory is described by Johnson, et al. [14] and Kolassa [17].

The series expansion represents an approximate density function for a standardized random variable that differs from the standard normal in having nonzero skewness and kurtosis. If the one period log-change ($\tilde{x}_{t+1}$) in the spot exchange rate e has conditional mean μ and standard deviation σ, the standardized variable is:

$$\tilde{\omega} \;=\; (\,\tilde{x}_{t+1} - \mu\,)/\sigma\,. \tag{8}$$

A truncated Gram-Charlier series expansion defines the approximate density for $\tilde{\omega}$ by

$$f(\tilde{\omega}) \;\approx\; \varphi(\tilde{\omega}) \,-\, \gamma_1 \frac{1}{3!} D^3 \varphi(\tilde{\omega}) \,+\, \gamma_2 \frac{1}{4!} D^4 \varphi(\tilde{\omega}) \tag{9}$$

where

$$\varphi(\tilde{\omega}) \;=\; \frac{1}{\sqrt{2\pi}}\,\exp\left(-\tilde{\omega}^2/2\right) \tag{10}$$

is the standard normal density and D^j denotes the jth derivative of what follows.

Using the Gram-Charlier expansion in (9), Backus et al. [4] solved the pricing equation (7) and obtained the following result for the price of a European currency call option:

$$\begin{aligned} cc_t(e_t, K) &= e_t \exp(-r_t^f) N(d) - K \exp(-r_t^d) N(d-\sigma) \\ &+ e_t \exp(-r_t^d) \varphi(d) \sigma \left[\frac{\gamma_1}{3!}(2\sigma - d) - \frac{\gamma_2}{4!}(1 - d^2 + 3d\sigma - 3\sigma^2) \right] \end{aligned} \tag{11}$$

where

$$d = \frac{\ln(e_t/K) - (r_t^f - r_t^d) + \sigma^2/2}{\sigma}. \tag{12}$$

$\varphi(.)$ is the standard normal density, $N(.)$ is the cumulative distribution of the standard normal, $\gamma_1 = \mu_3/\mu_2^{(3/2)}$ and $\gamma_2 = \mu_4/\mu_2^2$ are the Fisher parameters for skewness and kurtosis, and μ_i is the i^{th} central moment.

To apply this pricing procedure at a non-leaf node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ of the scenario tree, we first calculate the first four moments of the underlying exchange rate over the postulated outcomes on the immediate successor nodes, $\boldsymbol{S_n}$. These estimates of the moments, and the other parameters which are deterministic (current spot exchange rate e_t, exercise price K, interest rates r_t^f and r_t^d), are used as inputs in equation (11) to price the European currency call option. The price of a European currency put option with the same term and strike price K is determined by put-call parity:

$$pc_t(e_t, K) = cc_t(e_t, K) + K \exp(-r_t^d) - e_t. \tag{13}$$

This method of approximating the density of the underlying, has been applied by Abken et al. [1, 2], Brenner and Eom [5], Knight and Satchell [16], Longstaff [19], Madan and Milne [20] and Topaloglou et al. [28].

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