EXPLAINING BANKRUPTCY USING OPTION THEORY

by

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ABSTRACT

This study builds on, and extends, option-pricing theory to explain financial distress based on a sample of 420 distressed U.S. firms for the period 1986-2001. Our results indicate that the primary option variables, such as firm volatility, play an important role in explaining distress up to five years prior to bankruptcy filing. When the model is extended to account for the probability of default on interest and debt repayments due at intermediate times prior to debt maturity (due to voluntary equityholder default or due to cash flow problems), an option-motivated transformation of the cash flow coverage is shown to have incremental explanatory power, while the primary option variables remain statistically significant. Our theory-driven models have significant explanatory power in the years tested.

JEL codes: G33, G3, G0, M4
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I. INTRODUCTION

The economic and social costs of corporate failure to various stakeholders are substantial.[1] The suppliers of capital, stockholders and creditors, as well as management and employees, are severely affected from corporate financial distress. Auditors who fail to provide timely warning signals about troubled firms through the issuance of qualified audit opinions also face the threat of potential lawsuit. Despite the significant costs of business failure and the empirical research efforts of many academics over several decades, there has been little attempt to formulate a theoretical model that helps identify those financial variables that might explain financial distress or business failure more rigorously.[2]

The lack of a theoretical framework concerning the primary explanatory variables that are relevant in distinguishing between distressed and healthy firms has been a serious impediment to the development of a truly scientific approach to bankruptcy prediction. Without a solid economic understanding of the determinants of financial distress it is difficult to ascertain whether a model developed based on data from one set of companies in a particular time period is appropriate for explaining business failure in a different economic or temporal setting. To really understand business failure, rather than just attempt to predict

[1] Costs of default include not only the direct costs of restructuring and/or bankruptcy, but also the loss of value resulting from workers resigning, customers directing business to other potential sellers, trade credit being curtailed, and possible loss of growth options. These "indirect" costs are likely to be several times greater than the direct costs. Andrade and Kaplan (1998) find default costs to be 10%-20%. Evidence also shows that the market value of large firms that filed for bankruptcy recently dropped by more than 50% during the last two years prior to bankruptcy filing (see Bankruptcy.data.com 2004). Also results in Figure 1 in the present study show that the market value of bankrupt firms declined by 62% during the last three years prior to the bankruptcy filing (while the market value of healthy firms increased by more than 15% during the same period).

[2] Among the few noteworthy attempts are the KMV model, Hillegeist et al (2004), Leland (2004), Shumway (2001) and the structural bankruptcy models discussed later in the study (see also Crouhy at al., 2001).

it, it is important to impute economic interpretation to financial distress models based on a sound theoretical foundation.[3]

This study builds on, and extends, a theoretical model using option pricing or contingent claims analysis (CCA) to derive the factors associated with the probability of business default (see the seminal work of Black and Scholes (1973) and Merton (1973, 1974)). The basic intuition behind the standard option-pricing or contingent claims model (e.g., Merton, 1974, 1977) is that the equity of a levered firm can be viewed as a European call option to acquire the value of the firm's assets (V) by paying off (i.e., having as exercise price) the face value of the debt (B) at the debt's maturity (T).[4] From this perspective, a firm will be insolvent if the value of the firm's assets falls below what the firm owes its creditors at debt maturity (i.e., when $V_T < B$). In that event, equityholders will default on the debt (file for bankruptcy) and simply hand over the firm's assets to its creditors and walk away free (protected by their limited liability rights). The probability of default at debt maturity in this case, $Prob(V_T < B) = N(-d2)$, is driven by the five primary option-pricing variables: (the natural logarithm of) the book value of total liabilities (lnB) due at maturity representing the option's exercise price, (the ln of) the current market value of the firm's assets (lnV), the standard deviation of percentage firm value changes (σ), the (average) time to the debt's maturity (T) representing the option's expiration, and the difference between the expected asset return (μ) and the firm's payout yield (interest and dividend payments as proportion of asset value, D).

[3] The bankruptcy prediction literature is substantially based on the original work of Beaver (1966), Altman (1968), and the methodological improvements of Ohlson (1980). For more recent work on bankruptcy and financial distress see Hillegeist et al (2004), Leland (2004), Begley et al. (1996), Barth, Beaver and Landsman (1998), Shumway (2001), Cossin and Pirotte (2000), and Lehavy (2002), among others.

[4] Essentially, from an economic perspective it is the creditors who are considered to be the owners of the firm (rather than the equityholders, who are the legal owners), with equityholders having the right to acquire the firm

The standard option-based model is fairly parsimonious in that it uses only the aforementioned five primary option variables. This model, however, focuses on default at maturity only. We therefore extend it in a compound-option framework to also incorporate the probability of default on interest and debt repayments due at other intermediate times, before the debt maturity. Specifically, we account for the equityholders' option value of defaulting voluntarily or continuing (as a going concern) whenever an interest instalment or debt repayment comes due, as well as for the probability of intermediate involuntary default on an upcoming payment if the firm has insufficient cash or liquid assets. The latter is proxied using an option-motivated transformation of the cash flow coverage (CFC) to capture the probability of intermediate default.

A version of the aforementioned primary option model has been adapted by Vasicek (1984) (see also the Vasicek-Kealhofer model) and has been applied in practice by KMV (recently sold to Moody`s). The KMV model assumes equity is like a (perpetual) option on the firm's asset value which can trigger default when it goes below a given default point. Unlike the original Merton model which focuses exclusively on default on the principal payment (total liabilities) at maturity, both KMV and our model recognize that default may be triggered by nonpayment of any scheduled payment, either interest expense or principal repayment. To account for the probability of intermediate default, KMV adjust downward the default boundary at maturity based on their proprietary data base and experience, to (current liabilities + 0.5 x long-term liabilities). We instead preserve the original (theoretically-motivated) default boundary as being total liabilities (or a duration-weighted average of the firm's total debts) and explicitly capture

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after paying off what they owe.

the possibility of earlier (intermediate) default separately, via a transformation of the cash flow coverage (CFC) variable. KMV focus primarily on a distance to default measure, which they define as (V – default point)/Vσ. Although we also use our version of the distance to default (ln(V/B)/σ) for comparison, we primarily focus instead on (a transformation of) the probability of default at maturity, $-d_2$, and on our CFC-based proxy for the probability of intermediate default due to cash flow inadequacy.[5] Vassalou and Xing (2004) also rely on Merton's (1974, 1977) standard option pricing model, but instead of using the face value of debt at maturity (B) as the default point they adopt the arbitrary default boundary used by KMV, without explicitly accounting for the probability of intermediate default, as we do. They also do not adjust for any dividend-like payout, and their method for estimating the expected asset return in the probability of default often results in negative expected growth rates (which seems inconsistent with asset pricing theory).

Somewhat similar arguments in a related context have been made elsewhere in the literature. The possibility of early default, and differences between insolvency and illiquidity, have been analyzed in various types of capital structure models: static ones (e.g., Leland and Toft, 1996); dynamic ones (e.g., Goldstein, Ju and Leland, 2001); and strategic ones, in which shareholders can renegotiate the debt without formally defaulting (e.g., Mella-Barral and Perraudin, 1997). These "structural" models of optimal capital structure have implications for critical default boundaries (below which shareholders should default

[5] KMV also focus on estimating a default probability over the next (one up to five) year(s). However, they use a proprietary historical default database to derive an empirical distribution relating a given distance to default (e.g., for a firm being d standard deviations away from default) to a default probability. They do so as an indirect way to capture a presumed adjustment in firms' liabilities as they approach default. Crosbie (1999) and Crouhy, Galai and Mark (2001) provide a more detailed description of the KMV approach. We focus more on understanding the factors explaining bankruptcy.

whenever debt service payments are due) and for expected default probabilities. Leland (2004) compares the different implications for critical default boundaries and the relative performance of two structural models: the exogenous default boundary approach, represented by the standard Merton (1974, 1977) model, and the endogenous model where equityholders must decide whether it is worth meeting promised debt payments or defaulting, as in Leland and Toft (1996). Our paper is analogous to the endogenous structural model approach, except that we account for the possibility of early default through the inclusion of the option-motivated cash flow coverage (CFC) transformation based on our compound-option extension. Liquidity is not discussed explicitly in the above papers, e.g., no liquidity variable is used to calibrate the models in Leland (2004) or in the KMV credit risk model that practitioners reference routinely. Our paper differs from Leland (2004) in several ways. First, Leland (2004) is not an empirical study but a simulation-based investigation. He does not analyze real empirical data, but uses the firms' rating by Moodys to compare with the expected probabilities of default (from simulation). This relies on the accuracy of Moody's ratings and the practical estimation of his theoretical variables. Leland's (2004) simulation model relies heavily on the financial structure (leverage) of the firm. Our model accounts not only for leverage (via our ln(V) and ln(B) option variables), but also for the probability of intermediate default. The ability to meet upcoming debt payments through cash and cash equivalents or cash flow from operations is crucial in our approach.[6]

A number of studies have also addressed empirically the relevance of market

[6] Other related studies based on Contingent Claims Analysis include Jones, Mason and Rosenfeld (1985) and Vila and Schary (1995) who attempt to determine the default premium in corporate bond valuation. The former incorporate several features of standard bond covenants into a complex CCA model and compare their results to market bond values. The latter paper discusses valuation of risky debt based on both involuntary as well as

versus accounting based variables in explaining bankruptcy. Shumway (2001) uses a hazard model approach based on accounting variables identified previously by Altman (1968) and Zmijewski (1984) and finds that half of these variables are statistically unrelated to bankruptcy probability. Shumway (2001), as well as Chava and Jarrow (2001) and Hillegeist et al (2004), find that adding market variables to the previously used accounting variables helps improve forecasting accuracy. The present paper takes a different approach, showing that adding a transformation of the cash flow coverage (CFC) proxying for the probability of intermediate default (derived from our compound option extension of the primary option model) to the basic option-based financial variables that drive the probability of terminal default, brings about incremental explanatory power.

Our paper extends prior research by examining how the option model can be extended to account for the probability of default on interest and debt repayments due at intermediate times prior to debt maturity. To the best of our knowledge, prior studies have not examined the above issue. Hillegeist et al. (2004) use standard option pricing theory (OPT) in a discrete hazard model and examine the predictive ability of the Altman and Ohlson accounting-based variables. They find that traditional accounting-based measures of bankruptcy risk do not add incremental information beyond the standard option variable. They do not examine the probability of default at an intermediate stage. Their result is more a consequence of the poor performance of the accounting-based variables, rather than of their superior (hazard) model. The hazard model used in their study has the advantage of using more updated data observations and can circumvent

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voluntary bankruptcy (equity's put option to default).

some of the empirical problems associated with the single-period logit approach based on accounting variables (e.g., sample selection bias that arises from using only one non-randomly selected observation per firm, or a failure to capture time-varying changes in the underlying or baseline risk of bankruptcy). One limitation of the hazard model might be that distressed firms may not survive as many years as healthy firms and therefore they may have fewer firm-year observations. Another limitation of their study is that they focus on comparing the standard option-based model with the traditional default risk models of Altman (1968) and Ohlson (1980), which are based on data prior to the 1970's that may not be applicable for recent period data. In fact, Hillegeist et al. (2004) find significant only two of the accounting variables (profitability and leverage), and that five out of the eight significant variables in the Ohlson model have different (wrong) signs compared to their original counterparts. Specifically, they find that the probability of default is higher for larger firms, for more profitable firms, for more cash-flow rich firms, and for firms with higher working capital. These results are contrary to intuition and with what one would expect in the real world. Hillegeist et al (2004) modestly conclude that "the changed signs are not intuitive." Their evidence shows most clearly the inability of these old models based on ad hoc accounting variables derived purely from fit with (old) historical data to predict bankruptcy in more recent periods. Thus, despite their ability to handle more data observations through the hazard model, the content of their accounting variables and the predictive ability of their model is limited. Hillegeist et al (2004) do not provide time series prediction rates in the years prior to the default year. Those would probably be unreliable due to the inappropriateness of the historically-derived ad hoc accounting variables from the old models relied on. By contrast, we develop an extended compound-option bankruptcy model that incorporates the probability of intermediate default in a relatively simple and practical way. Our option-based model requires only current future-oriented data (on V, B, σ, μ-D, T, and CFC) and can be used in any year prior to bankruptcy to provide a time series analysis of firm performance which allows tracking the progress of the firm over time.[7] Thus the (historical) data-processing advantage of the backward-looking hazard model over the logit approach with regard to accounting variables is not present when using our forward-looking option model with its limited (current) data requirements.

In this study, we empirically test our option-based models using a sample of 420 financially distressed firms that filed for bankruptcy and corresponding control (healthy) US firms for the 15-year period between 1986-2001. Consistent with our predictions from option theory, our results indicate that the book value of total liabilities (lnB), the market value of the firm's assets (lnV), and the standard deviation of firm value changes (σ) that drive the probability of default at debt maturity, play an important role in explaining financial distress. Moreover, when we extend our option framework to take into consideration the probability of default on intermediate interest and debt payments due before the debt's maturity, the transformation of the cash flow coverage is found to also be a significant factor (beyond the above primary option variables) in explaining financial distress. Our option theory-driven models have significant explanatory power up to 4 years before bankruptcy filing. A main contribution of our paper has thus been to develop a compound-option cash-flow based extension and validate the extended option-based model variables in explaining financial

[7] Hillegeist et al (2004) used 1 year as the time to maturity (T) of the default option whereas we use the average debt maturity. They also used as the firm payout rate (D) only the stock dividends, whereas we also include the amount of interest paid to the debtholders. These assumptions may lead to different estimations of the probability of bankruptcy. Other option-related studies include Vasicek (1984), Cheung

distress.

The rest of the paper is organized as follows. The theoretical framework used to motivate the selection of the underlying default determinants based on option pricing theory is presented next. Section III describes our data set and methodology. The empirical findings are discussed in section IV. The last section concludes.

II. AN OPTION-PRICING FRAMEWORK FOR BUSINESS DEFAULT

This section discusses: a) the standard option-pricing (or CCA) model of business default, and b) the extended option-pricing framework that additionally accounts for the probability of default at intermediate times.

A. The Standard Option-pricing (CCA) Model of Business Default

Since the seminal work of Black and Scholes (1973) and Merton (1973, 1974, 1977) option valuation or Contingent Claims Analysis (CCA) has been applied to the valuation of various corporate securities seen as packages of claims or options on the total value of the firm's assets, V; the various corporate liabilities, such as the stockholders' equity, risky debt, warrants, and convertible bonds, could now be valued as claims contingent on V as the underlying asset.

The total market value of the firm's assets at time t, V_t, is assumed to follow a standard diffusion process of the form:

$$dV_t/V_t = (\mu - D)\,dt + \sigma\,dz \qquad (1)$$

where μ denotes the expected total rate of return on firm asset value (subsequently 'expected asset return') reflecting the business prospects (equal to the risk-free rate, r, plus

(1991), Kealhofer et al (1998), and Core and Schrand (1999).

an asset risk premium), D is the total payout rate by the firm to all its claimants (including dividends to equityholders and interest payments to debtholders) expressed as a % of V, σ is the business volatility or standard deviation of the firm asset returns (% asset value changes), and dz is an increment of a standard Wiener process.

Merton (1974, 1977) has shown that any claim whose value is contingent on a traded asset (portfolio) with value V, having a payout D and time to maturity τ ($\equiv$ T – t) must satisfy a certain fundamental partial differential equation (p.d.e.). Each individual contingent claim (corporate liability) is uniquely represented by specifying its particular terminal and boundary conditions, along with the payout it receives. Consider the case of a simple firm with only stockholders' equity of market value E and a single issue of coupon-paying debt (of market value MD). The promised face value of the bond, B, is due at maturity T, τ ($\equiv$ T - t) years from now. On the debt's maturity (t = T), $\tau = 0$, equity will be worth either (V - B) or zero, whichever is best for the equityholders, i.e., E(V, 0) = Max(V - B, 0).[8] The equity of such a levered firm is analogous to a European call option on the value of the firm's assets, V, with exercise price equal to the bond's promised payment, B, and time to expiration equal to the debt's maturity (τ).

The market value of stockholders' equity is given by the Black-Scholes solution for a European call option (on firm value V, after a transformation of variables) adjusted for a constant dividend-like payout D (see Merton, 1973):[9]

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[8] On the debt's maturity (T), if the value of the firm exceeds the face value of the debt, $V_T > B$, the bondholders will receive the full promised payment, B, and the equityholders will receive any residual claims, V - B. If $V_T < B$, the stockholders will find it preferable to exercise their limited liability rights, i.e., default on the promised payment and instead surrender the firm's assets V to its bondholders and receive nothing.

[9] If the debt promises regular (periodic) coupon interest payments (that are paid out or are lost for equityholders while they maintain alive their option to acquire the firm but are (re)captured once they exercise their option, analogous to "dividends"), equity in the presence of coupon-paying debt becomes analogous to a European call option on a *dividend-paying* asset. Generally, if the firm makes any form of "dividend" payments (e.g., coupon interest payments on the debt), its value will be reduced after each "dividend" payment so that it may become optimal to exercise the equityholders' call option early in order to capture the "dividend". "Dividends" generally affect the drift of the underlying stochastic process of firm value as well as the probability of default. Shortcuts are often used in adjusting for "dividend" effects that basically attempt to sidestep the complication arising from the possibility of early exercise. If the firm pays a continuous constant "dividend payout" D (which is lost for the equity option holder), then the Black-Scholes solution for the value of a call option can still be used,

$$E(V, \tau) = V\, e^{-D\tau}\, N(d_1) - B\, e^{-r\tau}\, N(d_2) \qquad (2)$$

where $d_2 = \{\ln(V/B) + [(r - D) - \frac{1}{2}\sigma^2]\tau\} / \sigma\sqrt{\tau}$; $d_1 = d_2 + \sigma\sqrt{\tau}$

$N(d)$ = (univariate) cumulative standard normal distribution function (from $-\infty$ to d)

B = face value (principal) of the debt

V = value of firm's assets

σ = standard deviation of firm value changes (returns in V)

τ ($\equiv$ T - t) = time to debt's maturity

r = risk-free interest rate

The first term in eq. (2) above is the discounted expected value of the firm if it is solvent (assuming a constant dividend payout D). $N(d_2)$ in the second term of eq. (2) is the (risk-neutral) probability the firm will be solvent at maturity, i.e., $\text{Prob}(V_T > B)$, in which case it will pay off the debt principal B (with a present value cost of $B\, e^{-r\tau}$). Analogously, 1 - $N(d_2)$ or $N(-d_2)$ in eq. (2) represents the risk-neutral probability of default at the debt's maturity.

It is worth noting that while the value of the option depends on the risk-neutral probability of default (where d2 depends on the value of the risk-free rate, r), the actual probability of default at the debt's maturity depends on the future value of the firm's assets and hence on the expected asset return, μ, instead. It is obtained simply by substituting the expected return on assets, μ, for the risk-free rate, r, in the above equation for –d2, i.e.,

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provided V is replaced by $V\, e^{-D\tau}$, as in eq. (2) This represents the current value of the asset minus the present value of the (stochastic) future "dividends" over the life of the option (debt maturity). That is, payment of a continuous "dividend yield" at the rate D reduces (or "drags down") the growth rate of firm value V at the constant rate D. Since the total return in a risk-neutral world must be r (including the "dividend yield" D), the expected growth rate in V must be (r - D).

Prob. default (on principal B at maturity T) = Prob($V_T < B$) = 1 - $N(d_2) = N(-d_2)$

where $-d_2(\mu) = -\{\ln(V/B) + [(\mu - D) - ½\sigma^2]\tau\}/ \sigma \sqrt{\tau}$. (3)

By analogy to KMV, we also use as an alternative measure of the risk to default our version of the distance to default (d2d), defined here as the distance of firm value (V) from the debt amount due (B), measured in units of standard deviation: $d2d = [\ln(V) - \ln(B)]/ \sigma$. It measures how many standard deviations it takes for firm value to move down before it can trigger bankruptcy filing.

The above standard option model has some interesting implications for the determinants of corporate distress. The probability of business default at the debt's maturity depends on the five primary option variables influencing $-d_2(\mu)$ in eq. (3). Namely, the actual probability of default, Prob($V_T < B$), measured by $N(-d_2)$ or simply by $-d_2(\mu)$, is higher when:

(1) the (natural logarithm of) current firm value V (lnV) is low;

(2) the (natural logarithm of the) face value of the debt B due at maturity (lnB) is high – alternatively, when ln(V/B) is low (or the firm's leverage B/V is high);

(3) the volatility of the firm's asset return σ is high;

(4) the (average) maturity of the debt τ is longer;

(5) the difference between the expected asset return, μ, and the firm's payout D (i.e., μ - D) is lower.

We empirically estimate the unobserved variables firm value (V) and firm volatility (σ) from market data based on the following two relations:

$$E = V e^{-D\tau} N(d_1) - B e^{-r\tau} N(d_2) \qquad (4)$$

$$\sigma_E = [N(d_1) e^{-D\tau} (V/E)] \sigma$$

with d_1, d_2 as defined above. The first equation is the Black-Scholes option pricing formula for equity E adjusted for a dividend payout on firm value D (eq. 2). The second is the relation between equity return volatility (σ_E) and firm (asset) return volatility (σ) connected via the equity/option elasticity. Using the identity that the total value of the firm equals the market value of equity plus the market value of debt (V = E + MD), the above can be rearranged into the following set of simultaneous equations for the market value of debt (MD) and firm volatility σ, which are solved through an iterative process (using MATLAB):[10]

$$MD = \frac{E\left(1 - e^{-D\tau} N(d_1)\right)}{e^{-D\tau} N(d_1)} + \frac{Be^{-r\tau} N(d_2)}{e^{-D\tau} N(d_1)}$$

$$\sigma = \frac{\sigma_E}{N(d_1) e^{-D\tau}} \left(\frac{E}{E + MD} \right) \qquad (4')$$

B. An Option-pricing Extension for Intermediate Default

The aforementioned simplified option model actually provides a lower-bound approximation for the true equity option value since it does not account for the option to default on intermediate coupon interest payments and debt repayments before maturity. More precisely, equity in the presence of coupon (and sinking fund) paying debt is more like a compound option where each interest and debt (re)payment made by the stockholders represents the exercise price that must be paid to continue with the next stage (maintaining an option toward eventual ownership of the firm), i.e., it is the exercise price that must be incurred when interest and debt repayments come due to acquire an option on firm value

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[10] The initial conditions used for the market value of debt (MD) and firm volatility (σ) were the respective average historical values of B and σ for each firm. Equity return volatility (σ_E) was estimated from monthly data over the previous 5-year period. Each market value of equity E corresponds to a market value of debt (MD) and a firm value estimate (V). Firm volatility (σ) in a new iteration is obtained from % changes in these V estimates. The iterative process is repeated until the combined error for firm value and firm volatility falls below 1.0e^{-5} % (over the period) or when the revised value of V and σ (V′ or σ') exceed twice their historical values, i.e., when the % difference between successive values of V or σ exceeds the limits:
$\frac{V' - V_{historical}}{V_{historical}} > 1$, $\frac{\sigma' - \sigma_{historical}}{\sigma_{historical}} > 1$

V.[11] In such an extended option formulation, equityholders may default on the debt not only at the debt's maturity T when $V_T < B$ (assuming that the firm has not previously defaulted on its interest and debt repayment I), related to the default probability $N(-d_2 (B, \tau))$ given by eq. (3), but now they may also default at an intermediate time T', just before the coupon interest and debt repayment I come due, if the value of the firm at that time falls below its cutoff option value as a going concern V* (where $E(V^*, \tau') = I$, with $E(V^*, \tau')$ as given from the earlier option solution in eq. (2)). This latter (marginal) probability of equityholders *voluntarily* defaulting on the interest and debt payment I at an intermediate time T' (with payout D) is given by

$$\text{Prob. voluntary default (on interest and debt repayment I at T')} = \text{Prob}(V_{T'} < V^*)$$

$$= \text{Prob}(E(V^*, \tau') < I) = 1 - N(d^*_2) = N(-d^*_2) \quad (6)$$

where $d^*_2 = d^*_2 (V^*, \tau') = \{\ln(V/V^*) + [(\mu - D) - ½\sigma^2]\tau'\}/ \sigma \sqrt{\tau'}$.

Of course, the higher the interest and debt repayment burden (I) from more leverage, the higher this probability of voluntary default at the intermediate time T'. The probability that equityholders exercise their call option to acquire the firm assets by paying off the principal B at the maturity of the debt T, given that they previously decide to keep alive their option to

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[11] For example, with just one interest and debt repayment, I, due at intermediate time T' (< T) giving equityholders the option to continue (provided they pay I) with an option to acquire the firm by debt maturity T, the value of equity (with no dividend payout) can be seen as a call on a call (or a compound call) option (e.g., see Geske (1979)) given by:

$$E(V, \tau) = V\, N(d^*_1 , d_1 ; \rho) - B\, e^{-r\tau'} N(d^*_2 , d_2 ; \rho) - I\, e^{-r\tau'}\, N(d^*_2) \quad (5)$$

where $d^*_2 \equiv d^*_2 (V^*, \tau') = \{\ln(V/V^*) + (r - ½\sigma^2)\tau'\}/ \sigma \sqrt{\tau'}$; $d^*_1 = d^*_2 + \sigma \sqrt{\tau'}$

$d_2 \equiv d_2 (B, \tau) = \{\ln(V/B) + (r - ½\sigma^2)\tau\}/ \sigma \sqrt{\tau}$; $d_1 = d_2 + \sigma \sqrt{\tau}$; $\tau \equiv T - t$; $\tau' \equiv T' - t$

$N(d)$ = (univariate) cumulative standard normal distribution function (from $-\infty$ to d)

$N(a, b; \rho)$ = bivariate cumulative standard normal distribution function with upper integral limits a and b and correlation coefficient ρ, where $\rho = \sqrt{\tau'/\tau}$

V* is the cut-off firm value, V, at the intermediate time T' when interest and debt repayments I come due, above which equity's call option (to pay the interest and debt repayment in order to continue with its option toward acquiring the firm) should be exercised, obtained from solving $E(V^*, \tau') - I = 0$, where $E(V^*, \tau')$ is obtained from the solution to earlier eq. (2). The Black-Scholes formula is actually a special case of above eq. (5), as can be seen by setting I = 0 (no intermediate interest payments) or $T = \infty$.

continue (by not defaulting on the interest and debt repayment I at intermediate time T') is given by the bivariate cumulative normal distribution $N(-d^*_2, -d_2; \rho)$.[12]

The above illustrates how the presence of intermediate debt payments opens up the possibility that equityholders may choose to default just before the next payment I comes due if their (option) value from continuing is not sufficient to cover the next interest and debt repayment (voluntary liquidation). However, default may additionally be triggered (this time initiated by creditors) if the firm (even when it is profitable and equity is valuable, i.e., ($E(V^*, \tau') > I$ or $V_T > B$) does not have sufficient cash flows or other liquid assets to make the next interest and debt repayment when due (involuntary liquidation).[13] Each period the firm may face two main interim payments: one payment for interest and another for debt due for repayment in the coming year. Cash flows must be found to cover both of these payments. Some cash and equivalents may already be in place for the coming year's payments, with the net balance to be paid out of cash flow from operations (CFO) and other sources. A variation of the interest coverage ratio that is close to this concept is the cash flow coverage (CFC) defined as:

$$CFC = \frac{\textit{Cash flow from operations}}{(\textit{Interest} + \textit{Debt repayment}) - \textit{Cash and cash equivalents}}.$$[14]

[12] The correlation among the two different events of default – on the principal B at maturity T and on interest I at T' – is related to the timing of the intermediate coupon payment T' relative to the principal (face value) repayment at the end, T, as captured by $\rho = \sqrt{\tau'/\tau}$. Note that when the intermediate payments occur rather early (τ'/τ and ρ are small) the two different probabilities that equityholders will default are relatively independent; but otherwise (if τ'/τ and ρ are large) they may interact so that the joint bivariate probability of default, $N(-d^*_2, -d_2; \rho)$, may approximately just equal the highest of the two marginal default probabilities, $N(-d_2)$ and $N(-d^*_2)$, given in eqs. (3) and (6).

[13] There may be various possibilities to support debt service in case of cash shortfall given sufficient time (e.g., asset sales, junior debt issue, equity issue or dividend reduction). These, however, are generally costly for the shareholders. In practice, even if cash shortages are predictable weeks or months in advance, when a firm is in financial distress equityholders may be reluctant to contribute additional capital as they may lose it all in case the situation deteriorates and the firm goes bankrupt (Myers underinvestment). It would also be difficult to borrow under these circumstances. Bond indenture covenants typically restrict sales or liquidation of operating assets to meet debt service payments (see also Leland, 2004). Such frictions may affect the relative magnitude but not the expected sign of the cash flow coverage variable in eqs. (7) and (8) that follow.

[14] Some firms may have negative cash flow from operations (numerator) and/or negative net balance (denominator). The CFC ratio is not well defined for negative values. For both positive numerator and

If the firm generates a constant proportion of its value as cash flow from operations, i.e., CFO = cV_T, involuntary early default would be triggered at time T' (< T) if ($c\,V_{T'}$ + cash) < I or $V_{T'}$ < (I – cash) /c, where 'cash' above represents cash and cash equivalents and I is the sum of interest plus debt repayments due. The (marginal) cumulative intermediate probability of default if the firm does not have sufficient cash to make the interest and debt payments due at T' is (by analogy to eq. 6) given by[15]

Prob. involuntary default (on interest and debt repayments, I, due to insufficient cash at T')

$$= \text{Prob}(c\,V_{T'} < \text{I - cash}) = \text{Prob}(\text{CFC} < 1) = N(-d''_2),$$

where $-d''_2 = -\{\ln(cV / (I - \text{cash})) + [(\mu - D) - \frac{1}{2}\sigma^2]T'\}/ \sigma \sqrt{T'}.$ (7)

$$= -\{\ln(\text{CFC}) + [(\mu - D) - \tfrac{1}{2}\sigma^2]T'\}/ \sigma \sqrt{T'}. \quad (7')$$

The above probability of intermediate (involuntary) default on interest and debt repayments due to cash flow inadequacy is higher the lower d''_2 and specifically (in addition to the other five primary option-pricing variables already discussed) the lower c V / (I – cash), i.e., the lower the proportion of the firm's cash flows from operations c, the higher the interest and debt repayments I, and the lower the firm's cash and equivalents.

Note again that the early or delayed timing of interest payments (T') relative to debt maturity (T) also affects how the different events of default are correlated (ρ), although this

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denominator, the CFC variable is simply calculated according to the above formula (first case). When the denominator is negative (and the numerator positive) the company has more than sufficient cash to cover its upcoming debt payments and CFC receives a high value. In contrast, a negative numerator with a positive denominator implies a distressed firm with insufficient cash to cover its debts and CFC receives a low value. Finally, both a negative numerator and denominator imply the firm has sufficient cash and equivalents despite low CFO, resulting in an intermediate value. The above formula is a special case of the following:

$$\frac{\textit{Cash flow from operations}}{\textit{Interest} + (\textit{Debt repayment} + \textit{Preferred dividends})/(1 - \textit{tax rate}) - \textit{Cash and cash equivalents}}$$

Here, preferred dividends are left out as they are optional and can be deferred without triggering bankruptcy. Marginal tax rates may also be reasonably assumed to be negligible for distressed firms. The results are qualitatively similar. Moreover, we also used funds flow from operations (EBIT + non cash expenses) in the numerator of the aforementioned cash flow coverage formulas. Results were again qualitatively similar (see Palepu et al., 2003).

[15] If we assume, as in Leland and Toft (1996), that debt is continuously being offered (with maturity T) under a stationary capital structure, so that debt maturities are uniformly distributed between 0 and T, the average maturity of the debt structure will be T' = T/2 (see also Leland 2004).

is harder to operationalize in empirical testing compared to the other bankruptcy financial distress determinants. This latter effect makes the impact of the debt's maturity (which in practice is a weighted average of the maturities of the various kinds of debt issued by the firm) less clear. Furthermore, firms in financial distress may have difficulty issuing long-term public debt and may opt for the greater flexibility of short-term bank financing, which may effectively reduce the average maturity (duration) of their debt.[16] Thus, the relationship between average debt maturity (T) and the risk of default may be endogenous (circular). The net effect of μ - D is expected to be negative if the influence of the expected asset return is greater than that of the firm payout rate.

The above extended compound-option framework motivates the use of (a transformation of) the cash flow coverage (CFC) variable in assessing the probability of default at an intermediary stage before debt maturity, in addition to the five primary option variables identified in equation (3) earlier.

Our option-motivated determinants of the probability of business default (with the predicted signs) can be summarized as follows:

$$\text{Prob. default} = f\,(\underset{-}{\ln V},\ \underset{+}{\ln B},\ \underset{+}{\sigma},\ \underset{?}{T},\ \underset{-}{\mu - D},\ \underset{-}{\ln CFC}) \tag{8}$$

In compact form,

$$\text{Prob. default} = f\,(\underset{+}{-d_2},\ \underset{+}{-d''_2}).^{17} \tag{8'}$$

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[16] Another reason why debt maturity may not have a strong effect on business failure is that in reality the default option is American rather than European. The Companies Law provides that default may be triggered (the inequality V < B is evaluated) not only at the debt's maturity (T) but in any year (effectively making the default option an American one).

[17] We use $-d_2$ and $-d''_2$, instead of $N(-d_2)$ and $N(-d''_2)$, to avoid an econometric problem from using a probability as a single explanatory variable in a logit regression model (restricting the logit probability values in the range 0.5 to 1).

III. DATA AND METHODOLOGY

This section discusses the dataset, control procedures and empirical models used in the study.

A. Dataset

The main dataset consists of 420 distressed U.S. firms that filed for bankruptcy and an equivalent sample of healthy control firms. The initial sample consists of 485 firms that filed for bankruptcy during the 15-year period 1986-2001. These firms had data available in the Compustat database (as of 23/11/2001) and were identified in the *Wall Street Journal Index* or in the *Internet Bankruptcy Library* as having filed a Chapter 11 bankruptcy petition. From the 485 identified firms, 444 were included in the Standard and Poor´s Research File while 41 firms in the Standard and Poor´s Active File of the Compustat database. From these, thirty utilities and financial institutions (SIC 6000-6999) were removed from the sample (as they had different financial structure). Finally, an additional thirty-five firms that changed their fiscal year during the first three years prior to bankruptcy filing were also removed from the sample, resulting in a total of 420 distressed firms.[18] To ensure the robustness of our results we employ three different matching procedures. In the first procedure, each of the 420 distressed firms was matched with a healthy control firm (giving a total sample of 840 US firms) based on same industry, size and year of bankruptcy. In the second procedure, the 420 distressed firms were matched with 420 randomly-selected healthy firms based on the year of bankruptcy only. Finally, in the third procedure we used as control all 6,560 available healthy firms included in the Compustat database (without matching).

[18] The names of all distressed and matched healthy firms as well as the year of bankruptcy filing are available upon request. Three and four-digit SIC codes were used to match the distressed with the control

The primary and extended option variables in models (8) and (8') were calculated using the following Compustat data items: The market value of the firm (V) and the firm standard deviation (σ) were calculated by solving the simultaneous equations (4) and (4'). In these equations, the market value of equity (E) and the volatility of equity returns (σ_E) were determined from the price, dividend payout and number of shares outstanding from Compustat.[19] The book value of total liabilities (B) was obtained from annualizing quarterly long-term debt (LTQ, item # Q54). The duration-weighted average maturity of the firm's debts was obtained from $T = \sum_{t=1}^{30} PV(DDt) * t / \sum_{t=1}^{30} PV(DDt)$

where *PV(DDt)* is the present value of debt due in year t.[20] For t = 1 to 5, *DDt* (debt due in year 1 …5) was obtained from data items # A44, A91-A94, and for t > 5 an approximation was used taking account of the total long-term debt (*DLTT*, item # A9).[21] In the determination of μ - D, the expected return on assets μ was calculated (as in Hillegeist et al, 2004) from:

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firms.

[19] For robustness purposes, the market value of the firm (V) and the firm value standard deviation (σ) were also calculated from historical data (instead of using the simultaneous option equations). Specifically, the market value of the firm (V) was calculated from Compustat items (annual MKVALM + annual LTQ), where MKVALM is a monthly item and LTQ (item # Q54) is quarterly. In order to calculate V, MKVALM and LTQ were transformed into both monthly and annual items. For the calculation of standard deviation (σ) the monthly MKVALM and the monthly LTQ were used. In order to transform the quarterly long-term debt (LTQ) into monthly values, an averaging method based on the two surrounding months was used to estimate the two missing months. The annual value of LTQ is the corresponding value at fiscal year end. The results based on historical data were qualitatively similar to the ones presented in the study (using the simultaneous option equations).

[20] The model implies that T should be the maturity date for all of the firms' debts B. However, it is not possible to calculate T for some liabilities. For example, currently operating liabilities typically turn over, and it is not possible to determine the maturity date for longer-term operating liabilities such as deferred income taxes. Hence, our implementation of the duration concept involves an approximation.

[21] First, we estimate the cumulative debt from year 6 onwards by subtracting the debt of the first years (sum of *DD2* to *DD5*) from the total long-term debt (DLTT); second, we determine the average annual debt (from the first years *DD2* to *DD5*) and apportion it to the remaining years until the cumulative debt is exhausted up to year 30.

$$\mu_t = \max\left[\frac{V(t) + D(t) - V(t-1)}{V(t-1)}, r\right] \quad (9)$$

where $V(t)$ is the total market value of the firm's assets at time t, $D(t)$ is the total payout by the firm (including dividends and coupon payments to debtholders) expressed as a % of V(t) at time t calculated from (XINT+DV) / V, where XINT is interest expense (item # A15), DV is cash dividends (item # A127), and r is the risk free rate.[22] In the determination of the extended option variable cash flow coverage (CFC), the cash flow from operations (CFO) was obtained from item #A308,[23] cash and cash equivalents from data item # A1 (with interest expense XINT from item # A15, and debt repayment due in one year *DD1* from item # A44).

B. Empirical Models

Logistic regression methodology was used to test the significance of the above option-motivated models, namely: a) the standard option-pricing model using the five primary option variables or their composite measure ($-d_2$), and b) the extended option model that includes a transformation of the option-motivated cash flow coverage (CFC) to also capture the probability of intermediate default. Specifically, the models used are the following:

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[22] To further examine the robustness of our results to the expected return on assets measure (μ), we used the risk-neutral probability of bankruptcy as it appears directly in the Merton (1973, 1974) option model, instead of the actual PB. We additionally used the following accounting-based proxy: $\mu_t = \frac{NI_{t-1} + IntExp_{t-1} * (1 - TR)}{TA_{t-1}}$, where *NI* is the reported net income, *IntExp* is the pretax interest expense, and *TR* is the tax rate (in an earlier version of Hillegeist et al (2004)). The results are qualitatively similar to those reported in the paper.

[23] For the years prior to 1987 if Compustat item #A308 was not available, CFO was calculated as IB+STA+LTA, where IB (item # A18) is income before extraordinary items and discontinued operations, STA is short-term accruals estimated as change in current assets except cash and cash equivalents minus changes in current liabilities except debt in current liabilities, and LTA is long-term accruals estimated as depreciation plus non-cash expenses.

a) Standard option-pricing models (default only at debt maturity):

Model 1: Prob. default = f (lnV, lnB, σ, T, $\mu - D$)

Model 2: Prob. default = f (d2d, T, $\mu - D$)

Model 3: Prob. default = $-d_2 \equiv -\{\ln(V/B) + [(\mu - D) - \frac{1}{2}\sigma^2]T\}/\sigma\sqrt{T}$

b) Extended option-pricing models (including intermediate default):

Model 1': Prob. default = f (lnV, lnB, σ, T, $\mu - D$, lnCFC)

Model 2': Prob. default = f (d2d, T, $\mu - D$, lnCFC)

Model 3': Prob. default = f ($-d_2$, lnCFC)

Model 4': Prob. default = $-d_2'' \equiv -\{\ln(CFC) + [(\mu - D) - \frac{1}{2}\sigma^2]T'\}/\sigma\sqrt{T'}$

Model 5': Prob. default = f ($-d_2$, $-d_2''$).

Models (1) through (3) use the five primary option variables that account for default at (average) debt maturity only, whereas models (1') through (3') also include the extended option-motivated variable CFC (to also capture intermediate default). Specifically, models (1) and (1') use the five independent variables appearing in the Black and Scholes model in eq. (3). Models (2) and (2') use the alternative risk measure (as in KMV), combining the difference between lnV and lnB in units of σ into the "distance to default" (d2d) variable, ln(V/B)/σ. Models (3) and (3′) combine the five primary option variables into a single explanatory measure which proxies for the probability of default at maturity, $-d_2$. Model (4') directly uses the nonlinear transformation of CFC appearing in the intermediate probability of default measure $-d_2''$. Our final extended option model (5') relies solely on the option-theory motivated compact expressions for the probabilities of intermediate default $-d_2''$ as well as terminal default $-d_2$.

The 5 primary option variables are as follows:

(1) the ln of the current market value of the firm′s assets, lnV, estimated from the simultaneous equations in (4'). The greater the current worth of the firm's assets, the lower the probability of default at maturity.

(2) the natural logarithm of the book value of total liabilities, lnB. The higher the principal amount owed at maturity (the exercise price of the equityholders′ option), the greater the probability of default.

(3) the (annualized) standard deviation (σ) of % changes in firm value (returns) estimated from the simultaneous equations (4'). The greater the firm's volatility, the greater the value of equityholders′ default option.

(4) the average time to debt′s maturity (T), measured as the average duration of all outstanding debt maturities.[24] From a strict option-theory perspective, the longer the maturity, the greater the default option value, other things constant.[25]

(5) the difference between the expected asset return and the firm′s payout rate, μ – D, where μ is obtained from eq. (9), and D is the coupon interest payments plus dividends as a proportion of the market value of the firm (V) at fiscal year end.

The above are combined in a nonlinear way as per above option-pricing theory into the terminal probability of default measure, $-d_2$. The higher this measure the higher the

[24] The primary Merton (1974) model assumes a simple capital structure. In practice, firm capital structures are complex. A typical approach is to replace the given debt structure by a zero-coupon "equivalent" structure. We assume that all debt has maturity equal to the duration-weighted average of all debt issues. KMV take a different approach, assuming equity is a perpetual option on the firm's asset value which can default at maturity when it reaches a specified default boundary (below the principal amount due). Leland (2004) points out a weakness of the distance-to-default measure of the KMV model in that it determines the probability that the asset value exceed the boundary *only* at the maturity and not at all times up until maturity.

[25] In general the (European) option is not monotonic in time to maturity. $\partial C / \partial T$ depends on ($r - \delta - 0.5\sigma^2$), so its sign depends on the relative magnitude of $r-\delta$ vs. $0.5\sigma^2$, as well as on T. This may be shifting over time. For practical purposes a change in sign might occur after several years. Furthermore, in practice firms facing financial difficulties are likely to have more difficulty in maintaining long-term debt, and so, by necessity, the sample of bankrupt firms may be associated with a lower duration of debt than healthy

probability of default at maturity.

The extended option-motivated variable for intermediate default, cash flow coverage (CFC), indicating the cash flow ability of the firm to cover its interest and debt repayment obligations, is in general negatively related to the probability of intermediate default. The nonlinear transformation of this variable, $-d_2''$, is positively related to the intermediate probability of default.

We test the models' significance and explanatory power using –2 log-likelihood statistics and pseudo-R^2. The significance of the difference among the distressed and healthy groups is examined via the parametric paired t-test (for means) and the non-parametric Wilcoxon signed-rank matched-pair test (for medians), while the significance of differences between the primary and the extended option models is examined via the log-likelihood ratio test. We test the predictive ability of the above option-motivated variables using a holdout sample from a later period. Our results are presented next.

IV. EMPIRICAL RESULTS

If option theory is valid, then the aforementioned option-motivated measures should contain significant explanatory power in determining the risk of business default. The analysis and empirical results that follow shed light on the validity of this contention.

Figure 1 illustrates graphically the trends in the major primary and extended option variables over the five-year period prior to bankruptcy filing for both distressed and healthy firms. Descriptive statistics (mean, median, and standard deviation) along with parametric (paired t-test) and non-parametric (Wilcoxon) tests of the significance of the mean and

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firms.

median differences across the distressed and healthy groups are given in Table I. In general, the trends presented in Figure 1 and Table I are broadly consistent with option analysis. Specifically, the median firm market value (lnV) for distressed firms is slightly lower and declines significantly as the year of bankruptcy filing approaches. It is intuitive that as distress increases the effected firms lose market value. As expected, the median amount of debt (lnB), firm volatility (σ), and the probability of terminal default ($-d_2$) are all higher for distressed firms than for healthy firms in all years examined.[26]

Regarding the extended option variables, cash flow coverage (lnCFC) is lower for distressed firms and declines as the year of bankruptcy filing approaches, while the probability of intermediate default ($-d_2''$) is higher for distressed firms and increases as the year of bankruptcy filing approaches. The above confirms that troubled firms have relatively lower cash flows and have more difficulty in servicing their debt obligations. In summary, the above trends combined are generally consistent with our expectations from option theory and prior bankruptcy-prediction literature.

Table II shows Pearson correlations among the major explanatory variables for all firm-years examined. There seems to be a positive correlation among the default dummy (right-hand column) and the amount of debt due, ln(B), firm volatility, σ, and the probability of default measures,$-d_2$ and $-d_2''$. The correlation of default dummy with ln(V), ln(V/B) and d2d is negative. As far as the extended option variables are concerned, results are again consistent with our expectations. The cash flow coverage variable, ln(CFC), is negatively correlated with default, while the –d2" probability of intermediate default proxy is positively associated with default.

[26] The median debt maturity (T) declines for distressed firms as bankruptcy filing approaches, suggesting

Table III presents *univariate* results for all option-motivated variables used in the logistic regression for up to five years prior to bankruptcy filing. As expected, these results indicate that the distance to default (d2d) and the probability of default at maturity ($-d_2$) are statistically significant (at about $\alpha = 1\%$) in all 5 years tested, with the expected sign. All primary and extended option variables are statistically significant at least in the last year prior to bankruptcy filing (including T and μ-D). The ratio ln(V/B), firm volatility (σ), and distance-to-default are statistically significant in the 3 years prior to bankruptcy filing. The extended option variables, ln(CFC) and $-d_2''$, are also statistically significant with the expected sign in the last 4 years prior to bankruptcy, lending support to extended option theory.

Table IV shows the *multivariate* logistic regression results for the *five* primary option variables, while table V shows the results for *all* option-motivated variables, primary and extended, in each year up to five years prior to bankruptcy filing.[27] Consistent with option theory, all models tested are statistically significant at the 1% level (based on the –2 log-likelihood test) in at least the last four years prior to bankruptcy. In model (1) all individual primary option variables are statistically significant (mostly at 1%) in the first two years, while lnV, lnB and σ are significant in at least the first three years prior to the bankruptcy filing. As expected, the probability of default is higher the lower the value of the firm (lnV), the higher the amount of debt

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that perhaps firms in trouble have more difficulty borrowing long-term.

[27] As noted, models (1) through (3) present results for the primary option variables only, whereas models (1') through (3') also include the extended option-motivated variables (lnCFC or $-d_2''$). Model (2) or (2') combines the three main option-motivated variables (namely, ln(V), ln(B), and σ) into the single variable distance-to-default (d2d). Model (3) or (3') combines the five primary option variables into a single measure which proxies for the probability of default at maturity (d_2). Model (4) focuses on the probability of intermediate default alone ($-d_2''$), while model (5) combines both the terminal and intermediate default probability measures, $-d_2$ and $-d_2''$.

owed (lnB), and the higher the firm volatility (σ). The average debt maturity (T) seems to have a negative sign, partly because firms in financial distress have more difficulty in raising long-term debt and so they tend to hold more short-term loans. As expected, the model's pseudo-R^2 and the testing results increase as bankruptcy filing approaches. As far as model (2) is concerned, the distance-to-default (d2d) variable is statistically significant (at 1%) in all five years tested. The pseudo-R^2 increases from about 5% to 25% and the forecasting ability of the model as measured by the testing results increases from 53% to about 67% as bankruptcy filing approaches.[28] When the effect of all five primary option variables above is combined into the single probability of default at maturity measure, $-d_2$, model (3) is statistically significant (mostly at $\alpha = 1\%$) in all years tested, validating the predictions of option theory. As expected, both the pseudo-R^2 and the predictive performance of the model increase (to 14% and 71%) as bankruptcy filing approaches. These findings provide support for the validity of option theory in explaining financial distress.

Table V (models 1' - 3') presents results when we expand the primary option model to include the extended-option variables that also account for intermediate default on interest and debt repayments. In model (1') the main option variables lnV, lnB and σ maintain their statistical significance in the three years prior to bankruptcy, while the explanatory power and predictive performance of the model improves as bankruptcy approaches. This suggests that the extended option-motivated cash flow coverage (lnCFC) variable has incremental informativeness beyond the primary option variables for all four years prior to bankruptcy filing with a negative coefficient, indicating that the probability of bankruptcy is higher the

[28] The 67% prediction rate compares favorably with a random model that would correctly categorize 50%

lower the cash flow coverage (lnCFC). The model's classification (testing) accuracy is 76%. The pseudo-R^2 of model (1') increases from 3% to 35% and the model's predictive ability increases from 52% to 76% as bankruptcy filing approaches. The extended option variables, lnCFC and $-d_2''$, tend to maintain their sign and significance when the effect of the five primary option variables is combined into the distance-to-default (d2d) measure in model (2'), and even when combined into the single measure of the terminal default probability ($-d_2$) in model (3'). The results of the full models (1'), (2') and (3') constitute an improvement compared to the reduced primary option models (1), (2) and (3), confirming that the extended option variables have incremental information content beyond the primary option variables. Log-likelihood ratio tests presented in later Table VIII (panel A) confirm that the extended full model provides significantly more explanatory information than the corresponding reduced (primary) model. Model (4') shows that the non-linear transformation of CFC, $-d_2''$, is also statistically significant with its pseudo-R^2 increasing from less than 1% to 16% as bankruptcy approaches. Finally, model (5'), which is the most direct test of OPT's measures of the probabilities of intermediate and terminal default, confirms that $-d_2$ and $-d_2''$ remain significant in the presence of each other in most years. This constitutes the most powerful confirmation of the validity of option pricing theory in explaining distress. The pseudo-R^2 increases from 3% to 25%, with the predictive ability of the model rising to over 74% as bankruptcy filing approaches.

In order to examine the robustness of our results to the matching procedure, we present in Tables VI and VII logistic regression results based on different datasets. In Table VI, results are based on a random sample matched only by event year (in contrast to the

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of the firms.

previous results of Table V where matching was based on industry, size and year). In Table VII, results are based on a control sample of all healthy firms included in the Compustat database with available data.[29] In general, the results shown in these two tables are qualitatively similar to those presented thus far in the study. The conclusions drawn earlier regarding the significance level of the primary and extended option variables remain basically the same. Specifically, the results are qualitatively similar regarding the sign and significance of the coefficients of all variables tested, the explanatory power (pseudo-R^2), the predictive performance (testing results) of the models, and the incremental significance of the full/extended vs. the reduced/primary model.

To further examine the sensitivity of our results we repeated the tests using the same sample of bankrupt firms for all years tested. Results (non tabulated) were again qualitatively similar to those presented in the study. We also used alternative proxies for our main variables. For example, μ was also estimated as an accounting-based expected return on assets (see footnote 22) as in an earlier version of Hillegeist (2004). We also replaced μ with the risk free rate, r, to obtain the risk-neutral probability of default directly. Moreover, we used alternative transformations of the probability of default variables, such as $N(-d_2)$ and $N(-d_2^{''})$ or the logistic inverse of $N(-d_2)$ and of $N(-d_2^{''})$, instead of $-d_2$ and $-d_2^{''}$. Once again, the sensitivity results (non tabulated) were

[29] The rationale behind these two alternative datasets is as follows. The control dataset based on a random sample (matched only by event year) in Table VI does not match by size and industry since it could be argued that the size variable is one of the inputs into our option measure ($-d_2$) and this option variable may not be specific to any industry. We use all available healthy firms as a control sample in Table VII to avoid possible sample selection bias in non-randomly choosing the bankrupt year that may cause the coefficients to be biased. By using all available firms, the power of the tests may increase as well (see Shumway, 2001, Ohlson, 1980, Zmijewski, 1984).

qualitatively similar to those presented in the study.

It is interesting here to compare our results with those of the empirical hazard literature and those of the structural bankruptcy models. The results of our study differ from the results of prior studies in several respects. First, to the best of our knowledge, no prior study has examined the contribution of these extended option variables beyond the standard option variables. Hillegeist et al (2004) employed a hazard model to examine the predictive ability of the ad hoc Ohlson (1980) and Altman (1968) accounting-based variables beyond the option variables. His results that the option variables play an important role in explaining bankruptcy beyond the ad-hoc accounting variables was mostly due to the poor performance of the latter. Shumway (2001), also using hazard models, finds that most ad hoc accounting variables used in the bankruptcy models of Altman (1968) and Zmijewski (1984) (including current assets to current liabilities as proxy for liquidity) were insignificant. The only two variables he found significant are profitability and leverage proxies. The "leverage" effect is captured implicitly in our model via ln(V/B). Shumway (2001) found that the volatility of stock returns was not significant. Chava and Jarrow (2001) validate Shumway's results using annual data with a larger sample, except that the volatility of stock returns was found to be significant. They do not include a cash flow/ liquidity proxy since Shumway excluded it from the broader model (with additional market variables). We instead find that the volatility of firm value changes (for which market equity prices and stock return volatility are inputs) is a significant variable, as predicted by option theory.

Leland (2004) compares structural models with exogenous and endogenous default boundaries. The two approaches give similar expected default probabilities. However, although the exogenous model implies the expected default probability is invariant to various parameters, in the endogenous approach it rises with default costs and leverage, and it falls with debt maturity. The KMV model differs from the above. The expected default probability increases with leverage and volatility, and declines with debt maturity (while being invariant to default costs). We find that the expected default probability increases with firm asset volatility and "leverage" ln(B/V), and declines with cash flow coverage. Overall, our results confirm the usefulness of extended option theory in explaining financial distress.

V. CONCLUSIONS

This study builds on, and extends, a theoretical model using option pricing to derive the factors associated with distress leading to bankruptcy filing. Using a sample of matched U.S. firms during the 1986-2001 period, our results indicate that both the standard primary as well as our extended option-motivated models are statistically significant in explaining business default. The significant primary option variables include the face value of debt owed at maturity (lnB), the current market value of the firm's assets (lnV), and the standard deviation (σ) of firm value changes (returns). The distance to default (d2d) and the probability of default at maturity ($-d_2$) were also found to be significant predictor variables. Moreover, when these primary option variables are included along with a nonlinear transformation of cash flow coverage used to also capture the probability of intermediate default on due interest and debt repayments, the above primary option variables maintain their sign and significance. The latter results indicate that the extended option variables based on cash flow coverage have incremental explanatory power beyond the primary option variables. Log-likelihood ratio tests confirm the incremental significance of the full/extended model over the reduced/primary option model. Robustness tests using a random matched sample and *all* available healthy firms corroborate the option-based distress prediction model. The overall results are consistent with the predictions of (extended) option theory. Our theory-driven model has significant explanatory power and predictive ability in the years tested, providing a deeper understanding of the factors determining firm distress and bankruptcy filing compared to previous ad hoc empirical approaches.

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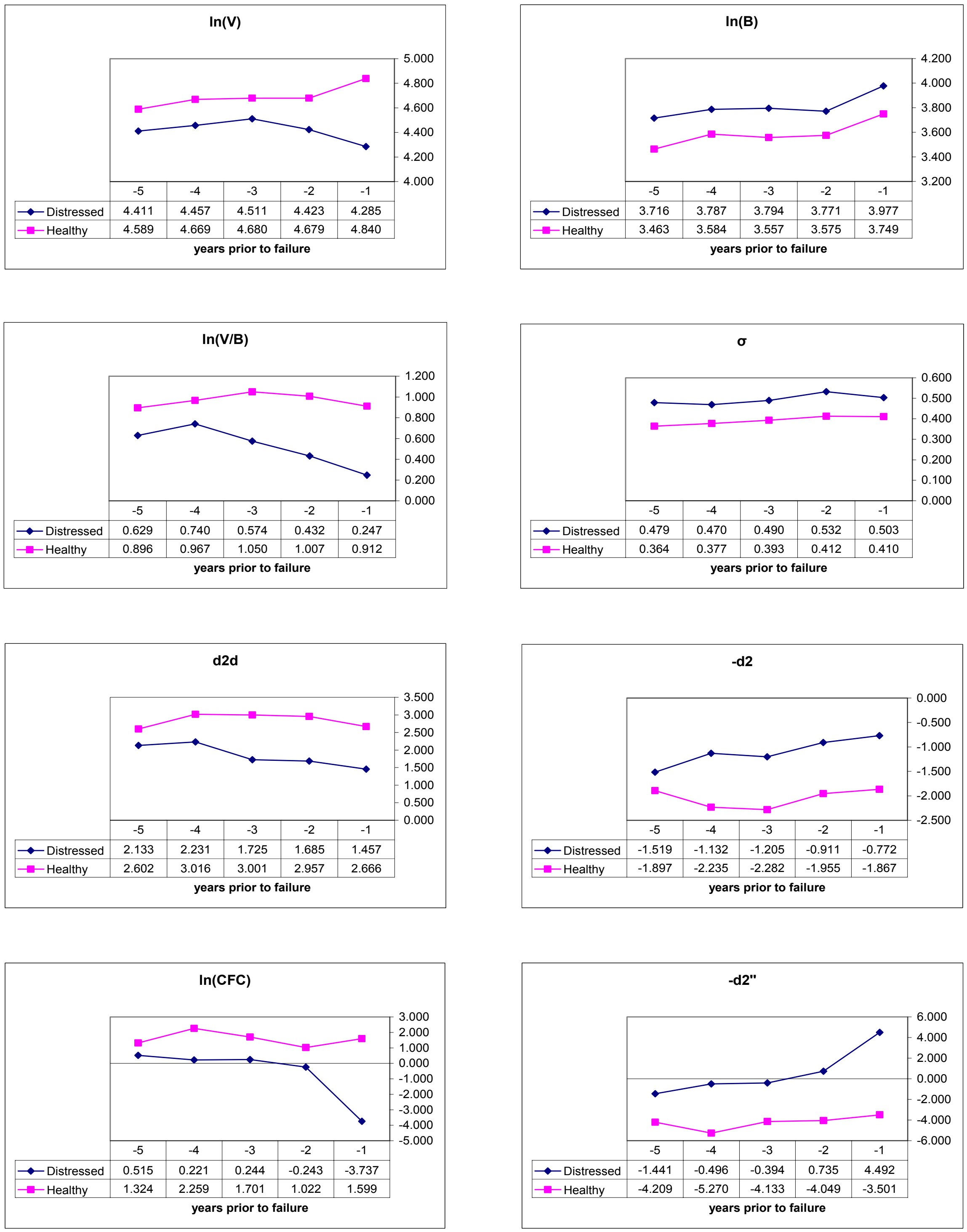
Figure 1
Trends of main option-motivated predictor variables for distressed and healthy firms up to five years prior to bankruptcy filing (based on median values)



ln(V)

years prior to failure	-5	-4	-3	-2	-1
Distressed	4.411	4.457	4.511	4.423	4.285
Healthy	4.589	4.669	4.680	4.679	4.840

ln(B)

years prior to failure	-5	-4	-3	-2	-1
Distressed	3.716	3.787	3.794	3.771	3.977
Healthy	3.463	3.584	3.557	3.575	3.749

ln(V/B)

years prior to failure	-5	-4	-3	-2	-1
Distressed	0.629	0.740	0.574	0.432	0.247
Healthy	0.896	0.967	1.050	1.007	0.912

σ

years prior to failure	-5	-4	-3	-2	-1
Distressed	0.479	0.470	0.490	0.532	0.503
Healthy	0.364	0.377	0.393	0.412	0.410

d2d

years prior to failure	-5	-4	-3	-2	-1
Distressed	2.133	2.231	1.725	1.685	1.457
Healthy	2.602	3.016	3.001	2.957	2.666

-d2

years prior to failure	-5	-4	-3	-2	-1
Distressed	-1.519	-1.132	-1.205	-0.911	-0.772
Healthy	-1.897	-2.235	-2.282	-1.955	-1.867

ln(CFC)

years prior to failure	-5	-4	-3	-2	-1
Distressed	0.515	0.221	0.244	-0.243	-3.737
Healthy	1.324	2.259	1.701	1.022	1.599

-d2"

years prior to failure	-5	-4	-3	-2	-1
Distressed	-1.441	-0.496	-0.394	0.735	4.492
Healthy	-4.209	-5.270	-4.133	-4.049	-3.501

Notes: lnV: ln of current market firm value; lnB: ln of book value of total liabilities; ln(V/B): ln of current market firm value to book value of total liabilities; (historical) σ: (historical) standard deviation of firm value changes; d2d: distance to default; -d2: proxy of default probability at maturity; ln(CFC): ln of cash flow coverage; -d2": proxy of intermediate default probability.

Table I

Descriptive statistics for all option-motivated variables

This table presents descriptive statistics (mean, median, and std dev.) for all option-motivated variables up to five years prior to bankruptcy filing for the sample of 420 highly distressed and corresponding healthy firms over the 15-year period 1986-2001, along with parametric (paired t-test) and nonparametric (Wilcoxon) tests of their mean (median) differences.

OPTION-MOTIVATED VARIABLES	SYMBOL		ONE YEAR BEFORE BANKRUPTCY				TWO YEARS BEFORE BANKRUPTCY				THREE YEARS BEFORE BANKRUPTCY				FOUR YEARS BEFORE BANKRUPTCY				FIVE YEARS BEFORE BANKRUPTCY			
			Distressed	Healthy	Paired t-test**	Wilcoxon test**	Distressed	Healthy	Paired t-test**	Wilcoxon test**	Distressed	Healthy	Paired t-test**	Wilcoxon test**	Distressed	Healthy	Paired t-test**	Wilcoxon test**	Distressed	Healthy	Paired t-test**	Wilcoxon test**
	N		279	2026			324	1822			286	1626			253	1446			203	1242		
A. Primary Option Variables																						
Ln of current market firm value	ln(V)	mean	4.376	4.705	(0.000)		4.555	4.725	(0.003)		4.650	4.692	(0.471)		4.572	4.635	(0.301)		4.494	4.529	(0.613)	
		median	4.285	4.840		(0.000)	4.423	4.679		(0.003)	4.511	4.680		(0.196)	4.457	4.669		(0.193)	4.411	4.589		(0.459)
		std	1.434	1.513			1.429	1.636			1.343	1.611			1.406	1.621			1.363	1.570		
Ln of book value of total liabilities	ln(B)	mean	3.967	3.642	(0.000)		3.876	3.587	(0.000)		3.707	3.522	(0.004)		3.593	3.474	(0.074)		3.535	3.400	(0.106)	
		median	3.977	3.749		(0.000)	3.771	3.575		(0.000)	3.794	3.557		(0.000)	3.787	3.584		(0.022)	3.716	3.463		(0.014)
		std	1.604	1.666			1.757	1.733			1.835	1.719			1.855	1.781			1.822	1.765		
Ln of (firm market value/BV of debt)	ln(V/B)	mean	0.409	1.063	(0.000)		0.679	1.138	(0.000)		0.943	1.170	(0.007)		0.979	1.160	(0.035)		0.959	1.130	(0.096)	
		median	0.247	0.912		(0.000)	0.432	1.007		(0.000)	0.574	1.050		(0.000)	0.740	0.967		(0.003)	0.629	0.896		(0.018)
		std	0.674	0.883			0.999	0.829			1.149	0.900			1.121	0.914			1.104	0.907		
Std deviation of firm value changes	σ	mean	0.582	0.458	(0.000)		0.611	0.473	(0.000)		0.573	0.466	(0.000)		0.559	0.451	(0.000)		0.552	0.461	(0.025)	
		median	0.503	0.410		(0.000)	0.532	0.412		(0.000)	0.490	0.393		(0.000)	0.470	0.377		(0.000)	0.479	0.364		(0.024)
		std	0.387	0.247			0.431	0.251			0.322	0.259			0.364	0.278			0.407	0.283		
Average time to debt's maturity	T	mean	4.262	5.249	(0.023)		5.338	5.503	(0.645)		5.479	5.272	(0.584)		5.696	4.893	(0.051)		5.595	4.755	(0.111)	
		median	2.653	3.699		(0.022)	3.770	3.822		(0.824)	3.881	3.810		(0.245)	4.201	3.617		(0.012)	3.757	3.528		(0.144)
		std	4.050	4.423			4.285	5.105			4.444	4.598			4.395	4.415			5.122	3.837		
Asset return minus firm payout	μ-D	mean	0.031	0.178	(0.000)		0.138	0.199	(0.021)		0.217	0.213	(0.896)		0.167	0.191	(0.381)		0.151	0.150	(0.957)	
		median	0.007	0.042		(0.000)	0.026	0.049		(0.000)	0.045	0.071		(0.209)	0.040	0.058		(0.060)	0.043	0.052		(0.944)
		std	0.140	0.338			0.331	0.343			0.391	0.311			0.323	0.288			0.258	0.254		
Probability of terminal default	-d2	mean	-0.741	-2.200	(0.000)		-1.270	-2.444	(0.000)		-1.676	-2.622	(0.000)		-1.942	-2.657	(0.023)		-1.801	-2.240	(0.168)	
		median	-0.772	-1.867		(0.000)	-0.911	-1.955		(0.000)	-1.205	-2.282		(0.000)	-1.132	-2.235		(0.000)	-1.519	-1.897		(0.001)
		std	1.013	2.138			1.903	2.464			2.049	2.011			4.391	2.324			3.094	1.834		
Distance to default	d2d	mean	1.589	2.968	(0.000)		1.948	3.210	(0.000)		1.983	3.295	(0.000)		2.786	3.522	(0.170)		2.525	3.205	(0.094)	
		median	1.457	2.666		(0.000)	1.685	2.957		(0.000)	1.725	3.001		(0.000)	2.231	3.016		(0.000)	2.133	2.602		(0.001)
		std	1.763	2.228			4.032	2.473			2.130	2.338			7.965	2.836			3.981	2.246		
B. Extended Option Variables																						
Ln of cash flow coverage	ln(CFC)	mean	-1.935	1.158	(0.000)		-0.689	2.627	(0.000)		0.345	1.877	(0.000)		-0.670	2.661	(0.000)		1.259	1.787	(0.110)	
		median	-3.737	1.599		(0.000)	-0.243	1.022		(0.000)	0.244	1.701		(0.000)	0.221	2.259		(0.000)	0.515	1.324		(0.158)
		std	2.565	2.171			2.406	4.941			2.911	2.245			5.488	3.542			3.252	2.699		
Probability of intermediate default	-d2"	mean	25.084	-3.523	(0.000)		7.333	-6.594	(0.000)		0.042	-5.408	(0.000)		4.959	-6.287	(0.000)		-2.040	-5.414	(0.011)	
		median	4.492	-3.501		(0.000)	0.735	-4.049		(0.000)	-0.394	-4.133		(0.000)	-0.496	-5.270		(0.000)	-1.441	-4.209		(0.011)
		std	80.626	7.739			44.253	16.730			11.568	9.176			29.566	29.648			11.212	11.948		

** Test of the significance of the mean (median) differences: Paired t-test (mean) / Wilcoxon Z-test (median); p-values in parentheses.

Table II
Correlation matrix among all option-motivated variables

This table shows Pearson correlations among the major explanatory variables for all firm-years tested. lnV: ln of current market firm value; lnB: ln of book value of total liabilities; ln(V/B): ln of current market firm value to book value of total liabilities; σ: (historical) standard deviation of firm value changes; d2d: distance to default; T: average time to debt's maturity; μ-D: expected asset return minus firm payout, $\mu = \max\{[V(t) + D(t) - V(t-1)]/V(t-1), r\}$; -d2: proxy for the default probability at maturity; ln(CFC): ln of cash flow coverage; -d2'': proxy for the intermediate default probability; Default dummy is one for firms that filed for bankruptcy (0 for healthy firms). Correlations are determined based on the entire sample of distressed and all healthy firms during 1986-2001. Number of observations is 420 for distressed firms and 2030 for healthy firms.

	ln(V)	ln(B)	ln(V/B)	σ	T	μ-D	d2d	$-d_2$	ln(CFC)	-d2''	Default dummy
ln(V)	1.00	0.83	0.04	-0.27	0.24	0.06	0.15	-0.14	0.15	0.01	-0.04
		(0.00)***	(0.05)*	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.72)	(0.04)**
ln(B)		1.00	-0.52	-0.46	0.30	-0.05	-0.13	0.14	-0.03	0.09	0.06
			(0.00)***	(0.00)***	(0.00)***	(0.01)**	(0.00)***	(0.00)***	(0.22)	(0.00)***	(0.00)***
ln(V/B)			1.00	0.42	-0.18	0.20	0.46	-0.47	0.27	-0.15	-0.18
				(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***
σ				1.00	0.05	0.34	-0.12	0.07	0.02	-0.13	0.02
					(0.03)**	(0.00)***	(0.00)***	(0.00)***	(0.27)	(0.00)***	(0.27)
T					1.00	0.00	-0.11	0.21	0.04	-0.06	0.01
						(0.98)	(0.00)***	(0.00)***	(0.08)*	(0.00)***	(0.67)
μ-D						1.00	-0.05	-0.42	0.09	-0.10	-0.08
							(0.03)**	(0.00)***	(0.00)***	(0.00)***	(0.00)***
d2d							1.00	-0.70	0.18	0.08	-0.15
								(0.00)***	(0.00)***	(0.00)***	(0.00)***
$-d_2$								1.00	-0.24	0.14	0.20
									(0.00)***	(0.00)***	(0.00)***
ln(CFC)									1.00	-0.42	-0.33
										(0.00)***	(0.00)***
-d2''										1.00	0.19
											(0.00)***
Default dummy											1.00

***, **, *: significant at 1%, 5% and 10% level (respectively)

Table III
Univariate logistic regression results for all option-motivated variables

This table presents univariate logistic regression results for all option-motivated variables for each year up to five years prior to bankruptcy filing. Results are presented for all primary option variables and the extended option variables for intermediate default. Pseudo-R^2 = {[-2 Log L (reduced model with constant only)] - [-2 Log L (full model)]} / [-2 Log L (reduced model with constant only)]. Prediction refers to the overall correct classification of distressed and healthy firms within the training sample.

		ONE YEAR BEFORE BANKRUPTCY FILING					TWO YEARS BEFORE BANKRUPTCY FILING					THREE YEARS BEFORE BANKRUPTCY FILING				
OPTION-MOTIVATED VARIABLES	**SYMBOL**	Coefficient	Constant	P-value (of coeff.)	Pseudo-R^2	Prediction	Coefficient	Constant	P-value (of coeff.)	Pseudo-R^2	Prediction	Coefficient	Constant	P-value (of coeff.)	Pseudo-R^2	Prediction
A. Primary Option Variables																
Probability of terminal default	-d2	0.686	0.952	0.000	14.39%	67.5%	0.259	0.458	0.000	4.38%	60.9%	0.308	0.585	0.000	5.51%	63.8%
Distance to default	d2d	-0.381	0.883	0.000	9.38%	66.8%	-0.148	0.380	0.002	2.44%	62.8%	-0.326	0.841	0.000	8.07%	61.5%
Ln of current market firm value	ln(V)	-0.149	0.678	0.080	0.81%	56.8%	-0.067	0.304	0.359	0.17%	54.5%	-0.013	0.061	0.868	0.01%	53.6%
Ln of book value of total liabilities	ln(B)	0.146	-0.562	0.064	0.91%	53.9%	0.098	-0.359	0.130	0.47%	52.8%	0.079	-0.275	0.246	0.32%	53.6%
Ln of (firm market value/BV of debt)	ln(V/B)	-1.417	0.938	0.000	14.35%	69.6%	-0.472	0.416	0.000	3.36%	61.7%	-0.256	0.261	0.030	1.16%	58.2%
Std deviation of firm value changes	σ	2.207	-1.008	0.000	4.05%	60.7%	2.079	-1.001	0.000	4.36%	60.1%	1.549	-0.753	0.002	2.67%	57.9%
Average time to debt's maturity	T	-0.076	0.361	0.014	1.67%	57.9%	-0.029	0.149	0.228	0.30%	52.5%	0.018	-0.096	0.508	0.10%	51.6%
Asset return minus firm payout	μ-D	-4.923	0.343	0.000	8.73%	62.5%	-0.412	0.059	0.254	0.27%	53.4%	0.345	-0.061	0.357	0.21%	48.7%
B. Extended Option Variables																
Ln of cash flow coverage	ln(CFC)	-0.453	-0.098	0.000	23.19%	68.9%	-0.194	0.161	0.000	10.48%	64.2%	-0.204	0.231	0.000	5.02%	65.5%
Probability of intermediate default	-d2"	0.074	-0.176	0.000	16.08%	69.6%	0.032	0.034	0.000	5.72%	66.8%	0.055	0.158	0.000	4.45%	67.4%

		FOUR YEARS BEFORE BANKRUPTCY FILING					FIVE YEARS BEFORE BANKRUPTCY FILING				
OPTION-MOTIVATED VARIABLES	**SYMBOL**	Coefficient	Constant	P-value (of coeff.)	Pseudo-R^2	Prediction	Coefficient	Constant	P-value (of coeff.)	Pseudo-R^2	Prediction
A. Primary Option Variables											
Probability of terminal default	-d2	0.294	0.558	0.000	5.34%	63.3%	0.232	0.407	0.016	2.61%	59.0%
Distance to default	d2d	-0.245	0.654	0.000	4.94%	60.2%	-0.272	0.680	0.002	4.66%	59.0%
Ln of current market firm value	ln(V)	-0.004	0.018	0.963	0.00%	51.6%	0.004	-0.019	0.968	0.00%	50.0%
Ln of book value of total liabilities	ln(B)	0.036	-0.124	0.618	0.07%	51.2%	0.072	-0.241	0.396	0.29%	54.5%
Ln of (firm market value/BV of debt)	ln(V/B)	-0.110	0.114	0.364	0.23%	55.9%	-0.220	0.216	0.156	0.86%	57.3%
Std deviation of firm value changes	σ	1.342	-0.642	0.006	2.43%	59.8%	0.798	-0.381	0.178	0.76%	60.1%
Average time to debt's maturity	T	0.050	-0.266	0.108	0.75%	51.6%	0.052	-0.276	0.155	0.85%	54.5%
Asset return minus firm payout	μ-D	-0.295	0.051	0.482	0.14%	56.6%	0.055	-0.009	0.922	0.00%	50.6%
B. Extended Option Variables											
Ln of cash flow coverage	ln(CFC)	-0.155	0.149	0.000	9.22%	63.3%	-0.037	0.053	0.472	0.21%	56.2%
Probability of intermediate default	-d2"	0.017	0.014	0.014	2.73%	63.7%	0.018	0.059	0.199	0.74%	60.1%

Table IV

Multivariate logistic regression results for primary option variables: Matched pairs

This table presents multivariate logistic regression results for the primary option variables for each year up to five years prior to bankruptcy filing. -d2: proxy for the default probability at maturity; d2d: distance to default; ln(V): ln of current market firm value; ln(B): ln of book value of total liabilities; σ: standard deviation of firm value changes; T: average time to debt's maturity; μ-D: expected return on asset value minus firm payout, μ = max{[V(t) + D(t) - V(t-1)]/V(t-1), r}. Coefficients are presented on the top line and p-values are shown below in parentheses. p-value corresponds to the Chi-square test for the significance of individual coefficients based on the -2 log-likelihood test. Pseudo-R^2 = {[-2 Log L (reduced model with constant only)] - [-2 Log L (full model)]} / [-2 Log L (reduced model with constant only)].

PANEL A: ONE YEAR PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	CONSTANT	Model signif test	Pseudo-R^2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1			-2.263	2.433	5.158	-0.198	-6.270	0.488	(0.000)	28.67	78.2	67.59	24.07	40.74	194
			(0.000)	*(0.000)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.415)*							
			***	***	***	***	***								
2		-0.568				-0.154	-6.608	2.419	(0.000)	24.96	76.8	66.67	22.22	44.44	194
		(0.000)				*(0.000)*	*(0.000)*	*(0.000)*							
		***				***	***	***							
3	0.686							0.952	(0.000)	14.39	67.5	71.30	22.22	35.19	194
	(0.000)							*(0.000)*							
	***							***							

PANEL B: TWO YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	CONSTANT	Model signif test	Pseudo-R^2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1			-0.711	0.876	2.864	-0.087	-0.800	-0.484	(0.000)	7.43	64.5	63.33	28.89	44.44	269
			(0.000)	*(0.000)*	*(0.000)*	*(0.004)*	*(0.053)*	*(0.322)*							
			***	***	***	***	*								
2		-0.171				-0.046	-0.543	0.750	(0.000)	3.60	64.0	61.11	34.44	43.33	269
		(0.001)				*(0.070)*	*(0.138)*	*(0.001)*							
		***				*		***							
3	0.259							0.458	(0.000)	4.38	60.9	67.22	25.56	40.00	269
	(0.000)							*(0.002)*							
	***							***							

PANEL C: THREE YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	CONSTANT	Model signif test	Pseudo-R^2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1			-0.594	0.735	3.715	-0.025	-0.405	-1.148	(0.000)	6.92	64.8	56.76	41.89	44.59	226
			(0.000)	*(0.000)*	*(0.000)*	*(0.420)*	*(0.350)*	*(0.026)*							
			***	***	***			**							
2		-0.326				-0.004	0.170	0.834	(0.000)	8.12	62.5	58.78	40.54	41.89	226
		(0.000)				*(0.883)*	*(0.656)*	*(0.004)*							
		***						***							
3	0.308							0.585	(0.000)	5.51	63.8	61.49	45.95	31.08	226
	(0.000)							*(0.001)*							
	***							***							

PANEL D: FOUR YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	CONSTANT	Model signif test	Pseudo-R^2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1			-0.264	0.340	2.151	0.035	-0.704	-0.895	(0.046)	3.18	61.7	55.71	47.14	41.43	198
			(0.121)	*(0.045)*	*(0.007)*	*(0.283)*	*(0.160)*	*(0.101)*							
				**	***										
2		-0.239				0.038	-0.272	0.484	(0.000)	5.47	60.9	55.00	50.00	40.00	198
		(0.000)				*(0.239)*	*(0.536)*	*(0.095)*							
		***						*							
3	0.294							0.558	(0.000)	5.34	63.3	57.86	45.71	38.57	198
	(0.000)							*(0.003)*							
	***							***							

PANEL E: FIVE YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	CONSTANT	Model signif test	Pseudo-R^2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1			-0.370	0.413	1.606	0.033	-0.054	-0.581	(0.211)	2.89	59.6	53.06	55.10	38.78	138
			(0.091)	*(0.045)*	*(0.088)*	*(0.403)*	*(0.937)*	*(0.369)*							
			*	**	*										
2		-0.259				0.037	0.060	0.443	(0.006)	5.06	59.6	53.06	55.10	38.78	138
		(0.003)				*(0.331)*	*(0.920)*	*(0.224)*							
		***													
3	0.232							0.407	(0.011)	2.61	59.0	57.14	44.90	40.82	138
	(0.016)							*(0.070)*							
	**							*							

***, **, *: significant at 1%, 5% and 10% level (respectively)

Table V

Logistic regression results for all primary and extended option-motivated variables: Matched pairs

This table presents logistic regression results for all option variables up to five years prior to bankruptcy filing. $-d_2$: probability of default at maturity; d2d: distance to default; ln(V): ln of current market firm value; ln(B): ln of book value of total liabilities; σ: standard deviation of firm value changes; T: average time to debt's maturity; μ-D: expected return on asset value minus firm payout, μ = max{[V(t) + D(t) - V(t-1)]/V(t-1), r}; ln(CFC): ln of cash flow coverage; -d2": proxy for probability of intermediate default. Coefficients are presented on the top line and p-values are shown below in parentheses. p-value corresponds to the Chi-square test for the significance of individual coefficients based on the -2 log-likelihood test. Pseudo-R^2 = {[-2 Log L (reduced model with constant only)] - [-2 Log L (full model)]} / [-2 Log L (reduced model with constant only)].

MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
PANEL A: ONE YEAR PRIOR TO BANKRUPTCY FILING																	
1'			-1.646	1.853	4.474	-0.174	-4.968	-0.305		-0.100	(0.000)	35.22	75.4	75.93	20.370	27.778	194
			(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***	(0.000)***		(0.877)							
2'		-0.411				-0.128	-4.435	-0.322		1.711	(0.000)	32.96	77.1	73.15	27.778	25.926	194
		(0.000)***				(0.003)***	(0.001)***	(0.000)***		(0.000)***							
3'	0.466							-0.379		0.529	(0.000)	27.69	73.9	74.07	29.630	22.222	194
	(0.000)***							(0.000)***		(0.013)**							
4'									0.074	-0.176	(0.000)	16.08	69.6	71.30	46.30	11.11	194
									(0.000)***	(0.185)							
5'	0.611								0.051	0.623	(0.000)	24.83	74.6	74.07	25.926	25.926	194
	(0.000)***								(0.000)***	(0.002)***							
PANEL B: TWO YEARS PRIOR TO BANKRUPTCY FILING																	
MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1'			-0.380	0.591	2.396	-0.083	-0.596	-0.170		-0.689	(0.000)	13.84	64.5	68.33	23.33	40.00	269
			(0.022)**	(0.001)***	(0.002)***	(0.007)***	(0.154)	(0.000)***		(0.178)							
2'		-0.060				-0.040	-0.314	-0.183		0.561	(0.000)	11.55	64.8	67.22	26.67	38.89	269
		(0.139)				(0.129)	(0.417)	(0.000)***		(0.012)**							
3'	0.133							-0.172		0.376	(0.000)	11.43	63.4	68.33	25.56	37.78	269
	(0.037)**							(0.000)***		(0.015)**							
4'									0.032	0.034	(0.000)	5.72	66.8	68.89	25.56	36.67	269
									(0.000)***	(0.756)							
5'	0.189								0.024	0.347	(0.000)	7.71	65.1	73.89	23.33	28.89	269
	(0.003)***								(0.001)***	(0.022)**							
PANEL C: THREE YEARS PRIOR TO BANKRUPTCY FILING																	
MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1'			-0.415	0.614	3.523	-0.027	-0.324	-0.187		-1.244	(0.000)	10.50	66.1	68.24	27.03	36.49	226
			(0.018)**	(0.000)***	(0.000)***	(0.402)	(0.468)	(0.000)***		(0.019)**							
2'		-0.280				-0.002	0.280	-0.157		0.869	(0.000)	10.68	65.1	64.86	36.49	33.78	226
		(0.000)***				(0.941)	(0.478)	(0.001)***		(0.003)***							
3'	0.249							-0.160		0.656	(0.000)	8.27	61.5	64.19	41.89	29.73	226
	(0.000)***							(0.001)***		(0.000)***							
4'									0.055	0.158	(0.000)	4.45	67.4	70.27	27.03	32.43	226
									(0.000)***	(0.209)							
5'	0.251								0.039	0.580	(0.000)	7.56	62.5	64.19	41.89	29.73	226
	(0.001)								(0.009)***	(0.001)***							
PANEL D: FOUR YEARS PRIOR TO BANKRUPTCY FILING																	
MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1'			-0.056	0.166	2.023	0.061	-0.612	-0.164		-1.180	(0.000)	12.49	64.5	61.43	42.86	34.29	198
			(0.751)	(0.340)	(0.014)**	(0.084)*	(0.238)	(0.000)***		(0.045)*							
2'		-0.160				0.062	-0.050	-0.144		0.244	(0.000)	12.39	62.5	60.00	48.57	31.43	198
		(0.015)**				(0.071)*	(0.911)	(0.000)***		(0.426)							
3'	0.229							-0.139		0.571	(0.000)	12.21	61.7	61.43	45.71	31.43	198
	(0.002)***							(0.000)***		(0.003)***							
4'									0.017	0.014	(0.002)	2.73	63.7	65.71	45.71	22.86	198
									(0.014)**	(0.913)							
5'	0.262								0.010	0.494	(0.000)	6.49	63.3	58.57	44.29	38.57	198
	(0.001)***								(0.103)	(0.009)***							
PANEL E: FIVE YEARS PRIOR TO BANKRUPTCY FILING																	
MODEL	-d2	d2d	ln(V)	ln(B)	σ	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Training results (%)	Testing results (%)	Type I (%)	Type II (%)	Pair of companies
1'			-0.355	0.398	1.590	0.034	-0.093	-0.023		-0.555	(0.292)	2.97	60.1	52.04	53.06	42.86	138
			(0.108)	(0.056)*	(0.091)*	(0.391)	(0.892)	(0.657)		(0.392)							
2'		-0.257				0.037	0.050	-0.008		0.448	(0.014)	5.07	59.0	53.06	55.10	38.78	138
		(0.004)***				(0.327)	(0.934)	(0.878)		(0.221)							
3'	0.228							-0.021		0.429	(0.037)	2.68	57.9	55.10	46.94	42.86	138
	(0.019)**							(0.692)		(0.064)*							
4'									0.018	0.059	(0.177)	0.74	60.1	62.24	34.69	40.82	138
									(0.199)	(0.707)							
5'	0.217								0.011	0.414	(0.029)	2.88	60.1	56.12	46.94	40.82	138
	(0.028)**								(0.430)	(0.066)**							

***, **, *: significant at 1%, 5% and 10% level (respectively)

Table VI
Logistic regression results for primary and extended option-motivated variables: Random sample (matched by event year only)

This table presents multivariate logistic regression results for all primary and extended option variables for each year up to five years prior to bankruptcy filing. Each firm that filed for bankruptcy over the period 1986-2001 is matched with randomly selected healthy firms based on the event year only. -d_2: probability of default at maturity; d2d: distance to default; T: average time to debt's maturity; μ-D: expected return on asset value minus firm payout, μ = max{[V(t) + D(t) - V(t-1)]/V(t-1), r}; ln(CFC): ln of cash flow coverage; -d2": intermediate default probability. Coefficients are presented on the top line and p-values are shown below in parentheses. p-value corresponds to the Chi-square test for the significance of individual coefficients based on the -2 log-likelihood test. Pseudo-R^2 = {[-2 Log L (reduced model with constant only)] - [-2 Log L (full model)]} / [-2 Log L (reduced model with constant only)].

PANEL A: ONE YEAR PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R^2 (%)	Pair of companies
2		-0.618	-0.196	-6.135			2.912	(0.000)	34.51	268
		*(0.000)****	*(0.000)****	*(0.000)****			*(0.000)****			
2'		-0.484	-0.150	-4.974	-0.232		2.311	(0.000)	42.09	268
		*(0.000)****	*(0.000)****	*(0.000)****	*(0.000)****		*(0.000)****			
3	0.870						1.283	(0.000)	20.18	268
	*(0.000)****						*(0.000)*			
3'	0.648				-0.261		0.975	(0.000)	32.50	268
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.030	-0.087	(0.000)	8.43	268
						*(0.000)****	*(0.422)*			
5'	0.806					0.016	1.086	(0.000)	23.51	268
	*(0.000)****					*(0.001)****	*(0.000)****			

PANEL B: TWO YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Pair of companies
2		-0.283	-0.111	-0.248			1.362	(0.000)	10.05	314
		*(0.000)****	*(0.000)****	*(0.443)*			*(0.000)****			
2'		-0.114	-0.079	0.164	-0.354		1.020	(0.000)	22.16	314
		*(0.033)***	*(0.005)****	*(0.636)*	*(0.000)****		*(0.000)****			
3	0.391						0.722	(0.000)	7.51	314
	*(0.000)****						*(0.000)****			
3'	0.196				-0.354		0.700	(0.000)	20.80	314
	*(0.005)****				*(0.000)****		*(0.000)****			
4'						0.081	0.163	(0.000)	12.08	314
						*(0.000)****	*(0.135)*			
5'	0.249					0.063	0.566	(0.000)	14.48	314
	*(0.000)****					*(0.000)****	*(0.000)****			

PANEL C: THREE YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Pair of companies
2		-0.390	-0.087	-0.537			1.555	(0.000)	12.38	273
		*(0.000)****	*(0.001)****	*(0.253)*			*(0.000)****			
2'		-0.289	-0.077	-0.432	-0.142		1.577	(0.000)	17.07	273
		*(0.000)****	*(0.003)****	*(0.368)*	*(0.000)****		*(0.000)****			
3	0.386						0.706	(0.000)	7.27	273
	*(0.000)****						*(0.000)****			
3'	0.286				-0.167		0.938	(0.000)	14.57	273
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.036	0.217	(0.000)	6.14	273
						*(0.000)****	*(0.061)**			
5'	0.313					0.026	0.715	(0.000)	10.28	273
	*(0.000)****					*(0.001)****	*(0.000)****			

PANEL D: FOUR YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Pair of companies
2		-0.272	-0.064	0.075			1.063	(0.000)	7.35	244
		*(0.000)****	*(0.024)***	*(0.857)*			*(0.000)****			
2'		-0.185	-0.061	0.315	-0.144		0.944	(0.000)	13.56	244
		*(0.002)****	*(0.040)****	*(0.478)*	*(0.000)****		*(0.000)****			
3	0.291						0.537	(0.000)	4.09	244
	*(0.000)****						*(0.002)****			
3'	0.182				-0.150		0.517	(0.000)	11.36	244
	*(0.013)***				*(0.000)****		*(0.004)****			
4'						0.052	0.147	(0.000)	8.75	244
						*(0.000)****	*(0.232)*			
5'	0.160					0.044	0.412	(0.000)	9.75	244
	*(0.037)***					*(0.000)****	*(0.020)***			

PANEL E: FIVE YEARS PRIOR TO BANKRUPTCY FILING

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Pair of companies
2		-0.252	-0.056	-0.745			1.142	(0.000)	6.88	198
		*(0.000)****	*(0.080)**	*(0.122)*			*(0.000)****			
2'		-0.233	-0.056	-0.711	-0.063		1.173	(0.000)	7.47	198
		*(0.000)****	*(0.081)**	*(0.138)*	*(0.141)*		*(0.000)****			
3	0.282						0.578	(0.000)	3.93	198
	*(0.000)****						*(0.004)****			
3'	0.253				-0.070		0.620	(0.000)	4.66	198
	*(0.002)****				*(0.103)*		*(0.002)****			
4'						0.029	0.103	(0.008)	1.89	198
						*(0.021)***	*(0.438)*			
5'	0.248					0.018	0.571	(0.000)	4.67	198
	*(0.002)****					*(0.139)*	*(0.005)****			

***, **, *: significant at 1%, 5%, 10% level (respectively)

Table VII
Logistic regression results for primary and extended option-motivated variables: Control sample is all healthy firms

This table presents multivariate logistic regression results for all primary and extended option variables for each year up to five years prior to bankruptcy filing. The distressed sample includes all firms that filed for bankruptcy over the period 1986-2001. The control sample includes all available healthy firms in the event year (no matching). $-d_2$: default probability at maturity; d2d: distance to default; T: average time to debt's maturity; μ-D: expected return on asset value minus firm payout, μ = max{[V(t) + D(t) - V(t-1)]/V(t-1), r}; ln(CFC): ln of cash flow coverage; -d2": intermediate default probability. Coefficients are presented on the top line and p-values are shown (below) in parentheses. p-value corresponds to the Chi-square test for the significance of individual coefficients based on the -2 log-likelihood test. Pseudo-R^2 = {[-2 Log L (reduced model with constant only)] - [-2 Log L (full model)]} / [-2 Log L (reduced model with constant only)].

MODEL	-d2	d2d	T	μ-D	ln(CFC)	-d2"	CONSTANT	Model signif test	Pseudo-R2 (%)	Pair of companies
PANEL A: ONE YEAR PRIOR TO BANKRUPTCY FILING										
2		-0.623	-0.142	-6.369			0.905	(0.000)	26.06	279/2026
		*(0.000)****	*(0.000)****	*(0.000)****			*(0.000)****			
2'		-0.479	-0.108	-5.030	-0.185		0.328	(0.000)	32.32	279/2026
		*(0.000)****	*(0.000)****	*(0.000)****	*(0.000)****		*(0.134)*			
3	0.563						-0.865	(0.000)	12.89	279/2026
	*(0.000)****						*(0.000)****			
3'	0.407				-0.224		-1.090	(0.000)	24.17	279/2026
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.031	-1.814	(0.000)	10.07	279/2026
						*(0.000)****	*(0.000)****			
5'	0.505					0.022	-1.032	(0.000)	18.45	279/2026
	*(0.000)****					*(0.000)****	*(0.000)****			
PANEL B: TWO YEARS PRIOR TO BANKRUPTCY FILING										
2		-0.355	-0.071	-0.404			-0.135	(0.000)	10.40	324/1822
		*(0.000)****	*(0.000)****	*(0.116)*			*(0.413)*			
2'		-0.270	-0.062	-0.223	-0.138		-0.294	(0.000)	14.56	324/1822
		*(0.000)****	*(0.003)****	*(0.364)*	*(0.000)*		*(0.081)**			
3	0.334						-0.883	(0.000)	6.79	324/1822
	*(0.000)****						*(0.000)****			
3'	0.234				-0.152		-0.898	(0.000)	12.25	324/1822
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.037	-1.456	(0.000)	6.49	324/1822
						*(0.000)****	*(0.000)****			
5'	0.276					0.028	-0.948	(0.000)	10.46	324/1822
	*(0.000)****					*(0.000)****	*(0.000)****			
PANEL C: THREE YEARS PRIOR TO BANKRUPTCY FILING										
2		-0.409	-0.070	-0.885			0.080	(0.000)	11.86	286/1626
		*(0.000)****	*(0.001)****	*(0.009)****			*(0.667)*			
2'		-0.337	-0.061	-0.707	-0.114		0.078	(0.000)	14.66	286/1626
		*(0.000)****	*(0.003)****	*(0.032)***	*(0.000)****		*(0.673)*			
3	0.299						-0.939	(0.000)	5.48	286/1626
	*(0.000)****						*(0.000)****			
3'	0.213				-0.140		-0.780	(0.000)	10.06	286/1626
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.039	-1.300	(0.000)	5.35	286/1626
						*(0.000)****	*(0.000)****			
5'	0.231					0.028	-0.922	(0.000)	8.08	286/1626
	*(0.000)****					*(0.000)****	*(0.000)****			
PANEL D: FOUR YEARS PRIOR TO BANKRUPTCY FILING										
2		-0.275	-0.043	-0.378			-0.470	(0.000)	6.45	253/1446
		*(0.000)****	*(0.037)***	*(0.275)*			*(0.011)***			
2'		-0.184	-0.036	-0.212	-0.141		-0.637	(0.000)	11.94	253/1446
		*(0.000)****	*(0.094)**	*(0.531)*	*(0.000)****		*(0.001)****			
3	0.286						-0.978	(0.000)	4.39	253/1446
	*(0.000)****						*(0.000)****			
3'	0.184				-0.152		-1.008	(0.000)	10.96	253/1446
	*(0.000)****				*(0.000)****		*(0.000)****			
4'						0.045	-1.420	(0.000)	6.99	253/1446
						*(0.000)****	*(0.000)****			
5'	0.214					0.038	-1.038	(0.000)	9.14	253/1446
	*(0.000)****					*(0.000)****	*(0.000)****			
PANEL E: FIVE YEARS PRIOR TO BANKRUPTCY FILING										
2		-0.248	-0.035	-0.796			-0.575	(0.000)	5.25	203/1242
		*(0.000)****	*(0.147)*	*(0.046)***			*(0.007)****			
2'		-0.167	-0.029	-0.522	-0.133		-0.594	(0.000)	8.83	203/1242
		*(0.000)****	*(0.236)*	*(0.171)*	*(0.000)****		*(0.005)****			
3	0.247						-1.083	(0.000)	3.34	203/1242
	*(0.000)****						*(0.000)****			
3'	0.148				-0.147		-0.971	(0.000)	7.82	203/1242
	*(0.009)****				*(0.000)****		*(0.000)****			
4'						0.046	-1.353	(0.000)	5.46	203/1242
						*(0.000)****	*(0.000)****			
5'	0.144					0.038	-1.099	(0.000)	6.35	203/1242
	*(0.015)***					*(0.000)****	*(0.000)****			

***, **, *: significant at 1%, 5% and 10% level (respectively)

Table VIII
Log-likelihood ratio test of significant differences between full and reduced models

This table presents the log-likelihood ratio (LR) test for significant differences between the reduced and full models (without vs. with extended option variables) using each of the 3 matching procedures for up to five years prior to bankruptcy filing: Panel A: matched pairs (Table IV vs. V); Panel B: random sample (matched by event), and Panel C: control sample with all healthy firms (no matching). LR = [-2 Log L (reduced model)] - [-2 Log L (full model)]. The LR is obtained from the Chi-square distribution $\chi^2(1)$. Its critical values at the 1%, 5% and 10% level are 6.63, 3.84 and 2.71 respectively, so higher values imply rejection of the null (reduced model) in favor of the alternative (full model).

Panel A: Models in Tables IV vs. V (Matched pairs)					Panel B: Models in Table VI (Random sample)					Panel C: Models in Table VII (All healthy/no matching)				
YEAR	Reduced	Full	LR-Statistic		YEAR	Reduced	Full	LR-Statistic		YEAR	Reduced	Full	LR-Statistic	
	1	1'	25.442	***										
	2	2'	31.024	***		2	2'	39.522	***		2	2'	69.499	***
-1	3	3'	51.596	***	-1	3	3'	64.221	***	-1	3	3'	125.201	***
	3	5'	40.518	***		3	5'	17.373	***		3	5'	61.764	***
						4'	5'	78.620	***		4'	5'	93.119	***
	1	1'	31.839	***										
	2	2'	39.449	***		2	2'	71.517	***		2	2'	48.665	***
-2	3	3'	34.989	***	-2	3	3'	78.513	***	-2	3	3'	63.928	***
	3	5'	16.525	***		3	5'	41.176	***		3	5'	43.024	***
						4'	5'	14.186	***		4'	5'	46.530	***
	1	1'	15.084	***										
	2	2'	10.760	***		2	2'	24.876	***		2	2'	29.541	***
-3	3	3'	11.631	***	-3	3	3'	38.691	***	-3	3	3'	48.372	***
	3	5'	8.653	***		3	5'	15.943	***		3	5'	27.422	***
						4'	5'	21.906	***		4'	5'	28.787	***
	1	1'	33.034	***										
	2	2'	24.553	***		2	2'	28.258	***		2	2'	49.882	***
-4	3	3'	24.379	***	-4	3	3'	33.056	***	-4	3	3'	59.719	***
	3	5'	4.096	**		3	5'	25.748	***		3	5'	43.158	***
						4'	5'	4.547	**		4'	5'	19.551	***
	1	1'	0.198											
	2	2'	0.023			2	2'	2.188			2	2'	26.516	***
-5	3	3'	0.157		-5	3	3'	2.702	*	-5	3	3'	33.144	***
	3	5'	0.651			3	5'	2.752	*		3	5'	22.296	***
						4'	5'	10.339	***		4'	5'	6.599	**

***, **, *: significant at 1%, 5%, 10% level (respectively)