Risk Management for International Investment Portfolios using forward contracts and Options*

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Abstract

In this we introduce to the investment opportunity set of the portfolio management models options on stock indices as additional means to control risk exposures. Specifically, we consider two different types of European options on stock indices: unhedged (simple) and fully protected options (quantos). The first type of option has its strike price and payoffs expressed in the same currency as the stock index that constitutes the underlying; hence, it is also priced in the same currency units. This type of option is intended to hedge the market risk associated with the underlying stock index and does not account for changes in currency exchange rates. We use the term "simple option" to refer to stock index options of this type. On the contrary, quantos are options written on a foreign stock index that also protect against currency movements as they apply a prespecified exchange rate to convert the payoffs of the option to a different currency (i.e., the reference currency of the investor). The strike price and payoffs of this type of option are expressed in units of the base currency by using a prespecified exchange rate (usually the forward rate for the term of the option) as the conversion factor. Hence, the underlying of a quanto is the product of the value of the foreign stock index augmented by a fixed exchange rate. A quanto is an integrative instrument as it jointly protects against both the market risk of the stock index as well as the exchange risk between the index and the base currency; it is priced in units of the base currency.

The introduction of these options broadens the investment opportunity set and provides instruments geared towards risk control due to the asymmetric and nonlinear form of option payoffs. We suitably extend the portfolio optimization models so as to incorporate the options and we empirically investigate the ex ante and the ex post impact that these options have on the performance of international portfolios of financial assets. The residual currency risk from other foreign holdings (e.g., in bond indices) can be covered with forward currency exchange contracts that are also included in the models. The

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goal is to control the portfolio's total risk exposure and to attain an effective balance between portfolio risk and expected return. The results indicate that the inclusion of these types of options provides an efficient and effective way to control risk and to improve the performance of international portfolios. In fact, we find that the performance of the international portfolios is improved as we take a progressively integrated view towards total risk management; that is, as we progressively control more risk factors with appropriate hedging instruments.

In this paper we confine our attention to single-period models in order to simplify the notation and the presentation of the novel concepts. The problem addressed here concerns a single portfolio restructuring decision with a single-period planning horizon. The models take as primary inputs the composition of an initial multi-currency portfolio, along with the current prices of the assets, spot currency exchange rates, forward exchange rates for a term equal to the planning horizon, and the option prices. The stochastic elements concern the returns of the assets over the planning horizon - and consequently the asset prices at the horizon - the spot currency exchange rates and the option payoffs at the end of the horizon. The uncertainty in these parameters is depicted by means of discrete distributions (scenario sets).

1 Introduction

Derivative securities are being increasingly used for hedging purposes. Options, due to their asymmetric and nonlinear payoffs, provide the means to protect the value of holdings in an asset in the event of substantial variations in the market price of the underlying security. They can also be used for speculative purposes so as to generate profits in the event of large changes in the value of the underlying. A long position in a put option provides coverage against declines in the market value of the underlying security. Conversely, a long position in a call option generates its payoffs in the event of upside changes in the market value of the underlying. Combinations of options may be used to shape a payoff profile according to the preferences of an investor so as to yield a desired tradeoff between payoff risk and expected return.

An important aspect of risk in the context of our international portfolio management problem is the exposure to market risks in the various countries from holdings of stock indices in these countries. Options on these stock indices provide the means to mitigate these risks. In this paper we consider two types of European options on stock indices. The first type yields its payoffs in the currency of the underlying stock index; the strike price is also expressed in the same currency, as is also the price of the option. Hence, the sole concern of this option - which we call a "simple option" - is to hedge the market risk of the underlying stock index, without any consideration for the currency risk. The second type of option is designed so as to additionally cover the currency risk. Options of this type are known as "quantos"; they are written on a stock index but their strike price and payoff are expressed in units of a different currency (i.e., the base currency of the investor). This is accomplished by applying a predetermined exchange rate (usually the forward currency exchange rate for the term of the option) as the conversion factor. These options are priced and marketed in the base currency and jointly cover against the market risk and currency risk of an investment in a foreign stock index.

The inclusion of the options in the international portfolios is intended to cover the relevant risk exposures from positions in the stock indices. As the portfolio may also involve holdings in foreign bond indices our models additionally allow forward currency contracts in order to hedge the residual currency risk from these investments. Our aim is to control the total portfolio risk and to achieve a desirable balance between portfolio risk and expected return. Obviously, it is more effective to consider all the decisions in an integral manner rather than to separate the investment selection decisions from the risk hedging decisions. Our optimization models serve this purpose as they take a holistic view of the risk management problem for the international portfolios. The models incorporate in a unified framework the decisions for investments in various assets across countries and the positions in the appropriate hedging instruments (i.e., options on stock indices and forward currency exchange contracts). These decisions have typically been considered separately in practice.

A primary contribution of this work is that it unifies in a common optimization framework all the relevant decisions in the international portfolio management problem. The resulting benefits in the performance of international portfolios are demonstrated through empirical tests with real market data.

This study extends the work of Topaloglou et al. [16]. Similar to that paper, the current paper considers a single-period problem. However, the optimization model is extended in several ways. First, we consider a portfolio restructuring model that accounts for the initial composition of the portfolio as well as for transaction costs. Second, the decisions regarding forward currency exchanges are operationalized as the model includes separate decisions to explicitly specify the optimal amounts of forward currency contracts instead of approximating the level of currency hedging through forward exchanges. Most importantly, the novel features of the current models concern the additional inclusion of options on stock indices (domestic and foreign) for risk hedging purposes in the context of international asset allocation. The simple options provide the means to mitigate market risks, while quantos integrally control market and currency risk of holdings in foreign stock indices.

The problem is interesting because the asymmetric option payoffs provide the means to protect against adverse market movements. Hence, the inclusion of options as hedging instruments in international portfolio management models has the potential to provide effective means for risk control. In this paper we develop optimization models for the selection of internationally diversified portfolios including selective hedging decisions in appropriate instruments (options on stock indices and forward currency exchange contracts) in order to control the market and currency risks of the investments. We assess the relative performance of alternative risk hedging strategies through empirical tests. To this end, we also experiment with hedging strategies that impose choices among specific combinations of options in the portfolio.

An essential aspect of the portfolio management model is the specification of uncertainty in the random parameters (i.e., in asset prices, spot exchange rates and resulting option payoffs at the end of the planning period). In the stochastic programming models uncertainty is represented by means of discrete distributions (scenarios) that jointly depict the covariation of these random parameters. We do not impose any assumption concerning the functional form of the distribution of the random parameters. From a statistical analysis of historical data of stock index returns and currency exchange rates we observe that during the period under consideration they exhibited negative skewness and excess kurtosis (see Table 1) Jacque-Bera tests on these data series indicate that the normality hypothesis can not be accepted in most cases. This motivates us to generate scenarios without imposing a specific distribution for the random variables. Instead, we rather generate scenarios based on the empirical distribution of asset returns and exchange rates. The scenarios are generated so that the first four marginal moments and correlations of the random variables match their respective historical values over of the previous 10 years.

The stochastic optimization programs minimize the downside risk of the international portfolio's value at the end of the holding period. They also include a parametric constraint to enforce a minimum target level on the portfolio's expected return. The scenarios of asset returns, exchange rates and corresponding option prices are critical inputs to the optimization programs that determine jointly the asset holdings across markets amd the appropriate risk hedging decisions. Our models employ the conditional value-at-risk (CVaR) metric [12, 13] that minimizes the conditional expectation of portfolio losses above a prespecified percentile of the distribution (i.e., minimizes the expected losses beyond VaR). Our motivation for applying the CVaR risk measure stems from the observation that returns of international assets and proportional changes of exchange rates exhibit asymmetric distributions and fat tails. CVaR is a coherent risk measure, and is appropriate for asymmetric distributions because it aims to control the tail of the distribution of portfolio losses.

The contributions of our study are as follows: First, we develop and implement flexible models for international asset allocation that account for market risk of stock index holdings and currency risk of foreign investments in a unified manner. The models optimally select specific investments in domestic and foreign markets (stock and bond indices) and jointly determine appropriate risk hedging decisions via positions in options on stock indices (simple options or quantos) and forward currency contracts, so as to minimize the downside risk of the entire portfolio. In all previous studies (CITATIONS) in the literature the hedging strategy is either prespecified at the portfolio selection stage, or the hedging decisions are considered separately after the portfolio selection is made. Here all relevant decisions are incorporated in the same optimization model so as to take a holistic view of the problem. Hence, we provide an integrated and more effective tool for managing risk exposures of international portfolios.

Second, we price European options on stock indices and incorporate them in scenario-based optimization models for international asset allocation. The pricing method is general and flexible and does not depend on specific assumptions concerning the distributions of the underlying random variables. The options are priced consistently with the postulated scenarios for asset returns and exchange rates, that in this case reflect skewness and excess kurtosis characteristics as supported by historical data. We note particularly our adaptation of the pricing procedure to the case of quantos (see Appendix 6) and the novel consideration of such instruments in the context of international portfolio optimization.

These tools provide the means to investigate the performance of alternative risk hedging strategies, including popular strategies that enforce specific combinations of options (eg, strangle, straddle, strip, strap). We observe that the inclusion of options in the portfolio can materially reduce the downside risk. The returns of portfolios that include options have significantly lower tails and exhibit (more) positively skewed distributions in contrast to the distribution of portfolios without options.

Our empirical tests reveal that quantos provide particularly effective instruments for risk hedging purposes. This is due to the integrative nature of these instruments that cover both the market risk of the underlying security and the associated currency risk. Overall, we observe that progressively integrated views towards risk management are increasingly more effective. That is, incremental benefits in terms of reducing risk or generating cumulative profits can be gained as more risk factors are progressively controlled through appropriate hedging strategies that involve options and currency forward contracts. Hence, we demonstrate that integrated consideration of market and currency risks can yield substantial benefits for international investors. This result could possibly generalize to other portfolio management contexts that are governed by multiple risk factors.

The rest of this paper is organized as follows. In section 2 we formulate the optimization models for international portfolio selection. Two variants of the optimization model are present. One considers the use of simple options while the other incorporates quantos. Both models also include currency forward contracts. We also discuss the scenario generation procedure. In section 3 we discuss the hedging strategies employed in the empirical tests. In section 4 we describe our computational tests, and we analyze the empirical results. Section 5 concludes the paper. Finally in Appendix 6 we describe the methodology for pricing simple as well as quanto options.

2 The Stochastic Programming Models for Risk Management

2.1 Model description

We view the problem of portfolio (re)structuring from the perspective of a US investor who may hold assets denominated in multiple currencies. We consider, specifically, portfolios composed of one stock index and multiple bond indices in various countries. These portfolios are exposed to market risk in the domestic and foreign markets, as well as to currency exchange risk. To hedge the market risk in stock indices, the investor may buy (a combination of) options on these securities with a payoff that gives him protection against unfavorable movements in the price of the securities. The problem has a single time horizon. The options are of the European type and their maturity marches the planning horizon. To hedge the currency risk, the investor may enter into currency exchange contracts in the forward market. The amount of the forward currency transactions for each currency must be specified at the time of the decision (t=0) and hence, must be a deterministic quantity that can be implemented as a contract drown on the basis of the currently quoted forward exchange rate for the term of the planning period. These forward currency exchange contracts can hedge at least partially - depending on their relative magnitude against the investor's total exposure in the respective currency - the corresponding currency risk.

The scenario-based portfolio optimization models address the market and currency risk in an integrated manner. Their deterministic inputs are: the initial asset holdings, the current prices of the securities, the current spot exchange rates, the forward exchange rates for a term equal to the planning horizon and the option prices. The scenario dependent data that, together with the associated probabilities, specify the distribution of the random variables at the end of the planning horizon are: the final prices of the securities and the spot exchange rates at the end of the horizon. These, in turn, uniquely determine the option payoffs at the end of the horizon under each scenario. The European options can be purchased at the beginning of the time horizon and their maturity match the problem horizon.

The model's decision variables determine the required asset purchase and sale transactions that yield a revised portfolio. Thus, the optimal holdings of specific securities in each currency are completely specified. Additionally, the models jointly determine the optimal hedging decisions. These include the volumes of the forward currency contracts and the positions in the available options. Positions in specific combinations of options - reflecting certain hedging strategies - are easily enforced with appropriate constraints. The optimization models incorporate institutional considerations (no short sales for assets, transaction costs) and minimize the downside risk of final portfolio value at the level of a desirable target expected return. Thus, several interrelated decisions that were previously examined separately, are cast in the same framework with consequent benefits to the investors.

The models have a one stage investment horizon $[0, T]$. In this study the length of the horizon is one month. All decisions - portfolio rebalancing, forwards contracts and positions in options - are taken at the beginning of the period (t=0) taking into account the scenarios that describe the plausible changes in the values of the random variables during the planning period. The objective is to minimize a measure of risk for the total value of the revised portfolio at the end of the planning period [T]. As the risk measure we employ the conditional value-at-risk (CVaR) for portfolio losses at a prespecified percentile of the distribution. We parametrically impose a lower bound on the expected portfolio return.

The cashflow balance constraints in each country, as well as the final value of the portfolio under each scenario, are expressed differently for each type of options that is incorporated in the portfolio. This is because quanto options are issued in the investor's reference currency (USD), and thus the investor does not transfer funds in any foreign currency to purchase these options. Simple options, on the other hand, are issued in each currency. We present separately the models for each type of options that is considered.

We use the following notation:

Definitions of sets:

C_0	set of currencies (markets), including the base (reference) currency,
$\ell \in C_0$	the index of the base (reference) currency in the set of currencies,
$C = C_0 \backslash \{\ell\}$	the set of foreign currencies (i.e., excluding the base currency),
I_c	set of asset classes denominated in currency $c \in C_0$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term bond index in each country),
κ	the ordinal index of the stock index security in a set of assets I_c
N	the set of scenarios,
JS_c	the set of all available simple options in market $c \in C_0$ (differing in terms of their exercise price),
JQ_c	the set of all available quanto options in market $c \in C$ (differing in terms of their exercise price

Input Parameters (Data):

(a). Deterministic quantities:

b_{ic}	initial position in asset class $i \in I_c$ of currency $c \in C_0$ (in units of face value),
h_c	initially available cash in currency $c \in C_0$ (surplus if +ve, shortage if -ve),
T	is the time horizon (in our case 1 month),
δ	proportional transaction cost for sales and purchases of assets,
d	proportional transaction cost for currency transactions in spot market,
μ	prespecified target expected return of the revised portfolio,
α	the prespecified confidence level (percentile) for the CVaR measure,
π_{ic}	current market price (in units of the respective currency) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$,
e_c	current spot exchange rate for currency $c \in C$,
f_c	currently quoted forward exchange rate for currency $c \in C$ for a term equal to the horizon,
$S_{0,c}$	initial (t=0) price of the stock index S_c in currency $c \in C_0$ (in units of the respective currency),
$\bar{X}_c$	the fixed exchange rate for value translation of quanto on the stock index S_c of foreign market $c \in C$ (usually, $\bar{X}_c = f_c, \quad \forall c \in C$,
K_j	the strike price of an option (in units of the respective currency in the case of a simple option $j \in JS_c,\ c \in C_0$, or in units of the base currency in the case of a quanto option $j \in JQ_c,\ c \in C$,
$cs(S_c, K_j)$	price of European simple call option $j \in JS_c$ on stock index S_c of country $c \in C_0$ with exercise price K_j and maturity T,
$ps(S_c, K_j)$	Price of European simple put option $j \in JS_c$ on stock index S_c of country $c \in C_0$ with exercise price K_j and maturity T,
$cq(S_c, K_j)$	price of European quanto call option $j \in JQ_c$ on the underlying $\bar{X}_c S_c$, of foreign currency $c \in C$ with exercise price K_j and maturity T,
$pq(S_c, K_j)$	Price of European quanto put option $j \in JQ_c$ on the underlying $\bar{X}_c S_c$, of foreign currency $c \in C$ with exercise price K_j and maturity T.

(b). Scenario dependent quantities:

p_n obsective probability of occurrence of scenario $n \in N$,
$\bar{p_n}$ a risk-neutral probability associated with scenario $n \in N$
(used in the option pricing procedure),
e_c^n spot exchange rate of currency $c \in C$ at the end of the horizon under scenario $n \in N$,
π_{ic}^n market price (in units of the respective currency) per unit of face value of security $i \in I_c$ at the end of the horizon under scenario $n \in N$,
$S_{n,c}$ price of the stock index S_c in currency $c \in C$ at the end of the planning period, under scenario n.

All exchange rate parameters ($e_c, f_c, \bar{X}_c, e_c^n$) are expressed in units of the base currency per one unit of the foreign currency $c \in C$. Of course, the exchange rate of the base currency to itself is trivially equal to one, $e_\ell = f_\ell = \bar{X}_\ell = e_c^n \equiv 1, \quad \forall n \in N$. The stock index prices S_c are expressed in units of the respective currency $c \in C_0$. The prices cs and ps of simple call and put options respectively, are expressed in units of the local currency $c \in C_0$. Conversely, the prices cq and pq of quanto call and put options, respectively, on foreign stock indices, are expressed in terms of the base currency ℓ.

Computed Parameters:

V_ℓ^0 total value (in units of the base currency) of the initial portfolio

$$V_\ell^0 = h_\ell + \sum_{i \in I_\ell} b_{i\ell}\, \pi_{i\ell} + \sum_{c \in C} e_c \left(h_c + \sum_{i \in I_c} b_{ic}\, \pi_{ic} \right). \tag{1}$$

Decision Variables:

(a). Asset purchase and sale decisions, and resulting holdings after portfolio revision:
- x_{ic} units of security $i \in I_c$ of currency $c \in C_0$ purchased,
- v_{ic} units of security $i \in I_c$ of currency $c \in C_0$ sold,
- w_{ic} units of asset $i \in I_c$ of currency $c \in C_0$ in the revised portfolio.

(b). Currency transactions in the spot market:
- x_c^e units of the base currency exchanged in the spot market for foreign currency $c \in C$,
- v_c^e units of the base currency collected from sale of foreign currency $c \in C$,

(c). Forward currency exchange contracts:
- u_c^f units of the base currency collected at the end of the period from forward sale of foreign currency $c \in C$. This transaction is decided at the beginning of the period (t=0).

(d). Variables related to option transactions:
- $ncs(S_c, K_j)$ purchases of European simple call options $j \in JS_c$ on stock index S_c of currency $c \in C_0$, with exercise price K_j and maturity T,
- $nps(S_c, K_j)$ purchases of of European simple put options $j \in JS_c$ on stock index S_c of currency $c \in C_0$, with exercise price K_j and maturity T,
- $ncq(S_c, K_j)$ purchases of European quanto call options $j \in JQ_c$ on the underlying $\bar{X}_c S_c$ of foreign currency $c \in C$, with exercise price K_j and maturity T,
- $npq(S_c, K_j)$ purchases of European quanto put options $j \in JQ_c$ on the underlying $\bar{X}_c S_c$ of foreign currency $c \in C$, with exercise price K_j and maturity T.

Unlike the holdings of assets for which short positions are not allowed, the variables corresponding to positions in options are not restricted in the optimization models. Hence, short sales of options are permitted as the variables above are allowed to take negative values. Short sales of options can raise additional funds for investments. However, such a case has not been observed in our empirical tests.

Auxiliary variables:

- y_n auxiliary variables used to linearize the piecewise linear function in the definition of CVaR,
- z the VaR value of portfolio losses (at a prespecified confidence level, percentile α),
- V_ℓ^n the total value of the revised portfolio at the end of the holding period under scenario $n \in N$ (in units of the base currency),
- R_n holding-period return of the revised portfolio under scenario $n \in N$,
- $\bar{R}$ expected holding-period return of the revised international portfolio.

We investigate instances of the portfolio management model that incorporate either simple options or quantos, but not both. Hence, we need two slightly different model formulations that differ only in the expression of the cashflows. In the case of simple options, the cashflows (initial purchase of the options and collection of the payoffs at the end of the horizon) occur in the respective currencies. On the contrary, in the case of quanto options, all option-related cashflows (initial purchases as well as payoff collections at the end of the planning period) are in the base currency. Of course, when we use quanto options we also allow the purchase of options on the stock index S_ℓ in the base currency. (A quanto and a simple option on a stock index in the reference currency are of course equivalent, as $\bar{X}_\ell = 1$). We formulate now the two portfolio optimization models.

Portfolio optimization model with simple options

$$\texttt{min} \quad z + \frac{1}{1-\alpha}\sum_{n\in N} p_n y_n \tag{2a}$$

$$\texttt{s.t.} \quad h_\ell + \sum_{i\in I_\ell} v_{i\ell}\pi_{i\ell}(1-\delta) + \sum_{c\in C} v_c^e(1-d) = \sum_{i\in I_\ell} x_{i\ell}\pi_{i\ell}(1+\delta) + \sum_{c\in C} x_c^e(1+d)$$
$$+ \sum_{j\in JS_\ell} [ncs(S_\ell, K_j) * cs(S_\ell, K_j) + nps(S_\ell, K_j) * ps(S_\ell, K_j)] \tag{2b}$$

$$h_c + \sum_{i\in I_c} v_{ic}\pi_{ic}(1-\delta) + \frac{1}{e_c}x_c^e = \sum_{i\in I_c} x_{ic}\pi_{ic}(1+\delta) + \frac{1}{e_c}v_c^e$$
$$+ \sum_{j\in JS_c} [ncs(S_c, K_j) * cs(S_c, K_j) + nps(S_c, K_j) * ps(S_c, K_j)] \quad \forall\, c \in C \tag{2c}$$

$$V_\ell^n = \sum_{i\in I_\ell} w_{i\ell}\,\pi_{i\ell}^n$$
$$+ \sum_{j\in JS_\ell} [ncs(S_\ell, K_j) * max(S_{n,\ell} - K_j, 0) + nps(S_\ell, K_j) * max(K_j - S_{n,\ell}, 0)]$$
$$+ \sum_{c\in C} \Big\{ u_c^f + e_c^n \Big[\sum_{i\in I_c} w_{ic}\pi_{ic}^n + \sum_{j\in JS_c} [ncs(S_c, K_j) * max(S_{n,c} - K_j, 0)$$
$$+ nps(S_c, K_j) * max(K_j - S_{n,c}, 0)] - \frac{1}{f_c}u_c^f \Big] \Big\} \quad \forall\, n \in N \tag{2d}$$

$$\sum_{j\in JS_c} [ncs(S_c, K_j) * cs(S_c, K_j) + nps(S_c, K_j) * ps(S_c, K_j)] \le w_{kc}\pi_{kc},$$
$$\forall\, c \in C_0 \tag{2e}$$

$$0 \le u_c^f \le \sum_{n\in N} p_n (e_c^n \sum_{i\in I_c} w_{ic}\pi_{ic}^n), \quad \forall\, c \in C \tag{2f}$$

$$R_n = \frac{V_\ell^n}{V_\ell^0} - 1, \quad \forall n \in N \tag{2g}$$

$$\bar{R} = \sum_{n\in N} p_n R_n \tag{2h}$$

$$\bar{R} \ge \mu \tag{2i}$$

$$y_n \ge L_n - z \qquad \forall\, n \in N \tag{2j}$$

$$y_n \ge 0 \qquad \forall\, n \in N \tag{2k}$$

$$L_n = -R_n \qquad \forall\, n \in N \tag{2l}$$

$$w_{ic} = b_{ic} + x_{ic} - v_{ic} \quad \forall\, i \in I_c\,, \quad \forall\, c \in C_0 \tag{2m}$$

$$x_{ic} \ge 0 \qquad \forall\, i \in I_c\,, \quad \forall\, c \in C_0 \tag{2n}$$

$$w_{ic} \ge 0 \qquad \forall\, i \in I_c\,, \quad \forall\, c \in C_0 \tag{2o}$$

$$0 \le v_{ic} \le b_{ic} \qquad \forall\, i \in I_c\,, \quad \forall\, c \in C_0 \tag{2p}$$

This formulation minimizes the CVaR risk measure (2a) of the portfolio losses at the end of the horizon, while constraining the expected portfolio return. The derivation of the objective function when minimizing the CVaR metric of portfolio losses was given in the working paper [16].

Equations (2b) and (2c) impose the cash balance conditions in every currency; the former for the base currency ℓ and the latter for the foreign currencies $c \in C$. In each case we equate the sources and the uses of funds in the respective currency. Total availability of cash stems from initially available reserves, revenues from the sale of initial asset holdings and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of cash include the total expenditures for the purchase of assets and options and outgoing currency exchanges in the spot market. Note that the entire budget is placed in the available securities i.e., we don't have investments in risk-free interest rate (T-Bills), nor do we have borrowing. These could be simple extensions of the model. Note also that linear transaction costs are charged for transactions of assets as well as for currencies. In order to simplify the notation and the formulation, all currency transactions are made through the base currency. Without loss of generality, we do not allow direct transactions between foreign currencies. When reducing holdings in one asset in favor of assets in a different currency, the proceeds from the sale of assets are transfered between respective currencies always through an intermediate conversion to the base currency.

The final value of the portfolio under scenario $n \in N$ is computed in (2d). This equation expresses the total terminal value of the portfolio in units of the base currency. The total terminal value reflects the proceeds from the liquidation of all asset holdings at the corresponding market prices, the option payoffs and the proceeds of forward contracts in foreign currencies. The proceeds of investments in foreign markets (assets and option payoffs) are converted to the base currency by applying the respective spot exchange rates at the end of the horizon, after accounting for outstanding currency forward contracts.

Constraints (2e) limit the total expenditure for purchases of simple options in each currency. This expenditure is not permitted to exceed the value of the position in the corresponding stock index. This constraint is imposed in order to ensure that options maybe purchased so as to cover the exposure in the underlying stock index (i.e., for the intended hedging purpose) and not for speculative purposes. Similarly, constraints (2f) restrict the forward contracts that are used to hedge the currency risk to be up to the expected value of all assets in the respective foreign currency. We can exclude currency hedging decisions by eliminating the variables for the currency forwards contracts (i.e, setting $u_c^f = 0, \ \forall c \in C$).

Equation (2g) defines the return of the portfolio at the end of the horizon under scenario $n \in N$. Equation (2h) defines the expected return of the portfolio at the end of the horizon, while equation (2i) imposes a minimum target bound (μ) on the expected portfolio return. Constraints (2j), and (2k) are the definitional constraints for determining `CVaR`, while equation (2l) is the definition of portfolio loss as the negative return. Equation (2m) enforce balance constraint for each asset, in each market. These equations determine the resulting composition of the revised portfolio after the purchase and sale transactions of assets. Short positions in assets are not allowed. Constraints (2n) and (2o) ensure that the units of assets purchased, as well as the resulting units in the rebalanced portfolio are nonegative. Finally, constraints (2p) restrict the sales of each asset by the corresponding initial holdings.

Portfolio optimization model with quanto options

$$\texttt{min} \quad z + \frac{1}{1-\alpha}\sum_{n\in N} p_n y_n \tag{3a}$$

$$\texttt{s.t.} \quad h_\ell + \sum_{i\in I_\ell} v_{i\ell}\pi_{i\ell}(1-\delta) + \sum_{c\in C} v_c^e(1-d) = \sum_{i\in I_\ell} x_{i\ell}\pi_{i\ell}(1+\delta) + \sum_{c\in C} x_c^e(1+d)$$
$$+ \sum_{j\in JS_\ell} [ncs(S_\ell,K_j) * cs(S_\ell,K_j) + nps(S_\ell,K_j) * ps(S_\ell,K_j)]$$
$$+ \sum_{c\in C}\left[\sum_{j\in JQ_c} [ncq(S_c,K_j) * cq(S_c,K_j) + npq(S_c,K_j) * pq(S_c,K_j)]\right] \tag{3b}$$

$$h_c + \sum_{i\in I_c} v_{ic}\pi_{ic}(1-\delta) + \frac{1}{e_c}x_c^e = \sum_{i\in I_c} x_{ic}\pi_{ic}(1+\delta) + \frac{1}{e_c}v_c^e \quad \forall\, c\in C \tag{3c}$$

$$V_\ell^n = \sum_{i\in I_\ell} w_{i\ell}\,\pi_{i\ell}^n$$
$$+ \sum_{j\in JS_\ell} [ncs(S_\ell,K_j) * max(S_{n,\ell}-K_j,0) + nps(S_\ell,K_j) * max(K_j - S_{n,\ell},0)]$$
$$+ \sum_{c\in C}\left[\sum_{j\in JQ_c} \left[ncq(S_c,K_j) * max(\bar{X}_c S_{n,c} - K_j,0) + npq(S_c,K_j) * max(K_j - \bar{X}_c S_{n,c},0)\right]\right]$$
$$+ \sum_{c\in C}\left[u_c^f + e_c^n[\sum_{i\in I_c} w_{ic}\pi_{ic}^n - \frac{1}{f_c}u_c^f]\right] \quad \forall n\in N \tag{3d}$$

$$\sum_{j\in JQ_c} [ncq(S_c,K_j) * cq(S_c,K_j) + npq(S_c,K_j) * pq(S_c,K_j)] \le e_c w_{k_c}\pi_{k_c}, \quad \forall\, c\in C \tag{3e}$$

$$\sum_{j\in JS_\ell} [ncs(S_c,K_j) * cs(S_\ell,K_j) + nps(S_\ell,K_j) * ps(S_\ell,K_j)] \le w_{k\ell}\pi_{k\ell}, \tag{3f}$$

and also (2f), (2j), (2k), (2g), (2h), (2i), (2n), (2o), (2p).

This optimization model differs from the previous one primarily in the constraints that account for the cashflows, as the transactions of the quantos and their payoffs are now in the base currency. Equation (3b) imposes the cash balance condition for the base currency ℓ. Again, total availability of cash stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. The uses of cash include the total expenditures for the purchase of assets and simple options on the base stock index, the purchases of quanto options on foreign stock indices, and outgoing currency exchanges in the spot market. Equation (3c) imposes the cash balance constraints in foreign currencies $c \in C$. In this case, cash is used only for purchases of assets and outgoing currency exchanges in the spot market; in the case of quanto options there are no purchases of options in foreign currencies.

The final value of the portfolio under scenario $n \in N$ is computed in (3c). The total value of the revised portfolio at the end of the horizon accounts for the following: the proceeds from the liquidation of all assets (domestic and foreign) at the respective asset prices, the payoff of the option on the domestic stock index, the payoffs of quantos on foreign stock indices and the currency forwards contract. Again, the proceeds of foreign investments are valued in terms of the base currency by employing the applicable spot exchange rate at the end of the horizon, and after accounting for outstanding currency forward contracts.

Finally, constraints (3e) and (3f) limit the maximum expenditure for quantos in each foreign market, and simple options in the base currency respectively, to the value of the position in the respective stock index.

Quantos are integrative instruments that can hedge the market risk related to the movements of the underlying assets, jointly with the currency risk. In the stochastic programming models developed above, quanto as well as simple option prices are given as inputs. The pricing method used to value these options is given in Appendix.

2.2 Scenario Generation

The scenario generation is a critical step for the entire modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution in the values of the random variables (asset returns and exchange rates) and is consistent with market observations and financial theory. We employ an appropriate scenario generation method that conforms to fundamental financial principles. If the scenario generation does not satisfy non-arbitrage conditions then the solutions of the stochastic programs will be biased and will imply unattainable spurious profits. We generate scenarios such that the arbitrage free conditions are met in the scenario set.

We used the MSCI monthly return data for the stock indices of USA (USS), Great Britain (UKS), Germany (GRS) and Japan (JPS), and for spot and monthly forward exchange rates (UKtoUS, GRtoUS, JPtoUS). We also used return data for Government bond indices from Datastream International. We use Government bonds with three different maturity bands: 1-3 years (US1, UK1, GR1, JP1), 3-7 years (US3, UK3, GR3, JP3) and 7-10 years (US7, UK7, GR7, JP7). We collected 163 months of data that span the time period 05/1988 through 11/2001.

We analyzed the statistical characteristics of the historical data covering the period 05/1988–11/2001. As we can see from Table 1, both the domestic returns of the indices and the proportional changes of spot exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. In addition, several of the random variables exhibit excess kurtosis, implying heavier tails than the normal distribution. Jacque-Bera tests on these data indicated that the normality hypotheses can not be accepted for the majority of them.[1] Thus, we need to generate scenarios for asset returns and exchange rates that comply with historical observations, without relying on the normality assumption.

We rely solely on the observed asset returns and exchange rates in the market. We estimate the first four moments and correlations of the assets returns and proportional changes of exchange rates, and we generate scenarios based on these statistical characteristics.

We used the scenario generation method developed by Høyland and Wallace [6] and Høyland et al. [5]. The scenarios are generated so that the first four marginal moments and correlations of the random variables match their historical values. Applying this scenario generation method on historical values of the random variables we obtain scenarios for the

[1] The Jacque-Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jacque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

Statistical Characteristics of Monthly Domestic Returns of Assets					
Asset Class	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
USS	1.211%	4.097%	-0.413	3.649	6.11
UKS	1.025%	4.254%	-0.018	3.371	0.606
GRS	1.167%	5.807%	-0.561	4.301	13.92
JPS	-0.170%	6.881%	0.017	3.836	2.45
US1	0.567%	0.504%	-0.006	2.765	0.739
US3	0.654%	1.118%	-0.112	2.626	1.028
US7	0.710%	1.670%	-0.090	2.912	0.295
UK1	0.680%	0.714%	0.995	7.461	150.815
UK3	0.741%	1.349%	0.510	4.631	29.45
UK7	0.793%	1.880%	0.134	3.390	1.533
GR1	0.493%	0.480%	0.288	4.451	31.24
GR3	0.552%	0.930%	-0.255	3.162	4.58
GR7	0.583%	1.372%	-0.663	3.887	22.87
JP1	0.283%	0.481%	0.663	4.708	12.98
JP3	0.416%	1.121%	-0.099	4.401	26.37
JP7	0.503%	1.678%	-0.519	5.434	47.62
Statistical Characteristics of Monthly Proportional Spot Exchange Rate Changes					
Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
UKtoUS	-0.116%	2.894%	-0.755	5.672	102.39
GRtoUS	-0.124%	3.091%	-0.215	3.399	5.69
JPtoUS	0.030%	3.615%	0.942	6.213	105.27

Table 1: Statistical characteristics and Jacque-Bera test for normality of historical monthly data of domestic returns of assets and proportional changes of spot exchange rates over the period 05/1988–11/2001.

following quantities:

- r_{ic}^n monthly return (in domestic terms) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$ under scenario $n \in N$,
- g_c^n proportional change of spot exchange rate of currency $c \in C$ over one month under scenario $n \in N$.

The resulting scenarios are equiprobable in our tests, but generally they don't have to be.

Using these scenarios we compute the asset prices $\hat{\pi}_{ic}^n$ and exchange rates $\hat{e}_c^n$ for each scenario $n \in N$ as follows:

$$\hat{\pi}_{ic}^n = (1 + r_{ic}^n) \times \pi_{ic}, \quad \forall i \in I_c, \quad \forall c \in C_0 \quad \forall n \in N \tag{4}$$

$$\hat{e}_c^n = (1 + g_c^n) \times e_c, \quad \forall c \in C, \quad \forall n \in N \tag{5}$$

3 Hedging Strategies

From the statistics of the random variables in Table 1 we observe that the stock indices and the exchange rates exhibit higher volatilities than the bond indices; they also exhibit heavier than normal tails. This justifies our decision to focus in this study on appropriate instruments and strategies for hedging the stock market risks and the currency risks. The portfolio optimization models of the previous section aim exactly at controlling these risks either separately or jointly.

The models can flexibly accommodate alternative investment strategies. By setting the variables $u_c^f = 0, \ \forall c \in C$ we disallow currency hedging through forward exchange contracts. Similarly, by setting all the variables (ncs, nps) associated with purchases of options equal to zero in model 2 we disallow the decisions for hedging the market risks of positions in stock indices. To consider totally unhedged portfolios we eliminate from model 2 all the hedging decisions (i.e., option purchases and currency forward exchanges) by fixing their values equal to zero. In this manner, the optimization models provide the means to study the effect of strategies for controlling market risks and currency risks either integrally or in isolation.

Certain strategies that concern specific combinations of options in hedging applications are popular in practice. Here we consider the following option trading strategies: Straddle, Strip, Strap, and Strangle. All of these strategies involve combinations of long positions in call and put options. Each of them generates a different payoff profile. The choice among them is based on the investor's view regarding potential movements in the value of the underlying and his preferences for protection in the case of such movements.

Straddle strategy

This strategy suits investors who espouse the idea to "exit the market in the face of a storm//. It is designed to provide protection in the event of increased volatility and yields payoffs if there is a substantial movement in the price of the underlying security, either downside or upside. A long straddle consists of long positions in one call and one put option on the same underlying asset, at the same strike price, and with the same expiration date - the length of the planning horizon in our case. The buyer pays a premium on the call and a premium on the put in order to have the right, but not the obligation, to buy, or sell, a predetermined amount of the underlying asset (the number of the option contracts) at the exercise price. An investor enters a long straddle position when he anticipates an increase in volatility but is unsure about the direction of the movement, yet he wishes to be covered in the event of sharp changes in the price of the underlying in either direction. For a long straddle position to yield a profit, the underlying stock index must swing sufficiently low or high so as to cover the total cost of the option purchases. There is a linear profit against the movement of the stock index in either direction once the cost of the option premiums is cleared. If the price of the underlying stock index remains close to the strike price of the options the straddle results in a loss; the maximum loss is the sum of the purchase prices of the call and put options that constitute the straddle. However, in the event of a large movement of the stock index in either direction the straddle yields a substantial profit. Thus, the loss from a potential large decrease in the stock index is recovered from a long position in the straddle.

Strip strategy

A long strip consists of a long position in one call and two puts with the same exercise price and expiration date. An investor enters into a long strip position when he expects a large move in the stock index and considers a decrease in the index more likely than an increase. So, the investor again buys protection against large swings in the underlying stock index but gives preferential weight towards coverage against downward moves. Again, for a limited range of changes around the strike price the strip strategy results in a loss. But for large swings in either direction it yields a positive payoff; the payoff is twice as steep for downward moves than for upward moves.

Strap strategy

In a sense, the strap strategy is a mirror image of the strip strategy. A long strap consists of a long position in two calls and one put with the same exercise price and expiration date. An investor enters into a long strap position when he expects large moves in the stock index but considers an increase in the index more likely than a decrease. Again the investor reaps a profit in the event of a sharp movement of the stock index in either direction, but gives preferential emphasis to gains from potential upside moves.

The constituent options in these three strategies are typically at-the-money options; that is, their strike price is equal to the current value of the underlying index. Of course the closer that the strike price of a call or a put option is to the current value of the underlying asset, the more expensive the option is. Hence, these strategies are rather costly. Obviously, the strip and strap are costlier than the straddle as they involve an additional option.

Strangle strategy

Similar to the straddle, a long strangle consists of a long call and a long put option on the same underlying asset and with the same expiration date; but in a strangle the two options have different exercise prices. The strike price of the put is lower than the current price of the underlying, while that of the call is higher than the current price of the underlying. Again, a long straddle yields a profit when there is a substantial move in the stock index in either direction. The index must move farther in a strangle than in a straddle for the strategy to yield a profit. But the downside risk if there is only a small change in the value of the stock index is less with the strangle than with the straddle because the strangle is a cheaper alternative than the straddle as the prices of its constituent options are lower than those in the strangle. With a long strangle, an investor buys coverage against large movements in the stock index in either direction; that is, he covers against volatility. The payoff pattern resulting with a strangle depends on how close together the strike prices of the constituent call and put options are; if both of these strike prices approach the current price of the underlying stock index then the payoffs of the resulting strangle resemble those of the straddle. The farther apart the strike prices of the constituent options are the lower the cost of the strategy, but the farther the stock index must move for the strategy to realize a profit.

The payoff patterns of the four option strategies are illustrate in Figure 1. We investigate the effectiveness of all these strategies for risk hedging purposes in international portfolio management. In our tests, we design the strangle strategy by setting the strike price of the constituent call option, respectively the price of the put option, to a level 5% higher, respectively 5% lower, than the current value of the underlying stock index.

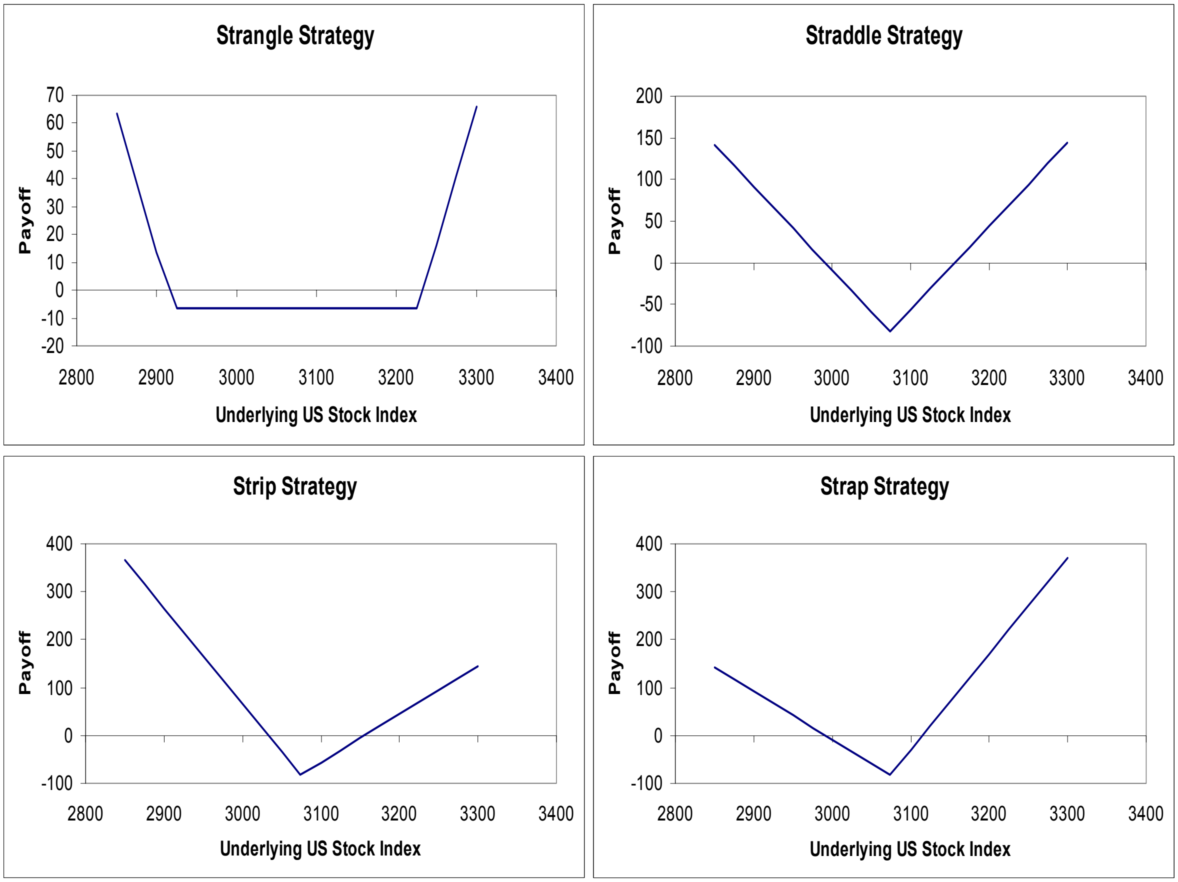

Figure 1: Payoff patterns of the option strategies.

4 Empirical Results

Having implemented appropriate tools to support systematic analysis, in this section we investigate the effectiveness of alternative strategies to control the most influential risk factors for the international portfolios; namely, the market risks of investments in stock indices (domestic and foreign) and the currency risk. We apply the optimization models so as to study the effects of the following investment strategies:

1. Totally unhedged portfolios. Optimal internationally diversified portfolios are determined in this case, but without any explicit regard to cover against either market or currency risk (i.e., neither options nor currency forward exchange decisions are considered).

2. Control of currency risk only by allowing currency forward exchanges.

3. Control of market risks only by incorporating in the portfolio optimization models simple options on the stock indices.

4. Joint protection against market risk with the use of simple options and against currency risk through currency forward contracts.

5. Use of the integrative options (quantos) to jointly protect against both market and currency risks of positions in stock indices, and coverage of the residual currency risk of the other foreign investments in bond indices through currency forward contracts.

The first strategy constitutes the basic benchmark against which the other strategies that incorporate risk hedging decisions are compared in order to determine the incremental benefits from risk hedging decisions. The second and third cases address separately each of the primary risk factors in order to assess the impact of controlling each type of risk in an international portfolio. The last two cases examine the effectiveness of alternative means for addressing both market and currency risks in an integrated manner. By comparing the results in these cases against those for the previous two cases, we can determine the incremental benefit from controlling both types of risk instead of just either of them alone. In the cases that involve consideration of options - either simple options or quantos - we apply the four option trading strategies that were discussed in the previous section. We study the performance of all these investment strategies both in static as well as in dynamic tests.

Previous empirical works on risk hedging strategies for international portfolios of financial assets considered separately the portfolio structuring decisions and the hedging decisions. For example, previous studies on the impact of currency hedging decisions for international investment portfolios have used either a unitary hedge ratio across all currencies, or currency overlays. Jorion [8] analyzes the effects of separating the investment and the currency hedging decisions. This is the case when the asset allocation is performed by one manager and the currency hedging decision is delegated to a currency overlay manager who treats the asset allocation as given ("partial optimization") and optimizes only hedging decisions on currencies. This is the approach typically used in practice. Jorion shows that conducting an asset allocation optimization for assets and currencies separately is clearly suboptimal. This conclusion motivate us to develop models that integrate the asset allocatio with the hedging decisions. Abken and Shrikhande [1] confirm that the efficient frontiers for a US investor choosing stocks and bonds of seven countries is unstable across portfolios and across periods. They show that hedging is the best policy for equities over the period 1980-1985, while no hedging is optimal over the period 1986-1996. They find that the two policies are complementary in the case of bond portfolios. Eun and Resnick [3] indicate that stock portfolios perform better when fully hedged. Moreover, Eun and Resnick [4] further consider international portfolios of stocks, bonds, and stocks and bonds. They show that when exchange rate risk is hedged with forward contracts, the risk-return relationship is improved over unhedged international portfolio investment for bond portfolios and stock and bond portfolios. Perold and Schulman [10] advocate the view that currency hedging is a "free lunch" implying that 100% of foreign currency exposure should be fully hedged. Other researchers propose universal hedge ratios different that one. Recently Beltratti et al. [2] use selective hedging, which is the more general approach, because the hedge ratio may change across currencies and take any value between zero and one. Topaloglou et al. [14] extend the work of Beltrati et al. [2] in that they employ a risk metric that accounts for asymmetric return distributions.

In these studies, the currency hedging strategy is either predetermined at the stage of optimal portfolio selection, or the hedging decisions are taken separately following the selection of an international diversified portfolio. This separate consideration of international portfolio management and risk hedging decisions is also the norm in practice. It should obviously be advantageous to optimally determine these interrelated decisions in a single unified framework, as we do in this study.

To our knowledge, the incorporation of selective hedging decisions for both the market and currency risks in the context of normative models for optimal international portfolio management is an entirely novel contribution. The introduction of options as risk hedging instruments in the portfolio optimization procedure is also novel. Here it is additionally considered, in a unified framework, in conjunction with currency forward exchanges. These novel developments provide effective means to address in an integrated manner the problem of optimally managing internationally diversified portfolios while controlling their total exposure to both market and currency risks.

4.1 Static tests

In these tests we examine the in-sample performance of the alternative investment strategies within a postulated scenario set for the random variables. Hence, we investigate the results of a single portfolio selection problem at a specific point in time. Specifically, we consider the international portfolio structuring problem on March 2001; the results at this specific time are typical of the performance of the alternative investment strategies. In the static tests we consider portfolio selection problems that start with a cash endowment in the base currency (USD) only; the initial portfolio does not have any holdings in any other asset.

The setup of the numerical experiments is as follows. The historical data of monthly asset returns (in domestic terms) and corresponding proportional changes in spot exchange rates over the previous ten years (120 observations) are used to determine the statistics of these random variables (i.e., their first four marginal moments and their correlations). These statistics are used to calibrate the scenario generation procedure. Using the moment matching scenario generation procedure of Høyland et al.[5] (see also Høyland and Wallace[6]) we generate 15000 scenarios that capture in terms of a discrete distribution the joint variation of these random variables. On the basis of these scenarios we compute the corresponding payoffs for the simple options and for the quantos under each scenario. The procedures described in Appendix are then applied to price the corresponding options. In this manner we have all the necessary inputs for the stochastic optimization models of the previous section.

For each of the investment strategies we described at the beginning of this section, we solve the corresponding parametric stochastic optimization model for several different values of target expected return () and we record the results. Thus we trace the risk-return tradeoff profile (efficient frontier) of each strategy for the postulated scenario set. As a measure of risk we use the CVaR of portfolio losses at the 95% threshold (or should it be 5% level since we refer to losses instead of returns?).

Figure 2 presents the efficient frontiers of optimal international portfolios of stock and bond indices for several investment strategies. We observe that the efficient frontier of totally unhedged portfolios is dominated by all the other efficient frontiers that correspond to risk hedging strategies. Hence, improvements in the risk-reward tradeoffs - in terms of higher expected return for the same level of risk, or conversely, reduced risk for the same level of expected portfolio return - are attained when risk hedging strategies are incorporated in the international portfolio optimization model. This holds for all risk hedging strategies we tested. For the strategies that employ options on stock indices (either simple options or quantos) we present in Figure 2 only the results for the strangle strategy. As we demonstrate subsequently, the strange strategy proved the most effective among the four option trading strategies we tested.

We note that the strategies that address the market risk of stock indices (via simple options or quantos) result in an upward rotation of the efficient frontier. The minimum risk portfolio is anchored at the same point in these cases. This is because the optimal minimum

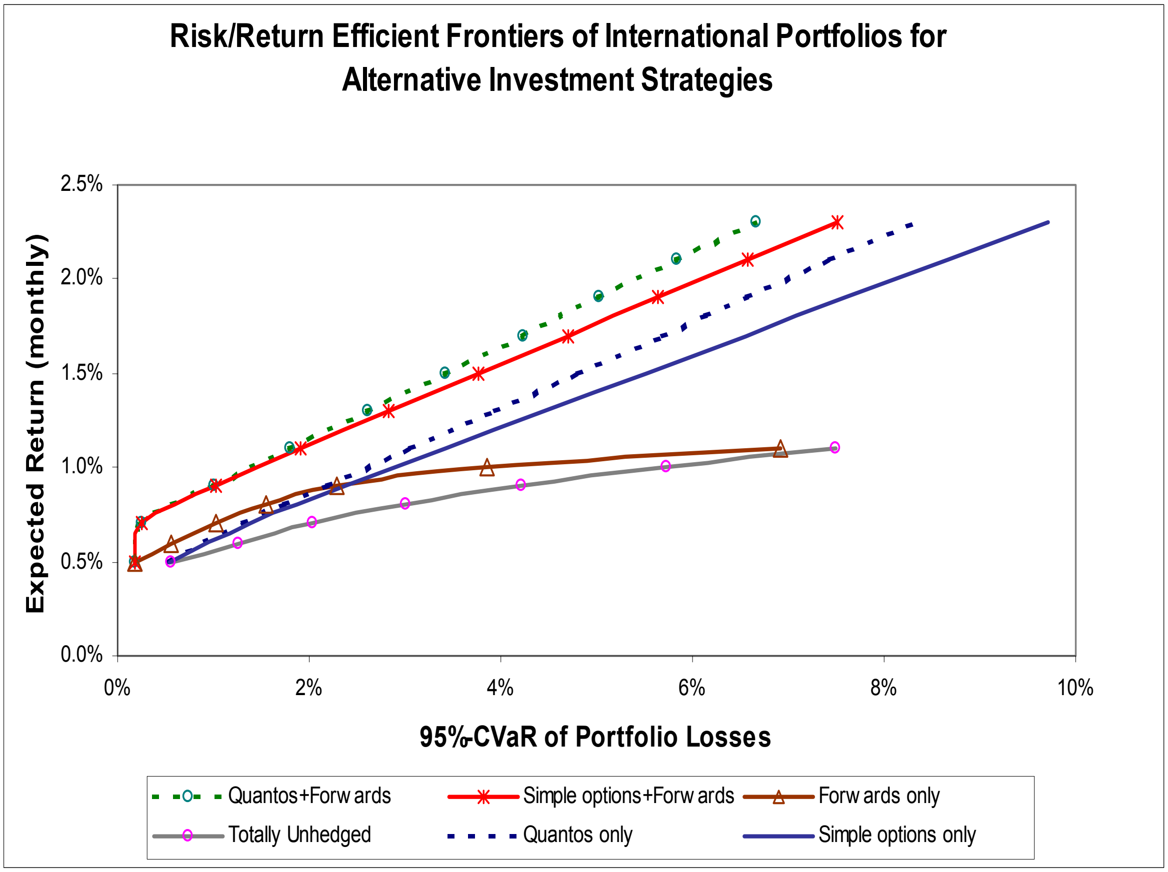

Figure 2: Efficient Frontiers of CVaR-optimized international portfolios of stock and bond indices with alternative risk hedging strategies.

risk portfolio solely contains holdings in bond indices, and thus the options do not enter the picture as there are no holdings in the stock indices. However, as higher expected returns are targeted the optimal portfolios gradually switch their positions to stock indices, and in these cases the incorporation of options in the optimal portfolios has a pronounced effect. By controlling the market risk of positions in stock indices through the optimal use of options, significantly higher expected returns can be achieved for the same level of total portfolio risk.

When viewing the impact of risk control of the two major risk factors (market risk of stock indices and currency risk) in isolation, we observe that controlling the market risk is increasingly important for the more aggressive strategies that target higher expected returns. However, for conservative investors controlling the currency risks of all foreign investments yields important benefits. The efficient frontier is shifted to the left when currency risks are addressed through the use of currency forward exchanges, thus reducing the total portfolio risk at each level of target expected return. This includes the minimum risk portfolio as well, as currency forward exchanges are used to cover the currency risk of holdings in foreign bond indices in this case.

The efficient frontiers of strategies that jointly address both market risks of stock index positions through options and currency risks of all foreign investments through currency forward exchanges exhibit the combined effect of the strategies that control just one of these risk factors; that is, both the leftward shift and the upward rotation of the efficient frontiers. Hence, the joint consideration of the market and currency risks in an integrated manner yields significant incremental benefits over the consideration of either of these risk factors in isolation. Lastly, we observe in Figure 2 that the use of quantos produces better results than the use of simple options to cover the market risks of stock holdings. This holds true regardless of whether or not currency forward exchanges are used to cover the currency risks of foreign investments, and the benefits are higher in the range of higher expected portfolio return (i.e., for more aggressive investments). The reason is that although both types of options protect against the market risks of stock indices, the quantos additionally protect the investments in these indices against the currency risks. As the proportion of holdings in the stock indices is increased in more aggressive portfolios, the incremental benefit from this additional risk coverage becomes increasingly evident.

Figure 3 contrasts the efficient frontiers of strategies that incorporate alternative combinations of options in the portfolios (i.e., staddle, strip, strap and strangle). This figure presents the results for optimal portfolios with simple options as well as those with quanto options. In these tests, selective forward currency exchanges are also used to cover the currency risks. The results indicate that the incorporation of options in the investment opportunity set leads to optimal portfolios with improved efficient frontiers in comparison to unhedged portfolios, regardless of the option trading strategy that is used. Hence, the use of options as hedging instruments in the international portfolios results in substantial improvement of their risk-return profiles. The strangle strategy produces the highest benefits.

Lastly, we examine the effects of option strategies on the distributional characteristics of international portfolios. Figure 4 shows the in-sample distributions (i.e, with respect to the postulated scenario set that is used in the optimization models) of returns of optimal portfolios that employ specific risk hedging strategies. Specifically, this figure shows the in-sample distributions of returns for optimal unhedged portfolios, and optimal portfolios that use quanto options and forward currency exchanges to control total risk exposure. We note that the minimization of the CVaR risk metric on portfolio losses - that is used as the objective function in the optimization models - aims to generate portfolios that minimize the downside risk; that is, that have a reduced lower tail of their return distribution. The results of Figure 4 highlight exactly two distinguishing characteristics of the optimal optioned portfolios in comparison to optimal unhedged portfolios, regardless of the option trading strategy: (a) the substantial reduction of the downside risk of the portfolio, as evidenced from the reduced left tail of the return distribution, and (b) the more positively skewed return distribution of optimal optioned portfolios. These characteristics are consequences of the asymmetric payoff profiles of the option trading strategies. The protection of the put options against downside risk result in the reduction of the lower tail of the portfolios' return distribution. Similarly, the call options included in the strategies tested induce upside potential as they produce payoffs in the event of upside market moves. These contribute, at least partly, to the generation of more positively skewed return distributions. The strangle strategy produces somewhat more positively skewed portfolio return distributions than the other option trading strategies.

4.2 Dynamic tests

So far we have studied the performance of alternative investment strategies in static tests. That is, we have examined the potential benefits of risk hedging strategies at a single (typical) point in time and against an in-sample discrete distribution (scenario set). Although these tests provide useful insights, they are not sufficient to convincingly compare the relative effectiveness of the alternative strategies for managing international portfolios of financial assets. To this end, in this section we conduct extensive dynamic tests in order to have more substantive comparisons of actual performance. These tests repeatedly apply the models in backtesting experiments using real market data on a rolling horizon basis during the period 05/1998 to 11/2001 (i.e., 43 months).

The simulations start on May 1998 with an initial cash endowment in the investor's reference currency (USD); the initial portfolio does not contain holdings in any other assets. At each month we use the historical observations of the asset returns and spot exchange rates during the previous 10 years in order to calibrate the scenario generation procedure as we explained in the previous section. The moment matching method is then applied to produce 15000 scenarios of asset returns (in domestic terms) and corresponding differentials of spot exchange rates. These scenario-dependent data are used to compute the projected final asset prices, the spot exchange rates and the respective option payoffs at the end of a one-month planning horizon under each scenario. The options are priced on the basis of the postulated scenario set. The scenario-based data are used in conjunction with the observed asset prices, spot and forward exchange rates and the option prices as inputs in the optimization model. The optimization model is solved and the optimal portfolio composition, as well as the forward currency transactions and option purchases, when applicable, are recorded. The clock is then advanced one month, and the actual asset prices and spot exchange rates are revealed. On the basis of these new observations, accounting calculations are carried out so as to settle outstanding investment decisions of the previous month. That is, the currency forward exchanges are settled, as well as potential execution of positions in options, and the resulting cash positions are updated accordingly. Starting with the new portfolio, the procedure is repeated for the next month. A new set of scenarios is computed and the optimization model is resolved using the data of the new scenario set and the new portfolio composition. The ex post realized returns are cumulatively recorded throughout the simulation that runs through November 2001.

These backtesting experiments were executed for several investment strategies that were discussed earlier. In these tests we use a target monthly return ($\mu = 1\%$) so as to induce the optimization models to include positions in stock indices. For conservative targets of expected returns (e.g., the minimum risk case with $\mu = 0\%$) the optimal portfolios are almost exclusively placed in bond indices and thus the options would have no effect.

Figure 5 contrasts the ex post realized performance of optimal portfolios for alternative risk hedging strategies. We observe that the totally unhedged portfolios exhibit considerable volatility, as well as declining performance from the beginning of the year 2000. When only currency risk is hedged with forward exchanges the portfolios achieve a much more stable performance; in this case, the portfolios generally produce small stable returns. A severe loss, is experienced only during the crisis of September 11, 2001, but is followed by a quick recovery. So, the consideration of only the currency risk results in much more stable returns, but does not yield substantial incremental profits, at least over this simulation period. On the contrary, we observe substantial additional profits from the outset and throughout the entire simulation period when options are introduced so as to manage the market risks of stock index positions, in addition to currency risk hedging through forward currency contracts. Thus, the impact of the market risk management strategies is substantial. This should be expected as the stock indices exhibit the highest volatility of all investment instruments in the portfolios. The results demonstrate the clear benefits from the joint management of both the market risks and the currency risks. The optimally hedged portfolios again suffer their most severe loss at the time of the September 11, 2001 crisis, but again they recover quickly.

We observe that quantos in conjunction with currency forwards that cover, at least partly, the residual currency risk of investments in bond indices in the portfolios achieve slightly better ex post performance than the portfolios with simple stock options and currency forwards. The difference between the two strategies is small because the investments in stock indices that are picked by the model are much higher in the US stock index (i.e., the base currency) for which currency risk does not play a role, than in foreign stock indices. Their small difference is due to some positions in quantos to cover exposure to the German stock index.

The results indicate that the rewards from the option payoffs, especially in the event of volatile markets, more than offset the cost for buying these options for risk hedging purposes. Note, for example, how the performance of the portfolios is tempered during the periods at which the corresponding unhedged portfolios suffer substantial losses. Moreover, the option trading strategies that have been tested in this study produce additional rewards in the event of large upside moves of the stock indices because of the long positions in call options. Hence, the gains in cases of significant market upswings can be further accentuated.

Figure 6 compares the performance of optimal optioned portfolios corresponding to the four option trading strategies. Currency risk is hedged through forwards in these tests. Clearly, the portfolios that contain options on stock indices - whether simple options or quantos - outperform the portfolios that do not contain options (i.e., do not hedge market risks). These results confirm that the inclusion of options as hedging instruments in the portfolios leads to improved performance, irrespective of the option selection strategy that is employed, as the static tests had implied Among the alternative option trading strategies, strangle yields the best performance. The relative performance of the alternative investment strategies in the backtesting experiments conforms to their respective relative performance in the static tests of the previous section.

The results in Figure 5 and 6 show that unhedged portfolios achieve the worst performance in the backtesting simulations (lowest cumulative return and highest volatility). Benefits are attained with strategies that control risk exposures, by leading to more stable return paths or, additionally, to higher realized returns. The introduction of options on stock indices, as means to manage market risks, clearly produces substantial benefits in terms of higher cumulative returns. The additional use of currency forwards so as to hedge currency risks yields further incremental benefits and more stability in the realized returns. The overall observation is that superior performance is achieved when both market and currency risks are jointly controlled through appropriate instruments. Portfolios with the integrative options (quantos) demonstrate better performance than corresponding portfolios with simple options on the stock indices, when currency risks are also hedged. These results confirm the anticipated improvements in the performance of international portfolios of financial assets when appropriate strategies for controlling market and/or currency risks are incorporated in the decision process. Moreover, the results illustrate the benefits achieved from the joint control of market and currency risks, and thus demonstrate the advantages of models that take a holistic view towards addressing the total risks from all factors in international portfolio management within a single integrative framework.

We also look at the total volume of portfolio turnovers - expressed in Table 2 in terms of the total transaction cost (in USD) for asset purchases and sales (excluding options) during the simulation period 05/1998 - 11/2001. We observe that the total turnover of optimal optioned portfolios is lower than with portfolios without options, indicating a greater stability in the portfolio compositions over time. We have used the following values for the transaction cost parameters: transaction cost for asset purchases and sales $\delta = 0.05\%$, transaction cost for spot currency exchanges $d = 0.01\%$.

Portfolios	Without options	Simple options	Quanto options
Turnover	15359	14411	14117

Table 2: Total cost for asset transactions (in USD) during the simulation period (05/1998-11/2001)

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Figure 8 shows the degree of currency hedging in each country (% of foreign investments hedged) when the strangle strategy is used. We observe that when the model decides to invest in a foreign market then most of the times the model also (almost) fully hedges the currency risk of these investments.

Finally, we examine how the results may be affected by the decision maker's risk aversion preferences. We use the parameter for the target expected return () as a proxy measure of risk aversion. Higher levels of target return imply lower risk aversion and lead to more aggressive investments (i.e., selection of riskier stock indices instead of bond indices because of their higher return potential). All previous tests in this section used a monthly return target =1.0% in the optimization models, which is a rather aggressive target. Figure 9 shows the realized returns of optimal portfolios for different values of target return. These portfolios use quanto options (with the strangle strategy) and currency forwards to control the market and currency risks, respectively.

For the target return μ=1%, the portfolios in the period of August-September 2001 involved a large position in the US stock index and smaller positions in intermediate- and long-term US government bond indices. Given the large position of the portfolio in the US stock market, it suffered a substantial loss at the time of the September 11 events, even though this loss was subsequently recovered quickly. On the contrary, for more conservative values of the target return the portfolios hold substantial proportions in government bond indices throughout the simulation period and only occasionally include small positions in stock indices. For the minimum risk case (μ=0%) the portfolio at the time of September 11, 2001, included a large allocation in the short-term US government bond index, smaller allocations in the German and Japanese stock indices, and some other smaller positions in other assets. As the returns of the bond indices were not materially affected by the events of September 11, this portfolio escaped any losses at that time, and even achieved gains during and after those events, as can be seen in the results of Figure 9. We observe that the minimum risk portfolios (μ=0%) exhibit the more stable returns; generally higher returns are achieved in more aggressive cases (i.e., as the value of target returns is increased) but at the expense of higher volatility.

5 Conclusions

In this study we developed and implemented scenario-based optimisation models to control market and currency risk exposures of portfolios of international stock and bond indices. We employed a moment matching procedure to generate scenarios for the underlying random variables (index returns and exchange rates); the scenario generation procedure was calibrated on the basis of historical observations.

Options on the stock indices served as the instruments for hedging market risks, while currency forwards served as the means for controlling currency risks. Additionally, quanto options were considered in order to jointly cover market and currency risk exposures of positions in stock indices. In order to incorporate the options in the stochastic programming models we adapted an option pricing procedure so as to price the options consistently with the postulated scenarios for the underlying assets, while at the same time satisfying the fundamental no-arbitrage condition.

The optimization models provided the necessary tools so as to investigate alternative investment strategies for managing risks in international portfolios. We studied the performance of alternative strategies in extensive empirical tests, both in a static as well as in a dynamic setting.

Empirical results indicate that the market risk affects more than the currency risk the performance of the portfolios, and hedging this risk using options is extremely beneficial. Both static as well as dynamic tests show that increasingly integrated views towards risk are more effective compared to unhedged positions. We observe that hedging both the market and currency risk really does pay. Regardless of the trading strategy of options that is selected, the incorporation of options in international portfolios improves significantly the performance of these portfolios. Although, it is not clear enough which of the alternative options (simple of quantos) is indeed more preferable, since their performance is almost the same, in all cases quantos exhibit slightly better results than simple options, and never worse. Thus, integration of market and currency risk into portfolios hedging strategies does pay. Finally, among the trading strategies of options, portfolios with strangle strategy yields the best ex ante and ex post performance.

Extensions to this approach are possible to allow portfolio managers to adjust this procedure to their specific investment circumstances. First of all, the choice of the risk measure that is used as well as the choice of the scenario generation procedure, are made for illustration purposes. The modelling framework we develop in this paper is not restricted to these choices. Alternative risk measures or utility functions, can be optimized, or they can be constrained in the model. Moreover, alternative scenario generation procedures to moment matching can be employed.

We can price and incorporate currency options to hedge the currency risk, together with options on stock indices. Further restrictions, like liquidity constraints, can be easily incorporated into the model.

The next step in this work is to incorporate derivative securities in multi-stage stochastic programming models. In that case, investors have to price new options in every stage with different maturities and exercise prices, and decisions must be made, concerning sales or purchases of all instruments, including options.

6 Appendix: Pricing of options

To price the options we use the methodology developed in the working paper [15].

6.1 Pricing simple options on stock indices

This type of option is relevant for the investor who wants to invest in a stock index, desires protection against losses in that asset, but is unconcerned about the translation risk arising from a potential drop in the exchange rate, in case this is an option on a foreign stock index. So, the investment in this option aims to cover market, but not currency risk. To price these options, we adopt the view point of the local-based option writer. The price of the option as well as the payoff, are in units of the respective currency. The option's pay-off under scenario n is:

$$CS^n(S_c, K_j) = max[S_{n,c} - K_j, 0], \quad \forall c \in C_0, \quad j \in JS_c \tag{6}$$

where K_j is the strike price in units of the respective currency. The underlying asset for this option is the stock index S_c. We assume that stock indices do not pay dividends.

We use the procedure described above to determine the set of risk neutral probabilities $\bar{p_{cn}}'$ associated with the set of scenarios, for each stock-index separately in the respective country. Once we find the risk-neutral probabilities, we can price the options. The "fair" value of the call option today, is the expected payoff in the risk-neutral measure of the option over all scenarios , discounted by the local riskless rate:

$$cs(S_c, K_j) = e^{-r_c T} \sum_{n=1}^{N} \bar{p_{cn}}'[max(S_{n,c} - K_j, 0)] \quad \forall c \in C_o, \quad j \in JS_c \tag{7}$$

And for put options, again we discount the expected payoff by the riskless rate. That is:

$$ps(S_c, K_j) = e^{-r_c T} \sum_{n=1}^{N} \bar{p_{cn}}'[max(K_j - S_{n,c}, 0)] \quad \forall c \in C_o, \quad j \in JS_c \tag{8}$$

6.2 Pricing quanto options on foreign stock indices

A quanto, often known by the more formal name of guaranteed exchange rate contract, is effectively a foreign exchange contract incorporated into an underlying foreign equity option which allows the investor to lock in a prespecified foreign exchange rate and eliminate currency risk. Thus, quantos are contracts that control market and currency risk in an integrated manner. This is particularly useful if the investor fears currency devaluation or when he believes that spot rates will be lower than forward rates suggest. Typically, the forward rate for term equal to option's maturity is used as the predetermined exchange rate. Reiner [11], Jamshidian [7] and Kat and Roozen [9] value quanto contracts that involve both foreign exchange and stock price risk, when both the underlying asset and the exchange rate follow Geometric Brownian Motion.

The investors' pay-off under scenario n, in units of the base currency now, for a call option, is that of a foreign equity call times a fixed exchange rate:

$$CQ^n(S_c, K_j) = \bar{X}_c max[S_{n,c} - K_j, 0] = max[S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0]$$

$\bar{X}_c$ is the rate at which the translation will be made and the two equivalent forms of the pay-off arise from the choice of expressing the strike in foreign or domestic terms (obviously the strike K_j is in units of foreign currency). We choose $\bar{X}_c$ to be the forward exchange rate with the same maturity T. The underlying asset for this option is the foreign stock price multiplied by the fixed exchange rate, i.e., $S_c\bar{X}_c$.

To price the quanto options, we discount by the riskless rate of the base currency this time, the expected payoff of the option over all scenarios, in the risk-neutral measure. In this case, we find the risk-neutral distribution $\bar{P}'$ for the US based investor (under which all underlying assets, $S_c\bar{X}_c, \forall c \in C$, are martingales) and we price the quanto options on each one of the foreign stock indices using these probabilities. All the cashflows, prices and payoffs are in the base currency. Thus, for a call quanto:

$$cq(S_c, K_j) = e^{-rT}\sum_{n=1}^{N}\bar{p}'_n[max(S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0)], \quad \forall c \in C, \quad j \in JQ_c \tag{10}$$

And for a put quanto:

$$pq(S_c, K_j) = e^{-rT}\sum_{n=1}^{N}\bar{p}'_n[max(S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0)], \quad \forall c \in C, \quad j \in JQ_c \tag{11}$$

References

[1] P.A. Abken and M.M. Shrikhande. The role of currency derivatives in internationally diversified portfolios. *Federal Reserve Bank of Atlanta Economic Review*, Third Quarter:34–59, 1997.

[2] A. Beltratti, A. Laurent, and S.A. Zenios. Scenario modeling of selective hedging strategies. *Journal of Economic Dynamics and Control*, (forthcoming).

[3] C.S. Eun and B.G. Resnick. Exchange rate uncertainty, forward contracts and international portfolio selection. *Journal of Finance*, 43(1):197–215, 1988.

[4] C.S. Eun and B.G. Resnick. International diversification of investment portfolios: U.S. and Japanese perspectives. *Management Science*, 40(1):140–161, 1994.

[5] K. Hoyland, M. Kaut, and S.W. Wallace. A heuristic for generating scenario trees for multistage decision problems. *Computational Optimization and Applications*, 24:169–185, 2003.

[6] K. Hoyland and S.W. Wallace. Generating scenario trees for multistage decision problems. *Management Science*, 47(2):295–307, 2001.

[7] F. Jamshidian. Option and futures evaluation with deterministic volatilities. *Mathematical Finance*, 3:149–159, 1993.

[8] P. Jorion. Mean-variance analysis of currency overlays. *Financial Analyst Journal*, May/June:48–56, 1994.

[9] H.M. Kat and H.N.M Roozen. Pricing and hedging international equity derivatives. *Journal of Derivatives*, 2:7–19, 1994.

[10] A. Perold and E. Schulman. The free lunch in currency hedging: Implications for investment policy and performance standards. *Financial Analysts Journal*, 44:45–50, 1988.

[11] E. Reiner. Quanto mechanics. *RISK*, 3:59–63, 1992.

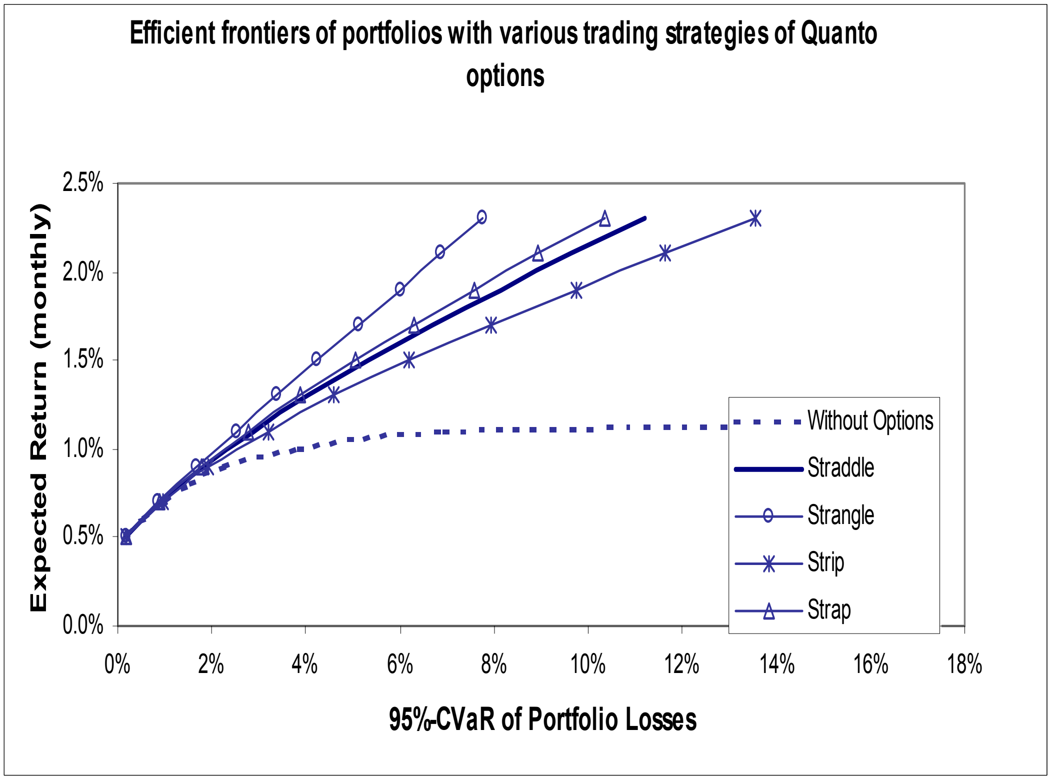
[12] R.T. Rockafellar and S. Uryasev. Optimization of Conditional Value-at-Risk. *The Journal of Risk*, 2(3):21–41, 2000.

[13] R.T. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general distributions. *Journal of Banking and Finance*, 26(7):1443–1471, 2002.

[14] N. Topaloglou, H. Vladimirou, and S.A. Zenios. CVaR models with selective hedging for international asset allocation. *Journal of Banking and Finance*, 26(7):1535–1561, 2002.

[15] N. Topaloglou, H. Vladimirou, and S.A. Zenios. Incorporating derivative securities in stochastic programs. Technical report, "Hermes" European Center of Excellence on Computational Finance and Economics, School of Economics and Management, University of Cyprus, Nicosia, Cyprus, 2003.

[16] N. Topaloglou, H. Vladimirou, and S.A. Zenios. Stochastic programming models for international portfolios. Technical report, "Hermes" European Center of Excellence on Computational Finance and Economics, School of Economics and Management, University of Cyprus, Nicosia, Cyprus, 2003.



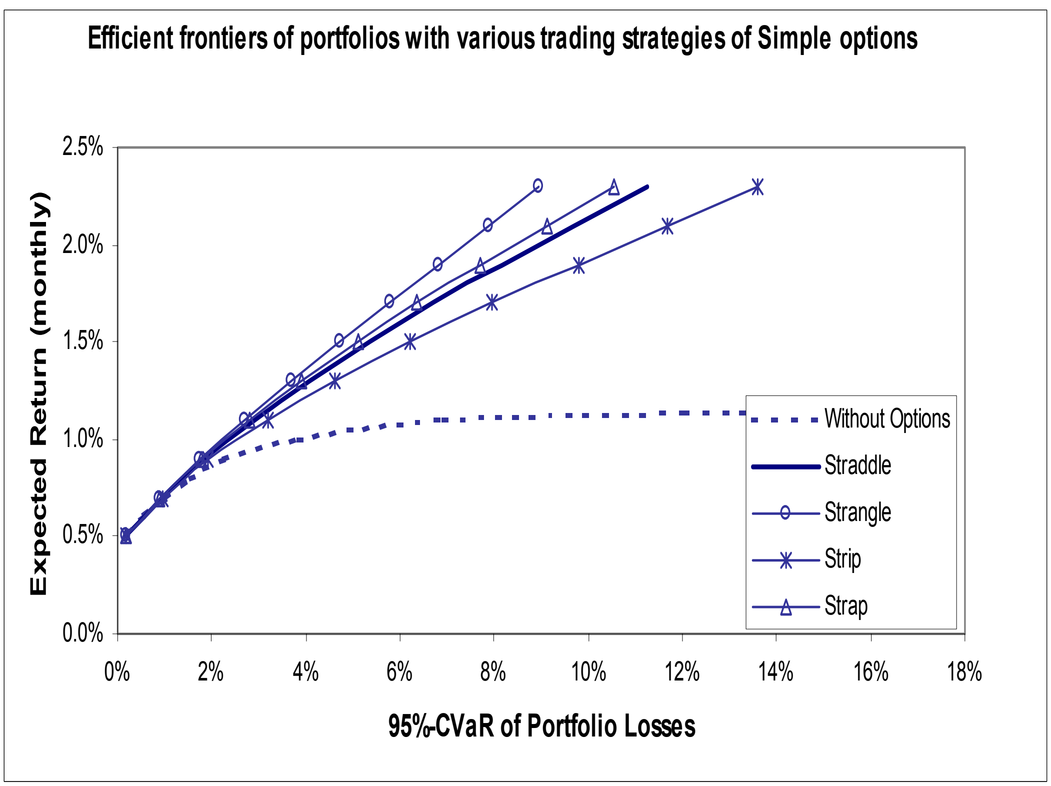


Figure 3: Efficient frontiers of CVaR-optimized international portfolios of stock and bond indices with alternative strategies for simple options and quantos.

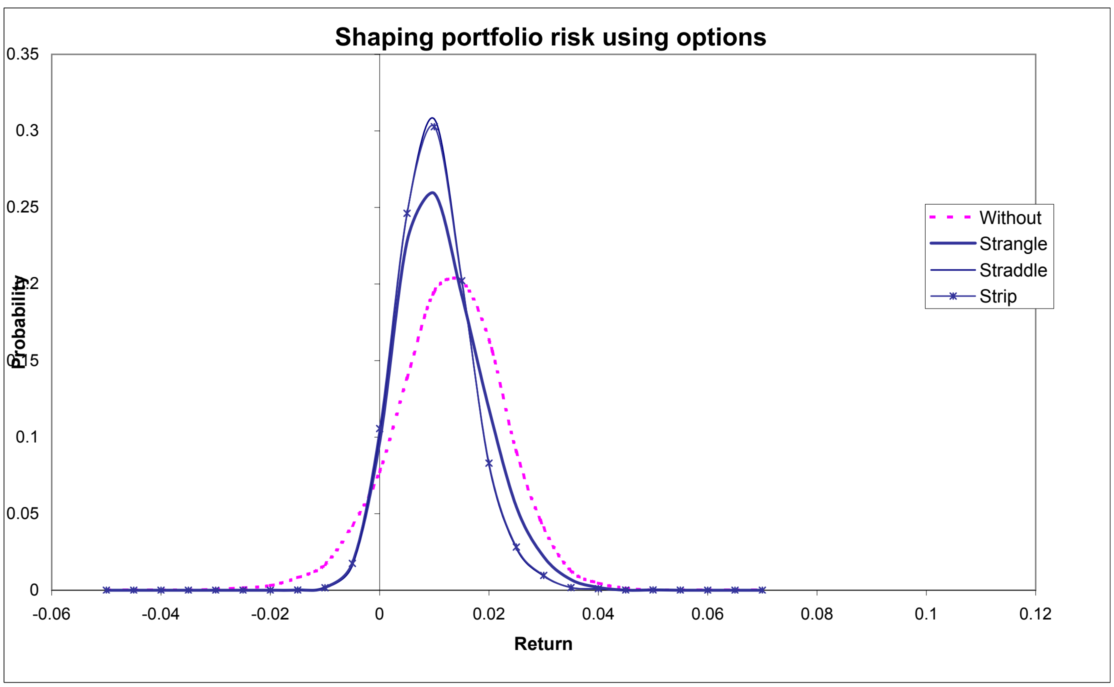


Figure 4: Comparison of portfolio return distributions of portfolios without options vs portfolios with alternative option trading strategies (March 2001).

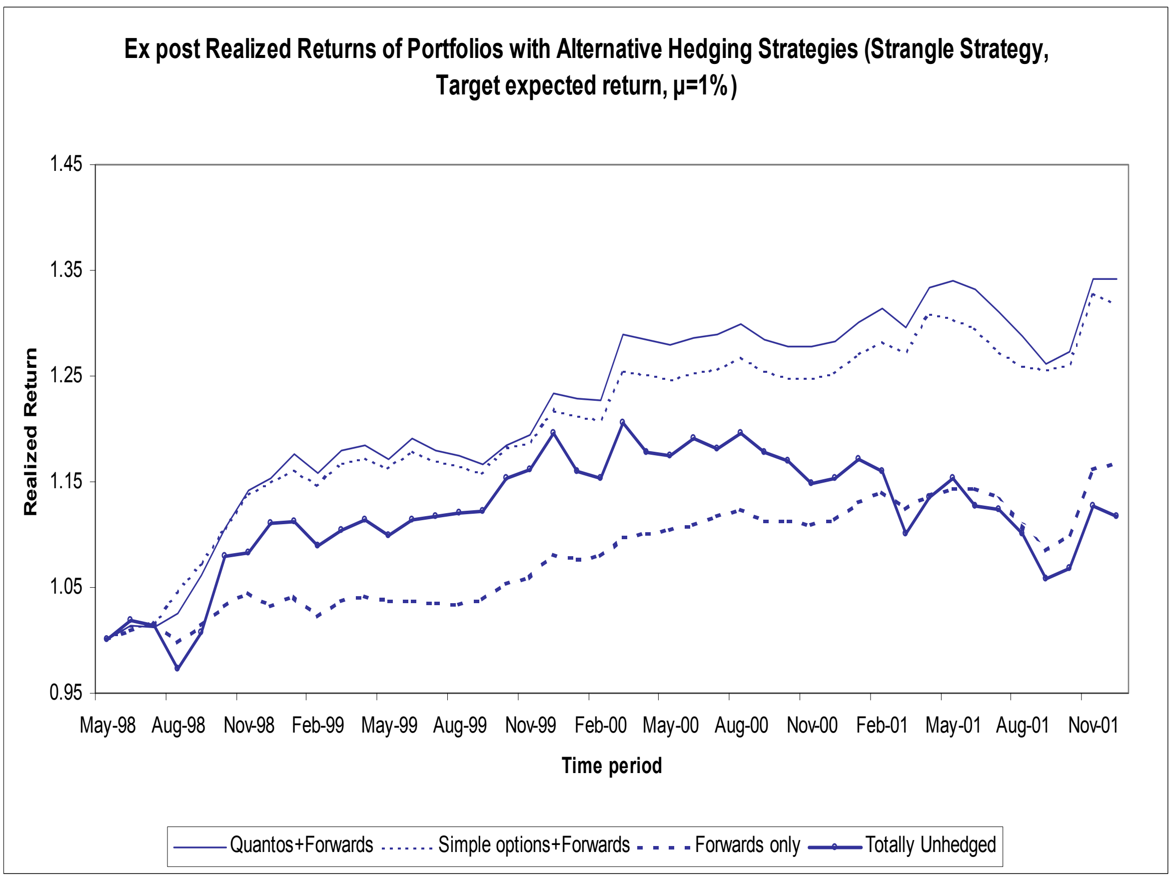
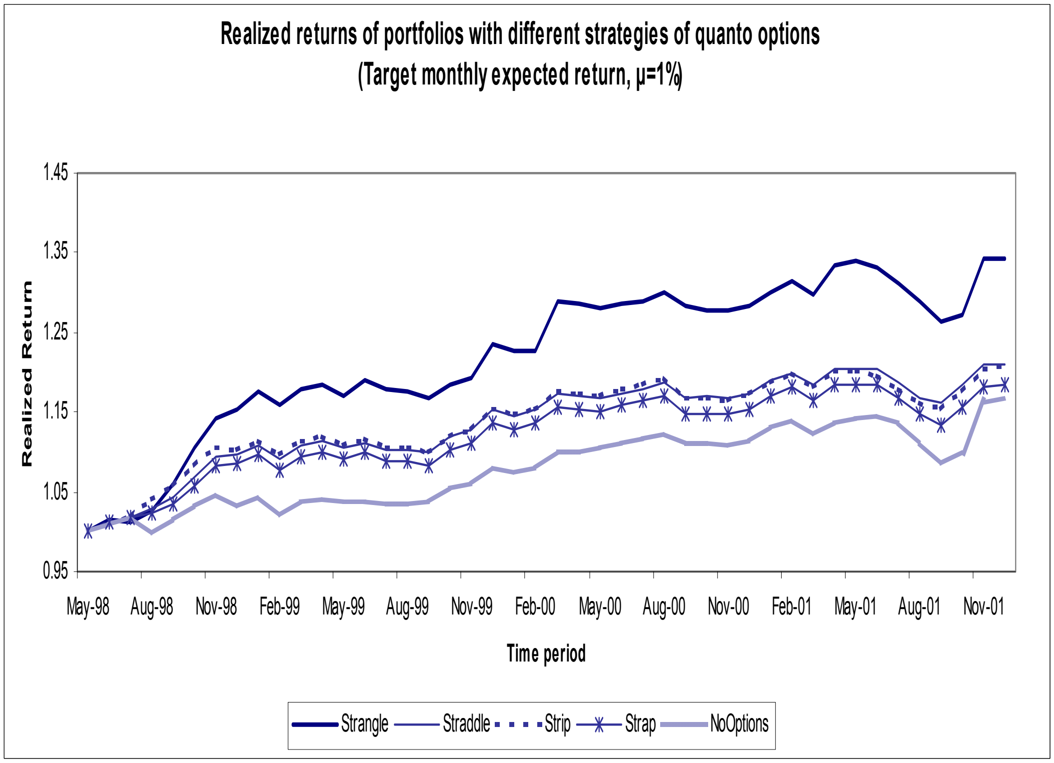


Figure 5: Ex-post realized returns of CVaR-optimized international portfolios of stock and bond indices including alternative strategies for hedging risks with simple options or quantos, and forward currency exchanges. The Strangle trading strategy is used for the options.



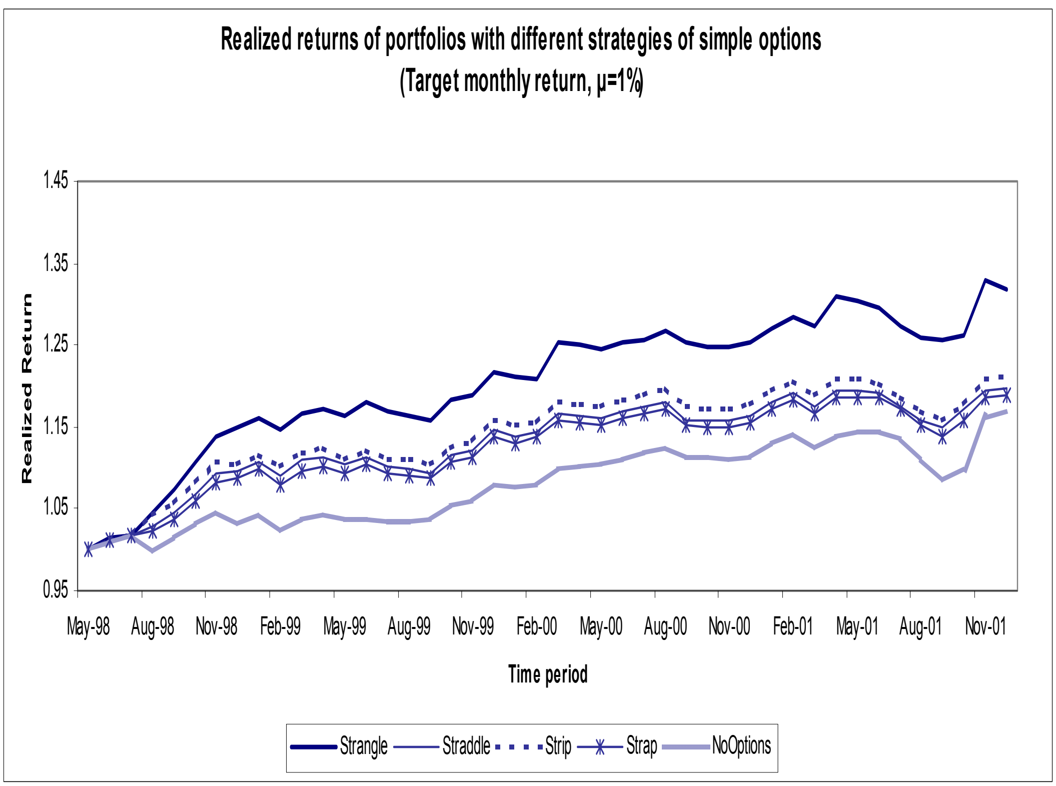
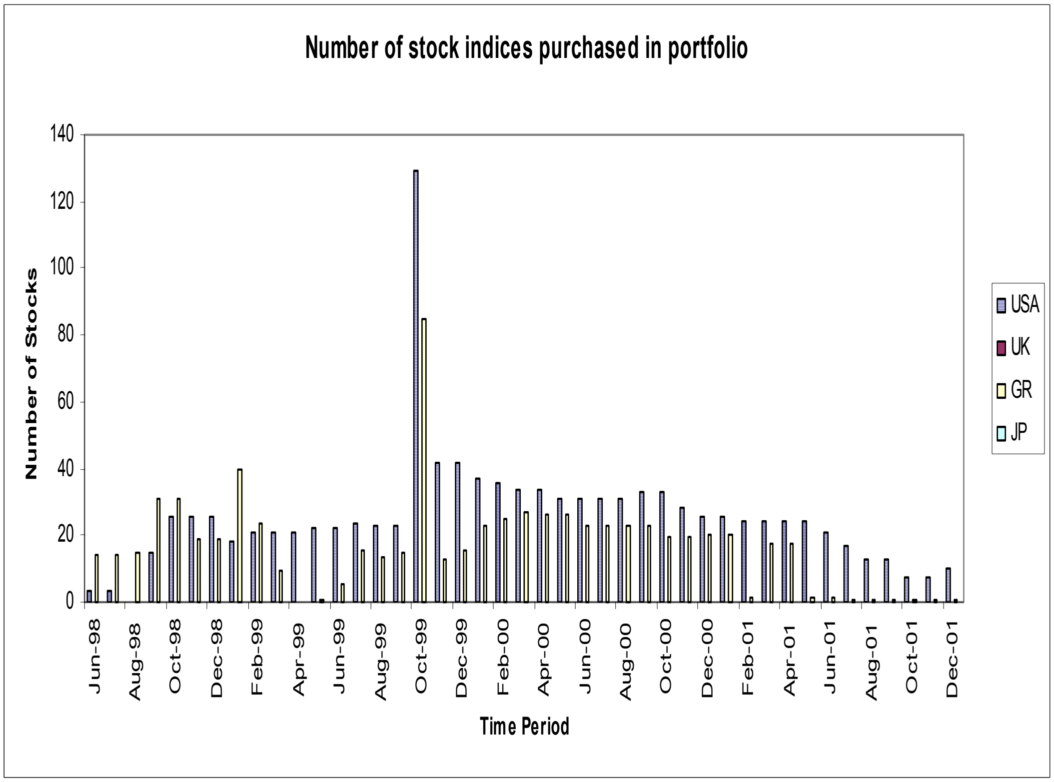


Figure 6: Ex-post realized returns of CVaR-optimized international portfolios of stock and bond indices with alternative option trading strategies.



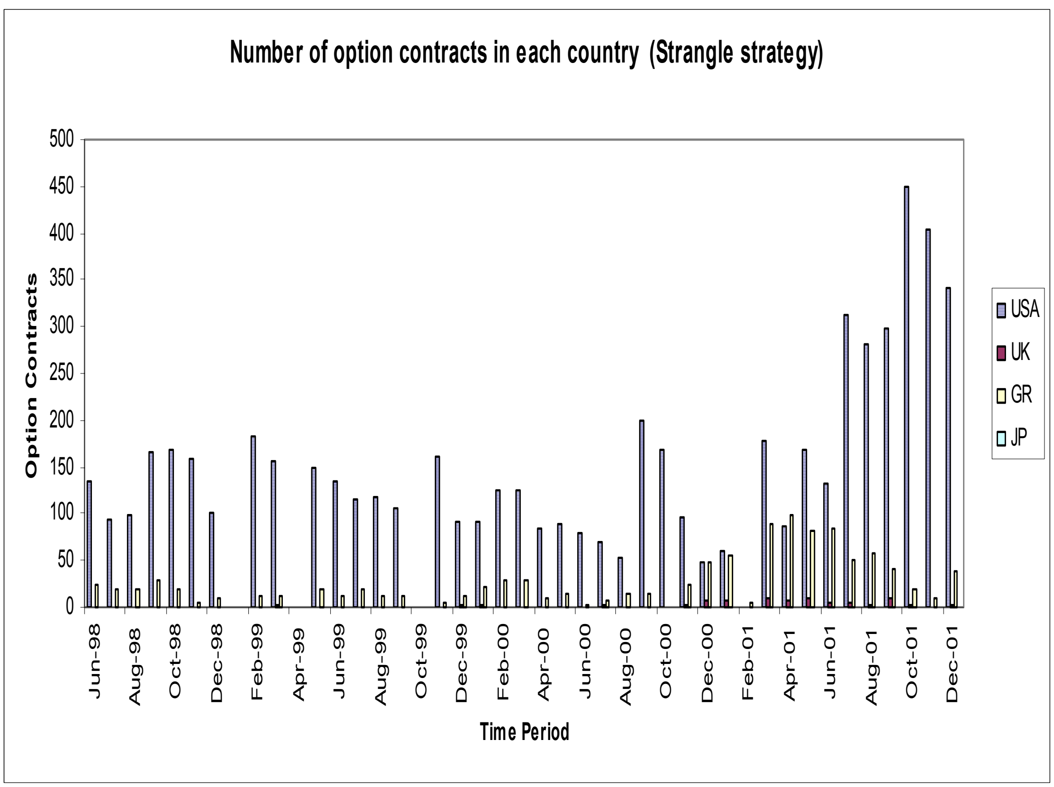


Figure 7: Optimal positions in stock indices and quanto options on these indices (Strangle strategy)

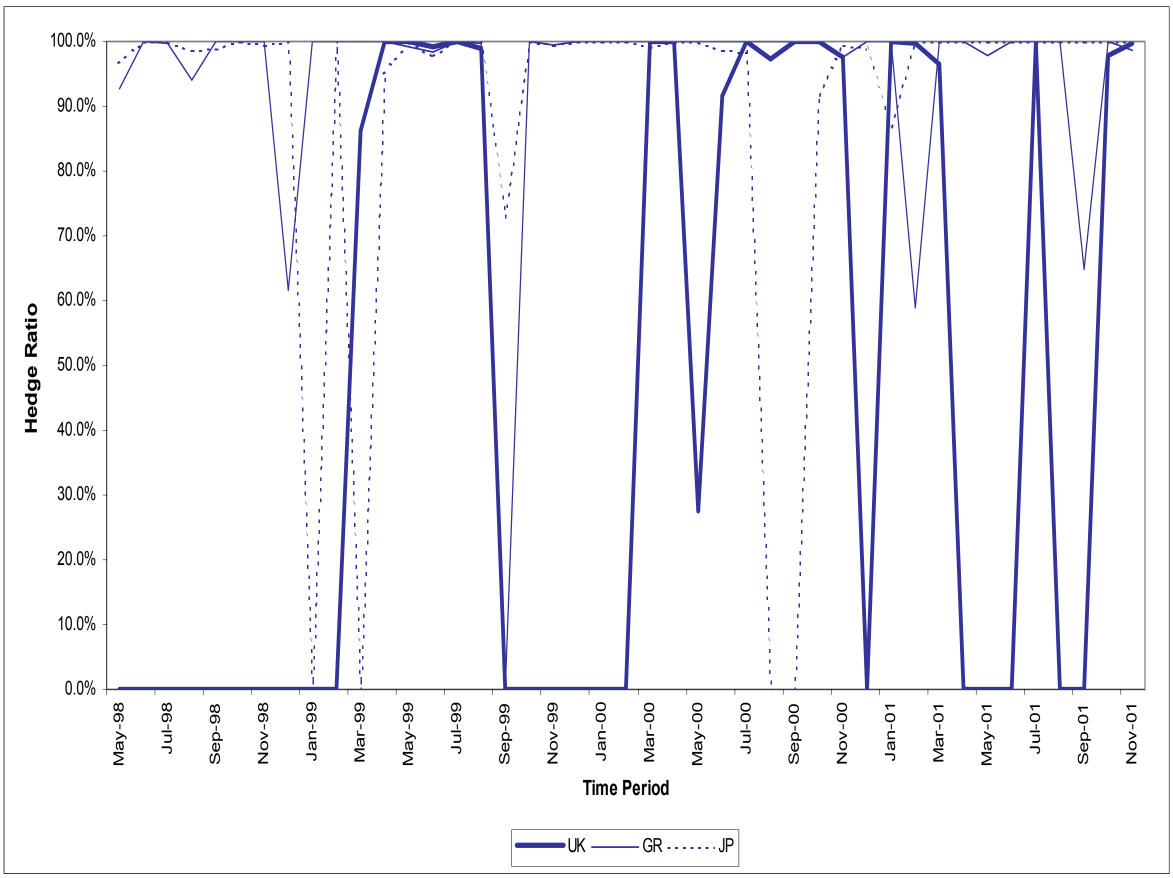


Figure 8: Hedge ratios of foreign investments in backtesting experiments (quanto options, strangle strategy).

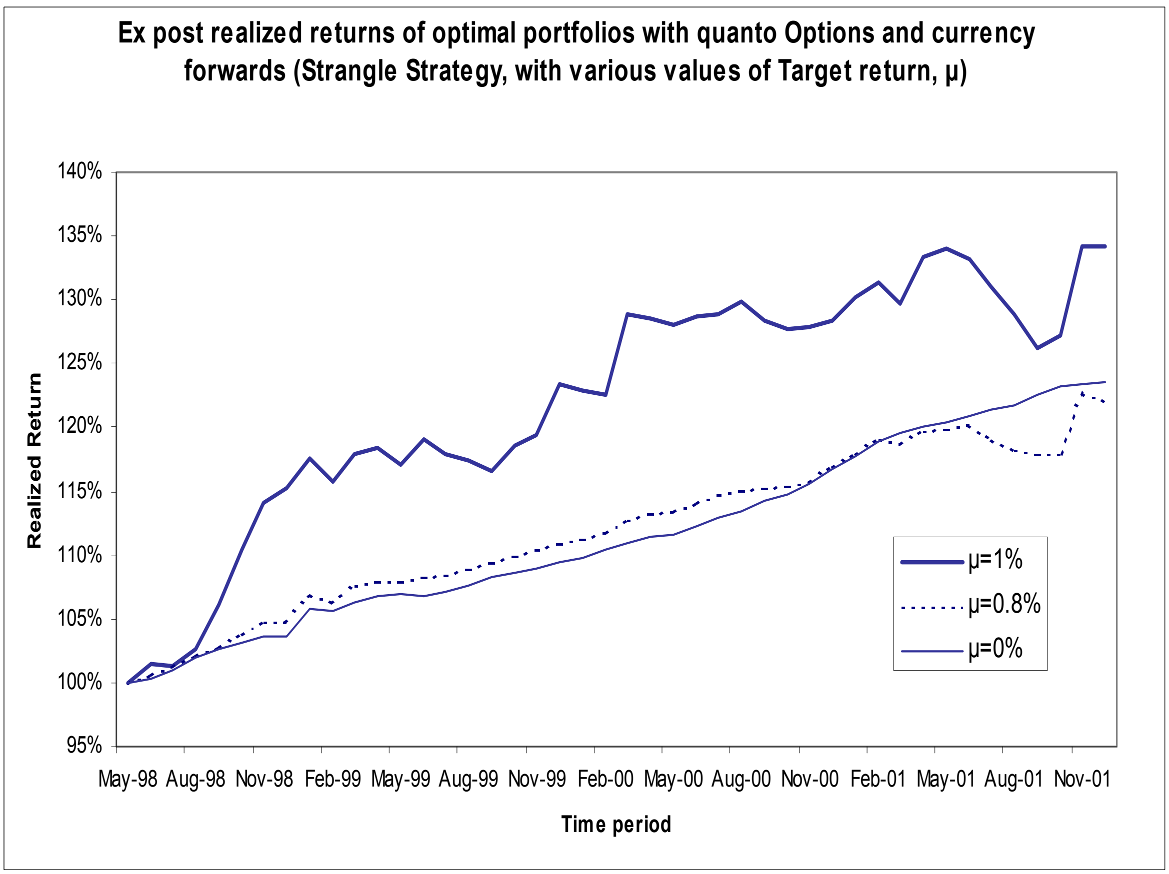


Figure 9: Ex-post realized returns of CVaR optimization models for internationally diversified portfolios of stocks bonds and simple Options. Comparison of different Target Returns.