CAPITAL INVESTMENT DECISIONS WITH PARTIAL REVERSIBILITY, OPERATING CONSTRAINTS, AND STOCHASTIC SWITCHING COSTS

by

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Abstract

We study dynamic investment strategy in a network of a discrete set of sequential and partially reversible decisions (i.e., optimal technology or capacity choice, optimal sequence of expansion, contraction, temporary shutdown, etc.) in the presence of hysteresis-inducing switching costs. We allow time intensive (time-to-build) decisions, and operating constraints (e.g., exhaustible resources or contractual limitations). More importantly, we incorporate a proper treatment of economic depreciation -- a mostly ignored factor in the contingent claims analysis of investments under uncertainty -- and we provide for switching costs and recovery (abandonment) values that are themselves path (utilization) dependent, and thus stochastic. We provide two illustrative examples, one with learning-by-doing in sequential investments, and one with a search for dominant technologies when introducing a new product/technology in shipping. It is seen that economic depreciation can be a very significant factor in valuation, with striking effects especially for the most important for decision-makers range of at- or near out-of-the-money investment options. Our other results are intuitive but sometimes non-conventional, e.g., in the presence of flexibility, an increase in uncertainty often can lead to investing earlier instead of waiting. Similar results are often observed with a decrease in the asset payout yield (a result not observed in earlier literature). In addition to previous literature we demonstrate that such non-conventional results may decrease or vanish in the presence of factors like operating constraints that limit flexibility.

Introduction

In this paper we implement a numerical solution framework with sequential investment and disinvestment strategies, optimal investment technology or capacity choice, costly switching, and path-dependent switching and abandonment costs. It also accommodates time-to-build, operating constraints (exhaustible resources), and interactions among investment timing and operating decisions. More importantly, we allow for path (utilization) dependent and thus stochastic switching costs and recovery (abandonment) values. Thus, we explicitly include economic depreciation as a significant element that affects valuation and decision making, a mostly ignored factor in real option analysis. To our knowledge this is the first paper that addresses stochastic switching costs and encompasses such a general solution framework. In contrast to the literature with analytic or even numerical solutions, when necessary we keep track of the complete path of decisions and not only the last one.

Contemporary models of partial irreversibility were first introduced in the literature by Brennan and Schwartz (1985) who valued natural resource investments and demonstrated that the classic Net Present Value (NPV) rule fails under uncertainty and irreversibility-inducing sunk costs. Dixit (1989a, 1989b, 1989c) extends their results and coines the term *hysteresis* to characterize the resulting zone of inaction within which optimal decisions are path-dependent. He studies optimal capital (de)allocation decisions in a two-sector economy with costly capital mobility and optimal entry or exit decisions in a foreign market with uncertain exchange rates. The emphasis in the original problem formulation is on infinite-horizon problems that admit analytic solutions. An extension in corporate finance with a finite horizon is given in Mauer and Triantis (1994) through the use of numerical partial differential equation (PDE) methods. Path-dependent problems with time-to-build and continuous learning-by-doing where the associated PDE is solved numerically are also treated in Majd and Pindyck (1987 and 1989).

The importance of a general framework with complex sequential investment decisions is discussed in Kulatilaka (1988), Kulatilaka and Marcus (1988), Triantis and Hodder (1990), and Trigeorgis (1993). Other authors have looked at more specific issues like capacity choice (Pindyck, 1988), sequential investment with time-to-build (Bar-Ilan and Strange, 1998), dynamic choice between two manufacturing locations in the presence of exchange rate risk (Kogut and Kulatilaka, 1994), intermediate inventories (Cortazar and Schwartz, 1993), the choice among mutually exclusive projects differing in scale (Dixit, 1993, and Dangle, 1999), interactions between time-to-build and capacity choice (Bar-Ilan, Sulem, and Zanello, 2002), optimal partially reversible investment (Hartman and Hendrickson, 2002, and Kandel and Pearson, 2002), etc.

We introduce a numerical solution framework that is general enough to allow the study of optimal scale, optimal expansion and contraction strategies (partial reversibility) allowing for more than two alternatives (unlike, for example, Kandel and Pearson, 2002), the impact of time-to-build and operating constraints, and a realistic treatment of economic depreciation. The last is very important since it has not been treated before, and it results in path (utilization) dependent switching costs and abandonment values that are hard to treat. Due to the generality and difficulty of the problem, we allow decisions only at (an arbitrary number of) points in time. We proceed as follows. We first describe the sequential investment problem with partial reversibility in its generality and the conceptual solution framework that allows us to study the impact of uncertainty, flexibility, and (potentially) stochastic switching costs. Subsequently we provide numerical results and discussion on two applications, namely learning-by-doing, and the search for a market niche in shipping. These are selected as motivational cases in order to demonstrate the variety of issues that can be addressed and the different results that can be encountered in practice. We then discuss the importance of allowing a realistic treatment of economic depreciation that produces utilization-dependent switching costs and abandonment values, which thus become path-dependent and in effect stochastic. Finally we conclude.

1. Investments as Networks of Decisions with Partial Reversibility

In this section we discuss the framework of complex investment problems in a multistage (sequential decision) setting with several alternative choice modes and partial reversibility to switch among these modes. This framework embeds the option to wait to invest, the option to choose among several investment alternatives (strategies, technologies, or operating scales), the option to switch back and forth among alternatives (at varying degrees of capital reversibility), and captures interactions between investment timing and operating decisions. Due to the presence of switching costs (and partial reversibility) the problem is inherently path-dependent. Complexity is further increased since switching costs and abandonment values themselves can be path-dependent due to explicit consideration of economic depreciation as a result of utilization of capital at varying degrees. Decisions are allowed at finitely many times (semi-American style embedded options). In addition, time-to-build and operating constraints are also built into it. Several aspects of the problem require keeping track of the complete path of decisions and not only the previous one.

Valuation of a claim (*a real option*) V is contingent on stochastic state-variables that follow risk-neutral Ito processes like in Black and Scholes (1973), Merton (1973a, b), and more importantly for the case of real option pricing like in McDonald and Siegel (1984 and 1986). Review of the literature on contingent claims valuation of capital investments can be found in Dixit and Pindyck (1994), and Trigeorgis (1996). In general, a continuous-time capital asset pricing model (see Merton, 1973b, or Breeden, 1979), the absence of market imperfections (taxes, etc.), dynamic market completeness (spanning), and an all-equity firm having monopoly power over its investments are assumed. In our applications and due to the great complexity of the problems addressed, we assume a single stochastic variable S that represents present value of cash flows received in each period of utilization of capital (an assumption that we can generalize to more stochastic variables at the expense of computational intensity). This stochastic variable follows in the risk-neutral measure the geometric Brownian motion process

(1) $$\frac{dS}{S} = \left(r - \delta\right) dt + \sigma dz$$

where r is the riskless rate of interest and σ^2 the instantaneous variance (of the continuous rate of change). The difference between the required return on asset S (or traded assets perfectly correlated with S) and its actual growth rate, is denoted by δ and represents a dividend-like opportunity cost of deferring investment in the revenue producing project (see McDonald and Siegel, 1984, and 1986); for a convenience yield interpretation, see Brennan and Schwartz (1985), and Brennan (1991).

Switching from mode i to j occurs at a switching cost $I^{i \to j}$ and provides a value V^j. At the beginning, there is a superset of admissible action paths M that defines what decisions are admissible given any realized sequence of decisions. At each point in time, M_t^- is a subset of M and includes the history of events up to time t. Given M_t^- and the decision at t, m_t, we denote by M_t^+ the remaining admissible decisions. The objective is to find the optimal claim value V^* over the set M_t^+ of remaining admissible choices that includes staying at the same mode i or switching to all other modes j including abandonment for salvage value A^i:

(2) $$V^*\left(S_t, t \mid M, M_{t-\Delta t}^-, m_{t-\Delta t} = i, S_{t-\Delta t}, S_{t-2\Delta t}, \ldots\right) = \max_{M_t^+} \left\{V - I\right\}$$

where M_t^+ includes switching from i to all other modes j, staying at the same mode i, becoming idle, or abandoning for salvage value A^i:

(3)

$$
\begin{gathered}
V^{j_1}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)-I^{i\to j_1}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\\
V^{j_2}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)-I^{i\to j_2}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\\
\cdots\\
\cdots\\
V^{i}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)-I^{i\to i}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\\
V^{i}_{idle}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)-I^{i\to i_{idle}}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\\
A^{i}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right).
\end{gathered}
$$

If a mode represents a state of inaction (a state of idleness, temporary shutdown or mothballing), the cash flows are determined by the proceeding mode *i*. Before any investment decision is made, the process starts in the mode wait to invest (*W*), which can also be maintained (option to wait to invest). The claim value at each mode *j* is a function of the cash flows at that mode plus the discounted optimal expected value of the claim at the next date:

(4)
$$
\begin{aligned}
&V^{j}\left(S_t,t \mid M,M^-_{t-\Delta t},m_{t-\Delta t}=i,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\\
&=R^j_t\left(S_t,t\right)-X^j_t\left(S_t,t\right)+e^{-r\Delta t}E_t\left[V^*\left(S_{t+\Delta t},t+\Delta t \mid M,M^-_t,m_t=i,S_t,S_{t-\Delta t},S_{t-2\Delta t},\ldots\right)\right]
\end{aligned}
$$

The operating revenues *R* minus the operating costs *X* capture the present value of cash flows till the next strategy revision. *R* is a deterministic function of state-variable *S* and allows us to consider different technologies and operating scales to be directly dependent on *S*. Of course, at the end of the time horizon the last term with the discounted expectation vanishes. Note also that at the boundary (critical threshold of *S*) that separates the regions where either of two decisions, *i* and *j*, are optimal, and assuming that the current state is *i*, the following value matching and smooth pasting conditions hold:

$$V^i = V^j - I^{i\to j}$$

and

$$\frac{\partial V^i}{\partial S} = \frac{\partial\left(V^j - I^{i\to j}\right)}{\partial S}$$

In many capital investment problems, operations cannot start until a first stage S_1 is completed that takes time to build (see Majd and Pindyck, 1987). In this case the attainable set of decisions differs significantly before and after completion of stage S_1. Before completion, it includes waiting W, S_1, and possibly abandonment for a resale value A^W; after completion the investment modes are as described earlier. The set of available decision modes and the admissible transitions (switching options) are shown in Figure 1 that shows a network of four technologies C, B, E_1, and E_2. These may differ in terms of installed capacity and they are mnemonics for contracted (small) scale, base-case capacity, expanded, and very expanded scale (alternatively they may simply represent different operating technologies).

[Enter Figure 1 about here]

In addition to the above, operating constraints can be included (that account for example, for exhaustibility of resources, etc.) Such constraints imply that the economic life of the investment is limited. In our formulation, the economic life of the investment in the presence of such constraints is not a function of time since capital was installed but a function of time that operations were was actually on. Thus, if the firm gets into an idle (mothballing) phase after investment, we assume that during that time operations are off and exploitable resources are not depleted. The extent to which an operation can stay in an active mode should be part of set M. During optimisation, M_t^- and m_t, keep track of past operations, and M_t^+ includes the set of actions that are further admissible. Importantly, switching costs, I, and/or abandonment values, A, can be utilization-dependent and in effect path-dependent and stochastic. M_t^- and m_t, (like in the case of operating constraints) keep track of past operations, and M_t^+ defines the further admissible actions, and the switching costs, I, and the abandonment values, A, as a function of the realized use of installed capital (true and not simply historical economic depreciation).

The switching matrix for costs must be logically (or economically) consistent. Consider for example $I^{1\to2}$, $I^{2\to3}$, and $I^{1\to3}$ the costs of switching from the first state to the second, from the second state to the third, and from the first to the third directly (in ascending order of scale or productive capacity, etc.) We must compare $I^{1\to3}$ with $I^{1\to2} + I^{2\to3}$. If $I^{1\to3} > I^{1\to2} + I^{2\to3}$cost efficiencies may be achieved due to learning-by-doing. If $I^{1\to3} < I^{1\to2} + I^{2\to3}$, there might be scale efficiencies. In general, a careful comparison of the total switching costs of all admissible paths is needed in order to avoid artificially inflating option values, and practically imposing or prohibiting certain transitions from one state to another.

Numerical PDE solutions for real option problems with path-dependency have appeared in Mauer and Triantis (1994), and Majd and Pindyck (1987 and 1989). Financial (Asian) options have also been solved via PDE methods, like for example in Ingersoll (1987), and Alziary, Decamps, and Koehl (1997), to mention just a few. They show that each decision point that contributes to path-dependency adds considerably to the dimensionality of the numerical solutions. The great complexity imposed by the problems we treat here and the need for a considerable number of intermediate decision points makes the choice of PDE methods practically infeasible. We therefore use a discretized (lattice-based) finite difference scheme as an approximation to the continuous state-space, and we allow decisions to be made at discrete points in time. Then we solve a discrete multi-stage optimization problem through a forward-backward looking algorithm of exhaustive search. This takes the path-dependency and the early exercise (semi-American) features of the problem properly into account (see also Hull and White, 1993, and Thompson, 1995, for conceptually similar, lattice-based, approaches). Path-dependency is accounted for among the several decision points. Between each two decision points, the lattice allows for an arbitrary number of steps in order to improve accuracy if and when necessary. Between decision points, lattice steps only provide discounting since no decisions are permitted in these regions, thus the dynamic programming (sequential) method afforded by the lattice can be replaced by an equivalent one-step method similar to that described for example by equation (6) (chapter 5, p. 177) in the classic book by Cox and Rubinstein (1985). When dense lattices are needed in some applications, this can improve computational efficiency considerably (say, a factor of two, three or more). Finally we need to consider only the lattice-points at all decision times and for all relevant values of the state-variable as given by the lattice construction. For each such point we calculate the relevant option values for every feasible path of past decisions by exhaustive search. We thus need to create auxiliary variables that keep track of all decisions and paths. This increases considerably memory requirements, but, given the high complexity of the problem, keeps the computational burden within the reach of the contemporary personal computers. The algorithm for exhaustive search is described below.

A. Forward run:

At each decision node (starting from time zero and proceeding to the option maturity), for each lattice point (from low to high values of the state-variable), we activate each admissible decision. By the time we reach maturity all decision combinations have been activated (operating revenues, fixed and operating costs have been determined); auxiliary variables keep track of the path of past decisions for each feasible path. At each time when decisions are permitted and for each lattice point, there are many paths of admissible decisions that are created and saved in computer memory (the exponential increase of the number of these paths gives rise to the computational intensity of the exhaustive search solution method).

B. Backwards run:

Option values are calculated (starting from option maturity and going backwards towards time zero) at each lattice point for each decision path; given the previous decision path, the optimal decision is determined and thus the optimal option value. Option values are calculated as functions of the cash flows and fixed (switching) costs (given a decision and the previous path) plus the expected continuation value as a (probability) weighted average of the optimal values at lattice points at the next decision node.

Going backwards we find optimal decisions at each lattice point for each decision path. Finally, at time zero we can determine the optimal decision (when there exist many possible alternatives suboptimal decisions may differ by orders of magnitude). A similar grid search above and below the starting lattice point at time zero provides also the critical thresholds for different decisions.

We now illustrate for pedagogical purposes possible realized optimal decisions in a simple case, without time-to-build assumptions and without mothballing. To make the illustration feasible we consider just for this section two decision stages, allowing just three investment decision-points (including the one at the end of the time horizon). In the numerical results that will be discussed in more detail in the next section, six decision points have been used. The number of decision points can be increased at the expense of computational intensity (but we have found that our implementation of a semi-American option with six decision points captures the most significant part of the early exercise premium of a fully American option).

[Enter Figure 2 about here]

Figure 2 demonstrates the path-dependent nature of the optimal solution path. In reality, the lattice *is* recombining in respect to the asset value *S*. But the path that *S* follows determines the optimal investment, disinvestment or abandonment decisions. Optimal solutions at any time differ according to the realized path (hysteresis) of *S*, and the previously realized decisions. The optimal investment decisions (and option values) are sensitive to the level of asset *S* and to the value of parameters like uncertainty, etc. This is the conceptual and solution framework that we adopt. In the following, we use it to study the impact of stochastic switching costs (due to utilization-dependent economic depreciation), operating constraints, etc. in the context of two applications that deserve merit on their own.

2. Two Applications: Learning-by-Doing, and Searching for a Market Niche in Shipping (Tanker Choices)

In this section we discuss two applications. The first is a general implementation of the generic problem shown in Figure 1. The cost parameters are chosen such that this case will simulate a *learning-by-doing* application with economic and technical efficiencies resulting from sequential (phased) development. By construction, the cumulative capital cost of installing the highest capacity (E_2) will be lower if built in a sequential fashion than if built all at once. This assumption of added value to sequential investment due to learning-by-doing is adopted on purpose in order to investigate a case with a high degree of embedded optionality. By incorporating operating constraints, we will see that the degree of optionality that we leave unconstrained affects values and investment decisions significantly. The second application is also based on the general setup of Figure 1 but made specific to the shipping application (optimal operation of a tanker with mothballing) described in Dixit and Pindyck (1994, pp. 237-242). However, it is extended to all four tanker technologies discussed there and not just the one actually analyzed by them. In this last case the flexibility to switch back and forth is reduced due to considerable switching costs resulting from reduced abandonment (resale) values. This setup is useful not only for a tanker operator that has the option to switch technologies, but also (and this is a novel approach) for a firm that considers *offering* in the market a new tanker technology while taking into account other competing technologies. By looking at the problem from the buyer´s point of view, the firm can investigate the extent to which the new technology will be the optimal one for adoption given the prevailing market conditions (and demand uncertainty), and also to what extent and under what circumstances it could become attractive in the future. Thus, the firm will know if a market niche can be captured, and can investigate whether and when to invest in building and offering the new product/technology.

Figure 3a provides the level of net revenues as functions of a price related state-variable S, and the capital costs for the first application. Similarly, Figure 3b provides the same information for the second application. In both applications, each time the firm is in an operating mode (technology) receives cash flows $R(S) - X$. The switching (capital) cost I to first enter a mode is paid once, and if the system afterwards gets into an idle mode, cost N (with subscript specific to the operating mode) is paid. When the time-to-build constraint is required, the system from the wait mode W must first enter an initial stage S_1. Abandonment happens at cost A (usually negative since part of the initial capital is hereby recovered). At first we will discuss the applications with I and A constant, and then we will consider them to be utilization dependent. In the first application the net revenue function is the simplest possible $R_t^j\left(S_t,t\right) - X_t^j\left(S_t,t\right) = f_S^j S_t - f_X^j X$ where the constant parameters f are expansion factors that depend on the operating mode j the system is in and for simplicity are both equal to f^j. In the last application and for consistency with the Dixit and Pindyck application, the state variable is price P (per ton), and $S_t = S_t\left(P_t\right)$ is a linear function of price.

[Enter Figures 3a, and 3b about here]

The remaining figures provide option values and an investigation of optimal exercise policy with sensitivity on S, the instantaneous standard deviation σ, and the asset payout or dividend yield δ, for the two cases. For the numerical results and without any loss of generality or any effect on the insights gained we assume that the decision maker has six decision points (and one step between decision points) to make or revise her opinion up to the end of the time horizon. From our experience that resulted from extensive experimentation, this affords a reasonable trade-off between accuracy (and generality) of the results, and calculation efficiency; due to the high computational intensity of the numerical method, without any effect on the insights derived. Note that we are not merely interested in the valuation of the investment options, but also on identifying numerically the critical thresholds where decisions change. This particular choice will also make it feasible to other researchers to exactly replicate our results. Using for example six steps instead of one between decision points improved option value accuracy for the most interesting cases of the near at-the-money options by 5% (on average), with an accuracy improvement even less than that for the determination of the critical thresholds, but the computational burden increased by 100!!!. Dense grids were more significant for very out-of-the-money options, and less significant for in-the-money options.

For the *first* application we assume for the base case parameter values $\sigma = 0.20$ per year, $\delta = 0.10$ per year, $r = 0.05$ per year, total time to maturity of 5 years, and $X = 100$. Here we observe (Figures 4a, 4b) the most striking results, which are consistent (in respect to σ) with observations reported in Bar-Ilan, Sulem, and Zanello (2002), and Brekke and Schieldrop (2000) (for an earlier debate on the sign of investment-uncertainty relationship, see Caballero, 1991, Cortazar and Schwartz, 1993, Abel, Dixit, Eberly, and Pindyck, 1996, Bar-Ilan and Strange, 1998, and more recently, Kandel and Pearson, 2002). In cases like the ones we study with heavy embedded future sequential flexibility, higher uncertainty often tends to speed up (rather than delay) investment, in contrast to the standard option literature. Investing earlier opens up new investment options whose value increases with uncertainty. The decision to invest earlier is often at a lower mode of operation (i.e., reduced capacity level, less expensive technology, etc.). Intuitively it makes sense to start small and expand later. Due to the sequential nature of the investment decisions, an increase in uncertainty (of S) increases also the value of the underlying asset, V, that includes the value of future prospects, which in effect represents a complex compound option. This may make it optimal to invest at lower levels of S since the increase in the critical threshold of V can be less than the increase in the actual value of V.

In contrast to any other literature so far, similar results (investing earlier) are also observed with a lower or zero dividend yield (whereas a zero dividend yield would never allow early exercise of the standard American call option). This is attributed to the sequential nature of the cash flows and the limited time horizon of the problem. A lower dividend yield is equivalent to a higher growth rate of the cash flows, so deferring investments penalizes investment value considerably due to the lost revenues. The same impact of volatility and dividend yield appear also when a time-to-build assumption is used (see for example Figure 4c). With the time-to-build (intermediate-stage) assumption production can only start after build-up (in our numerical examples one decision stage later), which effectively places an implicit constraint on operations and reduces the investment option value.

[Enter Figures 4a – 4e about here]

For the first application we also report results with explicit constraints on operations (Figures 4d, 4e), such as limited economic life or exploitation of a given amount of exhaustible natural resources, contractual limitations, etc. The more constrained the operations are the lower the option value, and the more investment is delayed (to be made later at potentially higher capacity levels). With operating constraints, flexibility (embedded optionality) to switch is reduced and usually higher uncertainty and a lower payout yield defers investment, which eventually is expected to occur at a higher operating level. Thus, the existence of operating constraints limits the value of future growth opportunities and enhances the irreversible nature of the investment.

The above results demonstrate that reversal of the sensitivity of the American call option to uncertainty and the payout yield depends on the tradeoff between factors that increase and factors that decrease flexibility and its value. The higher the degree of the embedded optionality the more we can expect such reversal to occur, and the opposite when flexibility is restricted, due for example to constraints on operation (like exhaustible natural resources or contractual limitations).

In the last application (from the shipping industry) we assume for the base case parameter values σ = 0.15 per year, δ = 0 per year, r = 0.05 per year, total time to maturity of 10 years, and X = 8.8. Here flexibility to switch is a-priori reduced because of relatively high switching costs. These high switching costs are due to the low resale (abandonment or scrap) value of each tanker type. The net revenues equal $f_S^j S_t\left(P_t\right) - f_X^j X = \dfrac{2 f_S^j 85000 P_t}{1000000} - f_X^j 8.8$ in millions USD for every two-year period, assuming for the base case like they do 85000 deadweight tons capacity. The price is in USD per ton. We have remained faithful to the extent possible to the information provided in Dixit and Pindyck (1994, pp. 237-241), retaining for technologies other than the base case the same constant of proportionality implied by them.

[Enter Figures 5a – 5b about here]

The numerical results (Figures 5a – 5b) are qualitatively similar to those reported earlier, although relatively high switching costs reduce the value of flexibility and thus the sensitivity of optimal policy to volatility is as in the standard American call option.

A careful observation of the results confirms the existence of *dominant* technologies, namely B and E_2. For the given investment problem configurations, the other two alternative modes (technologies C and E_1) even if they are technically sound, they are not attractive in terms of economics and they are practically never used. This results holds even if the value of the investment (as a function of S) increases, even if parameters like volatility, dividend yield, etc. are reasonably outside the present range. This demonstrates the value of the real options analysis to investigate whether there is a market niche for a new technology. *This relative attractiveness can be seen only in comparison with the competing alternatives as part of a network of investment decisions.* If these technologies were to be analyzed as isolated investment options, their value would appear to be significant, and such a naive real options analysis would be very misleading (for a general discussion of positive and negative option interactions, see Trigeorgis, 1993).

3. Stochastic (utilization-dependent) abandonment and switching costs

In the analysis so far, we have assumed that switching costs and abandonment values are constant. In many realistic applications though, it is more realistic to assume that these values are functions of the actual utilization of the technology that is being abandoned. Thus, *true* economic depreciation is accounted for. This makes these values path-dependent and in effect stochastic. In this section we assume that abandonment value depreciates according to $A_i e^{-c_i n_i}$ where A_i is the maximum possible recovery at immediate abandonment, counter n_i keeps a record of the actual usage of the exiting technology mode, and the depreciation parameter c_i determines the extent of recovery. If for example the depreciation parameter $c = 0.30$ and the technology has been in use for 3 periods, recovery is 40.70% of maximum abandonment value A. A parameter value $c = 0$ would imply that abandonment values are constant and do not depend on utilization of capital. In our implementation, mothballing does not add to depreciation, only actual utilization does. According to the industry experience, we could easily allow two depreciation factors, one for mothballing and one for actual utilization.

For the first application we provide numerical results with a stochastic abandonment value but with constant switching costs (see Tables 1 and 2). We see that results can vary significantly. When the constant c of economic depreciation is lower, option values can be significantly higher. Differences in valuation are particularly striking for at- or out-of-the-money investment options. This is very important, since the range of at- or near out-of-the-money is the most important for the economic consideration of new investments or the adoption of new technologies, addition of operating capacity, etc. by decision-makers (meaning that it is unlikely that rational managers will have forgone for too long extremely profitable opportunities; very in-the-money investment options are less interesting in practice). The effect of option values and critical decision thresholds depends on the extent to which the current level of S is in a range of values where investment is reasonably likely to be followed in the future by abandonment due to adverse movements in the market of demand, product prices, etc.

Similarly with the abandonment values, the switching costs $I^{i \to j}$ from mode i to mode j may also be path-dependent. We make use of this assumption in the shipping application for the components of the switching cost matrix

$$I^{i \to j} = I^{W \to j} - A_i e^{-c_i n_i} .$$

The switching cost (from i to j) is determined from the initial capital cost to get to mode j minus the stochastic abandonment value at mode i, where $A_i = I^{W \to i}$. For the shipping application numerical results are presented in Table 3. Again we see that when the depreciation parameter c is lower, option value can be significantly higher especially again for at- and out-of-the-money options.

The numerical results in both applications confirm that lower depreciation or higher recovery (which occurs when the depreciation parameter is lower, and when relevant it also reduces the switching cost) is associated with higher option values. This is again more apparent when asset value S is in a range where it is more likely that abandonment or switching (in future paths) will occur. Different assumptions on the extent of capital recovery will also affect the optimal investment thresholds and optimal decisions. If investment (or technological) alternatives have a varying degree of true economic depreciation, this should also be taken explicitly into account. The practice to ignore the true economic depreciation of installed capital in many real option models is likely to misprice investment options and mislead decision makers.

[Enter Tables 1 – 3 about here]

Conclusions

In this paper we study sequential investment decisions under uncertainty within a general framework that allows flexibility in a network of partially reversible capital decisions. The method accommodates stochastic recovery and switching costs (utilization-dependent economic depreciation), and various imperfections like time-to-build (production lags) and constraints on operation (e.g., exhaustibility of resources). Costly switching between operating modes induces path-dependency. This framework allows the study of operating or strategic investment decisions in alternative production technologies, different levels of operating capacity (scale), dominance of technologies, and can be easily adapted to study other problems like mutually exclusive technologies (with varying degree of flexibility, etc.). Our methodology allows us to keep track of the complete path of past decisions and not only the last one.

We have used this framework to study the impact of utilization-dependent depreciation that results in stochastic switching costs and recovery values. We have found non-constant recovery values and switching costs to affect investment option values and optimal investment decisions significantly. Among other important findings are, that, in the presence of flexibility, contrary to the standard options literature, an increase in uncertainty often leads to investing earlier instead of waiting. Similar results are observed with a decrease in the dividend yield in the presence of sequential cash flows. However, factors that limit flexibility, can reduce or eliminate these non-conventional results. Finally, as illustrated by the shipping application, the framework also allows the investigation of market niche availability for newly developed technologies. If new products or technologies are dominated by existing ones (under varying realizations of uncertainty and demand levels), it may not be fruitful to bring them in the market. Of course, and in a similar fashion, one can demonstrate the economic obsolescence of an existing technology in the emergence of new ones. Such investment (or disinvestment) decisions can only be investigated properly as part of a network of decisions, not in isolation.

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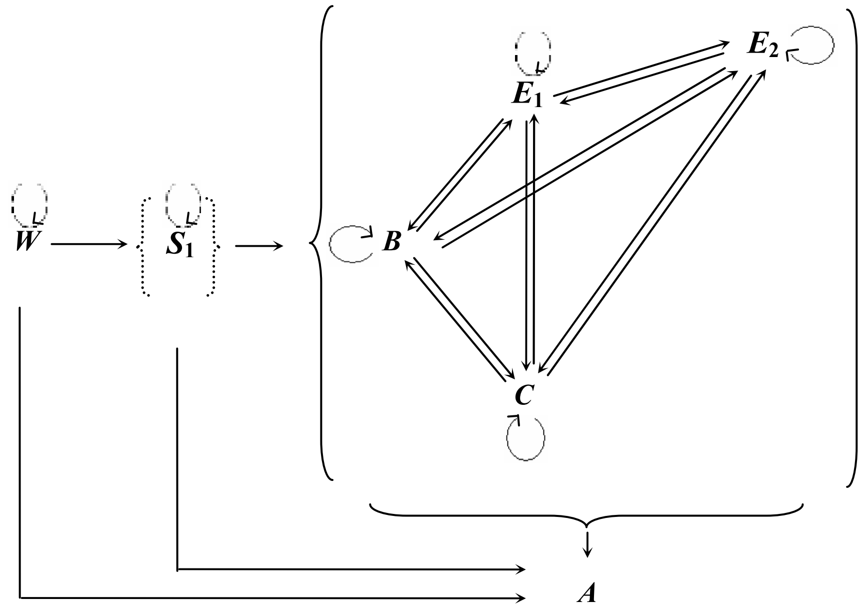
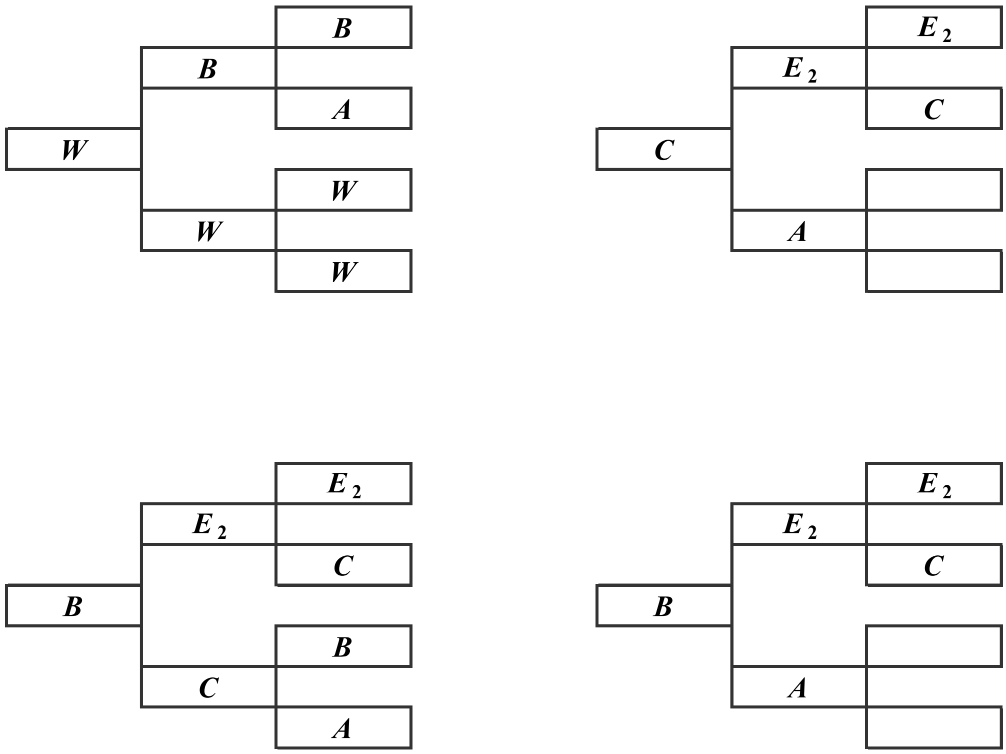
Figure 1
A decision network with high flexibility among four alternative operating modes



Note: The process starts from a wait-to-invest mode (W). Then can choose among alternative modes $\{C, B, E_1, E_2\}$. We can stay in each mode by keeping operations active or idle (mothballing). In mode W we can stay for as long as it is optimal to make the first investment decision. The decision set also includes the option to abandon (A) from any mode.

Figure 2
Illustration of optimal strategies with four flexible operating modes



Note: Illustrations of investment decisions with hysteresis for different paths of a state-variable S on a standard binomial lattice. At the end of the lattice, variable S takes only three values (the lattice reconnects), but for the middle value, there can be difference investment decisions due to path-dependency induced by switching costs and partial reversibility. The process starts from a wait-to-invest mode (W). Then can choose among alternative modes $\{C, B, E_1, E_2\}$. The decision set also includes the option to abandon (A) from any mode.

Figure 3a

Case 1: Capital costs and net operating revenues under four alternative operating modes with and without Time-to-Build

CAPITAL COSTS

FROM \ TO	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
						Initial Capital Costs						
W	-	10	-	12	20	65	170	-	-	-	-	0,00
S_1	-	-	2	2	10	55	160	-	-	-	-	-2,50
N_{S1}	-	-	-	5	13	58	163	-	-	-	-	-2,50
						Switching Costs						
C	-	-	-	-	5	35	85	2	-	-	-	-3,00
B	-	-	-	-5	-	30	75	-	2	-	-	-5,00
E_1	-	-	-	-35	-30	-	40	-	-	2	-	-16,25
E_2	-	-	-	-85	-75	-40	-	-	-	-	2	-42,50
N_C	-	-	-	3	8	38	88	-	-	-	-	-3,00
N_B	-	-	-	-2	3	33	78	-	-	-	-	-5,00
N_{E1}	-	-	-	-32	-27	3	43	-	-	-	-	-16,25
N_{E2}	-	-	-	-82	-72	-37	3	-	-	-	-	-42,50

NET OPERATING REVENUES

FROM \ TO	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
					Expansion factors for the Operating Revenues (S)							
	-	-	-	**-50%**	**0%**	**50%**	**100%**	-	-	-	-	-
Benchmark B	0,000	-	-	0,607	1,000	1,649	2,718	-	-	-	-	-
					Expansion factors for the Operating Costs (X)							
	-	-	-	**-50%**	**0%**	**50%**	**100%**	-	-	-	-	-
Benchmark B	0,000	-	-	0,607	1,000	1,649	2,718	0,050	0,050	0,050	0,050	-

Figure 3b

Case 2: Capital costs and net operating revenues under the shipping, mothballing and scrapping oil tankers application with and without Time-to-Build

CAPITAL COSTS

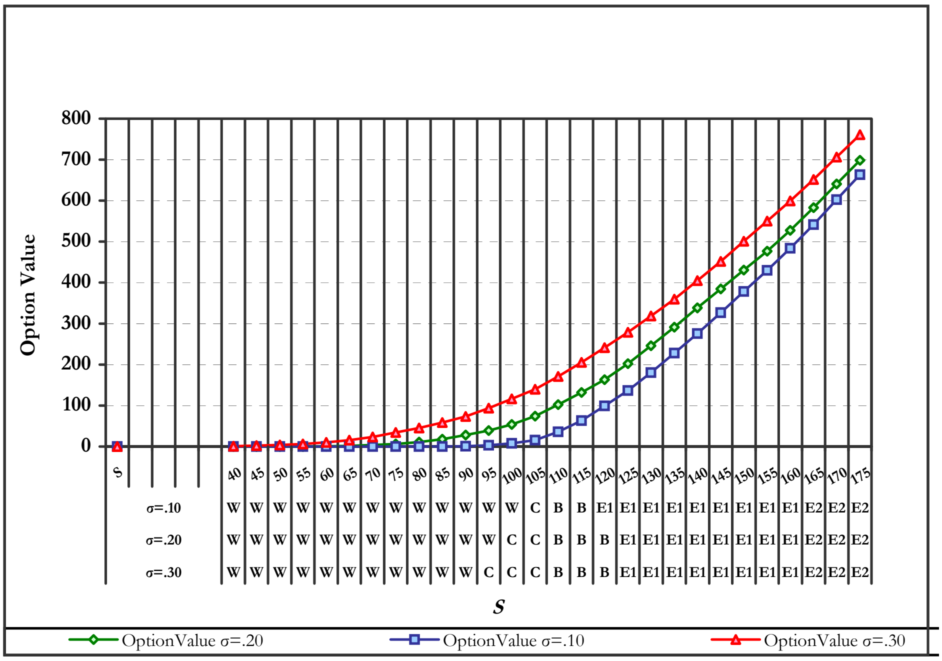
FROM \ TO	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
						Initial Capital Costs						
W	-	70,000	-	115,500	140,000	297,500	595,000	-	-	-	-	0,000
S_1	-	-	-	45,500	70,000	227,500	525,000	-	-	-	-	-5,950
N_{S1}	-	-	-	45,500	70,000	227,500	525,000	-	-	-	-	-5,950
						Switching Costs						
C	-	-	-	-	130,183	287,683	585,183	0,082	-	-	-	-9,818
B	-	-	-	103,600	-	285,600	583,100	-	0,200	-	-	-11,900
E_1	-	-	-	90,213	114,713	-	569,713	-	-	0,329	-	-25,288
E_2	-	-	-	64,925	89,425	246,925	-	-	-	-	0,635	-50,575
N_C	-	-	-	0,325	130,183	287,683	585,183	-	-	-	-	-9,818
N_B	-	-	-	103,600	0,790	285,600	583,100	-	-	-	-	-11,900
N_{E1}	-	-	-	90,213	114,713	1,301	569,713	-	-	-	-	-25,288
N_{E2}	-	-	-	64,925	89,425	246,925	2,509	-	-	-	-	-50,575

NET OPERATING REVENUES

FROM \ TO	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
					Expansion factors for the Operating Revenues (S)							
	-	-	-	**-88,7%**	**0,0%**	**49,9%**	**115,6%**	-	-	-	-	-
Benchmark B	0,000	-	-	0,412	1,000	1,647	3,176	-	-	-	-	-
					Expansion factors for the Operating Costs (X)							
	-	-	-	**-88,7%**	**0,0%**	**49,9%**	**115,6%**	-	-	-	-	-
Benchmark B	0,000	-	-	0,412	1,000	1,647	3,176	0,234	0,234	0,234	0,234	-

Figure 4a

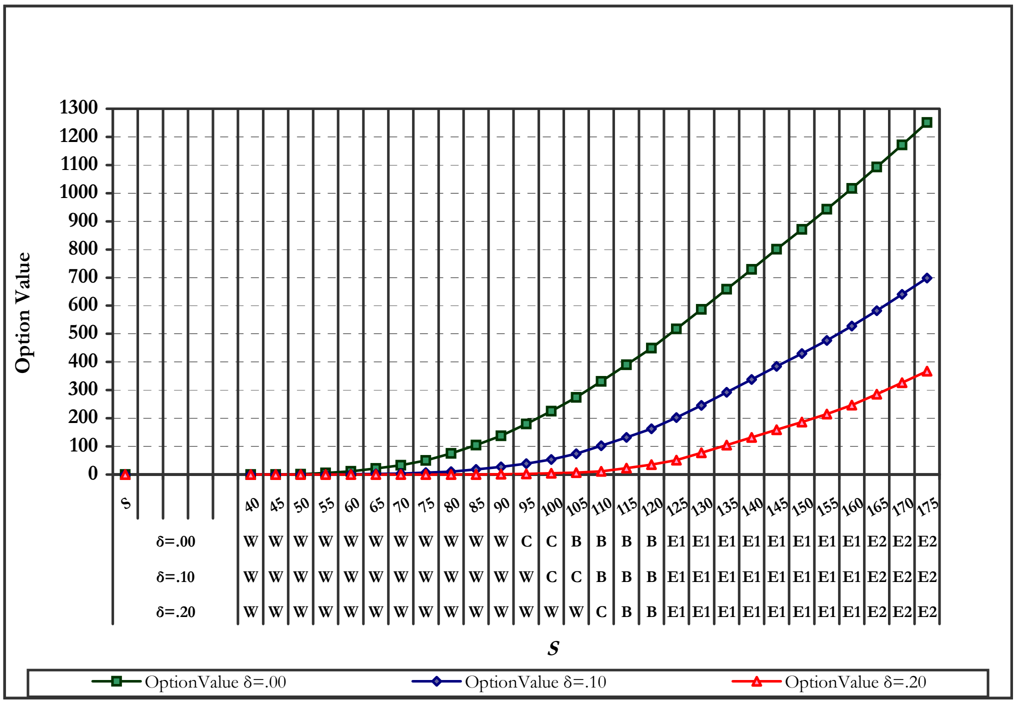
Sensitivity analysis of option value and optimal operating policy vs. uncertainty with four alternative operating modes without Time-to-Build.



Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model, without Time-to-Build. We denote with C the small operating scale, B the base case, E_1 *and* E_2 the expanded operating scales. Operating cost X = 100, riskless rate r = 5%, Δt = 1 per period.

Figure 4b

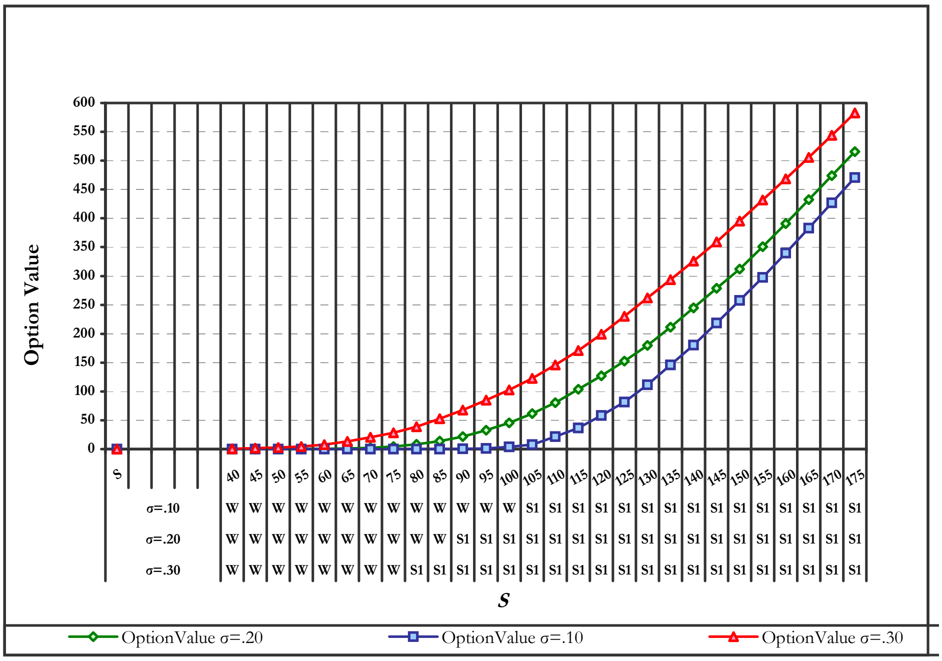
Sensitivity analysis of option value and optimal operating policy vs. payout yield with four alternative operating modes without Time-to-Build.



Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model, without Time-to-Build. We denote with C the small operating scale, B the base case, E_1 *and* E_2 the expanded operating scales. Operating cost X = 100, riskless rate r = 5%, Δt = 1 per period.

Figure 4c

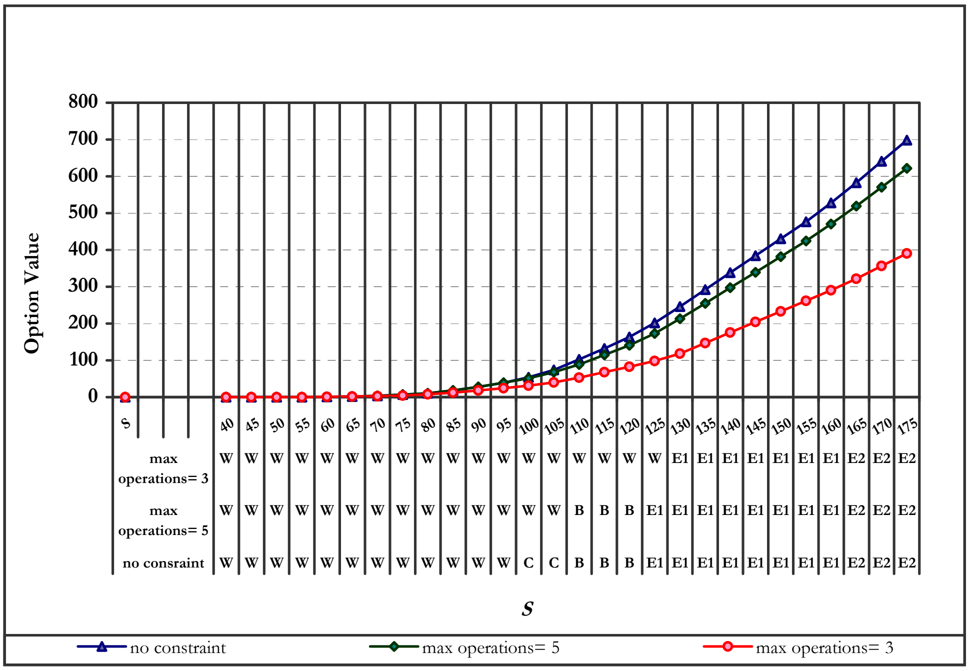
Sensitivity analysis of option value and optimal operating policy vs. uncertainty with four alternative operating modes with Time-to-Build.



Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model, with Time-to-Build. We denote with C the small operating scale, B the base case, E_1 *and* E_2 the expanded operating scales. Operating cost $X = 100$, riskless rate $r = 5\%$, $\Delta t = 1$ per period.

Figure 4d

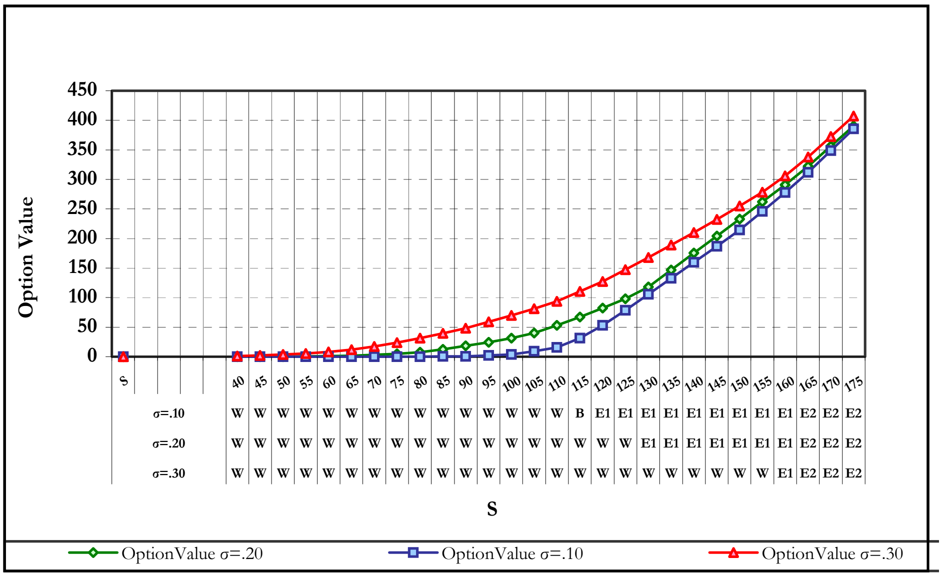
Impact of operating constraints on option value and optimal operating policy for four alternative operating modes without Time-to-Build.



Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model, without Time-to-Build but with constraints on the maximum number of operations till option maturity. We denote with C the small operating scale, B the base case, E_1 *and* E_2 the expanded operating scales. Operating cost $X = 100$, riskless rate $r = 5\%$, $\Delta t = 1$ per period.

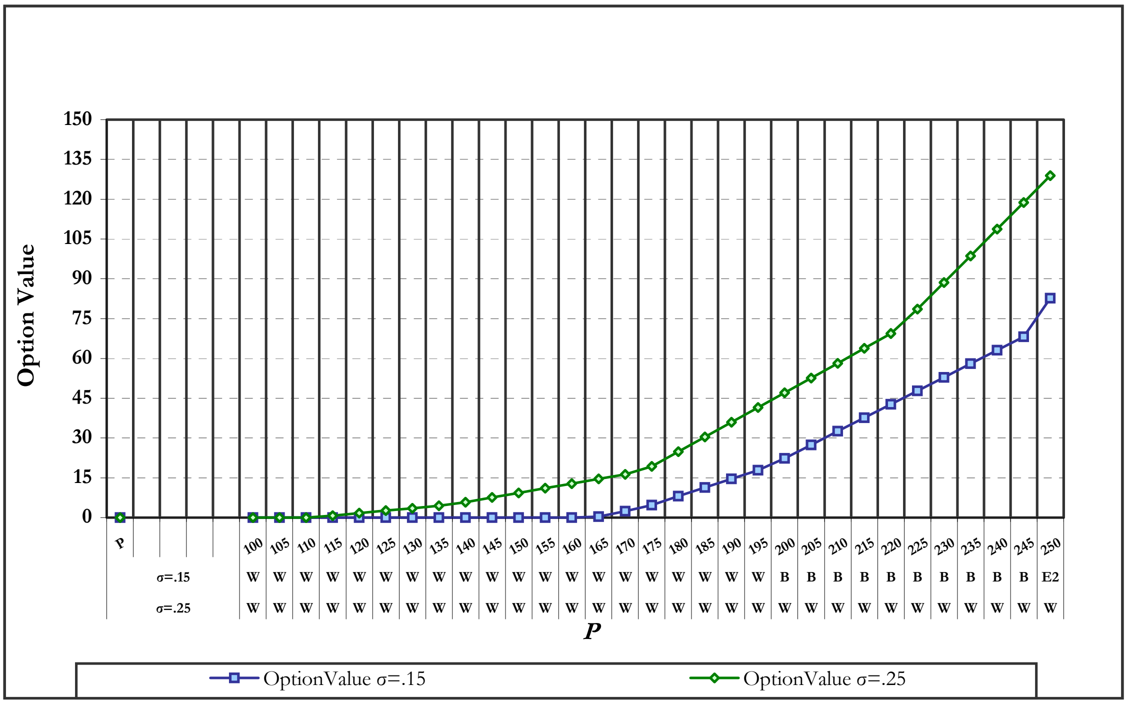
Figure 4e

Sensitivity of option value and optimal operating policy vs. uncertainty for four alternative operating modes without Time-to-Build but with an operating constraint.



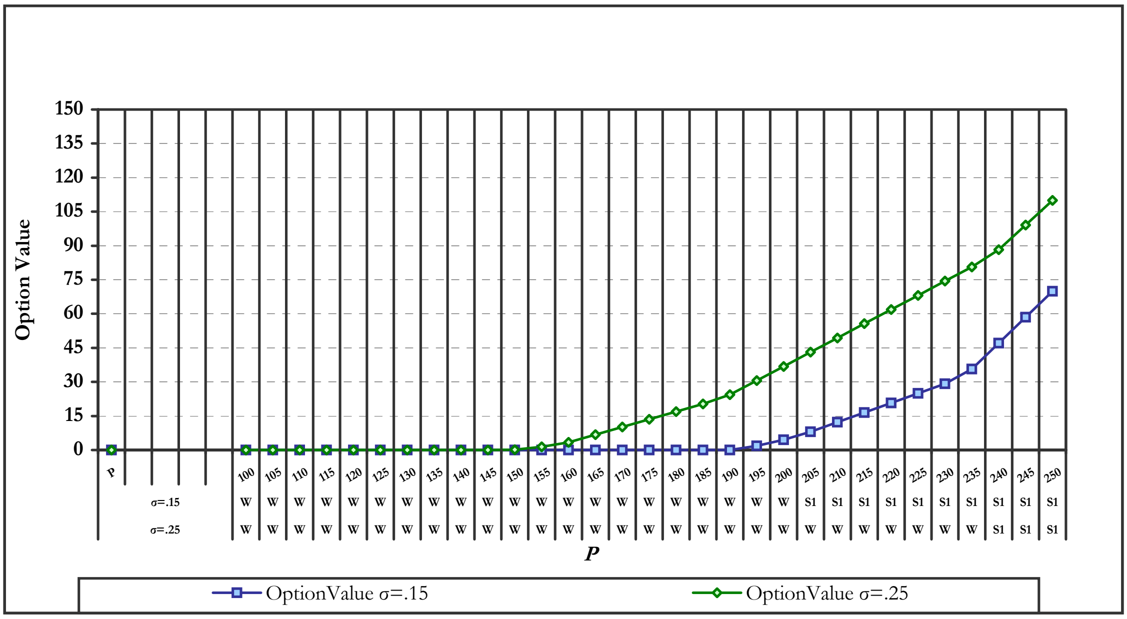
Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model, without Time-to-Build but with a constraint of maximum times of operation= 3. We denote with C the small operating scale, B the base case, E_1 and E_2 the expanded operating scales (having B as the benchmark). Operating cost $X = 100$, riskless rate $r = 5\%$, $\Delta t = 1$ per period.

Figure 5a
Sensitivity of option value and optimal operating policy vs. uncertainty for the shipping, mothballing and scrapping oil tankers application without Time-to-Build.



Shipping, Mothballing and Scrapping Oil Tankers model, for a (5+1)-stage model without Time-to-Build. We denote with C the small tanker, B the medium tanker, E_1 the large tanker, and E_2 the very large crude carrier. Operating cost $X = 8.8$, riskless rate $r = 5\%$, $\Delta t = 2$ per period.

Figure 5b
Sensitivity of option value and optimal operating policy vs. uncertainty for the shipping, mothballing and scrapping oil tankers application with Time-to-Build.



Shipping, Mothballing and Scrapping Oil Tankers model, for a (5+1)-stage model with Time-to-Build. We denote with C the small tanker, B the medium tanker, E_1 the large tanker, and E_2 the very large crude carrier. Operating cost $X = 8.8$, riskless rate $r = 5\%$, $\Delta t = 2$ per period.

Table 1

Sensitivity of option value and optimal operating policy vs. uncertainty with four alternative operating modes without Time-to-Build but with utilization-dependent abandonment values

	OPTIMAL INITIAL DECISION			OPTION VALUE		
S	σ= 10%	σ= 20%	σ= 30%	σ= 10%	σ= 20%	σ= 30%
	c= 0.70					
75	*W*	*W*	*W*	0.010	6.873	34.743
…	…	…	…	…	…	…
90	*W*	*W*	*W*	1.204	28.574	73.974
92	*W*	*W*	*C*	1.668	32.902	81.597
95	*W*	*W*	*C*	3.233	39.526	95.033
98	*W*	*C*	*C*	6.088	47.930	108.496
100	*W*	*C*	*C*	8.055	55.544	117.834
104	*C*	*C*	*C*	13.797	71.113	136.603
105	*B*	*C*	*C*	16.925	75.520	141.621
106	*B*	*B*	*B*	21.056	80.519	147.574
110	*B*	*B*	*B*	37.741	103.623	173.585
115	*B*	*B*	*B*	65.292	133.789	207.855
119	E_1	*B*	*B*	93.157	158.249	235.416
120	E_1	*B*	*B*	100.629	164.518	242.594
122	E_1	E_1	*B*	115.585	178.253	257.201
123	E_1	E_1	E_1	123.077	186.707	264.605
125	E_1	E_1	E_1	138.070	203.614	280.510
130	E_1	E_1	E_1	181.359	246.975	320.270
…	…	…	…	…	…	…
163	*E2*	*E2*	*E2*	517.351	559.954	630.414
	c= 0.30					
75	*W*	*W*	*W*	0.153	8.582	37.429
…	…	…	…	…	…	…
90	*W*	*W*	*W*	2.929	31.729	78.279
91	*W*	*W*	*C*	3.744	33.996	82.370
95	*W*	*W*	*C*	7.994	44.335	99.433
98	*W*	*C*	*C*	11.252	53.545	113.504
100	*B*	*C*	*C*	14.222	61.264	123.343
104	*B*	*B*	*B*	29.671	78.907	144.639
105	*B*	*B*	*B*	33.538	83.933	150.508
110	*B*	*B*	*B*	57.216	109.956	181.033
115	*B*	*B*	*B*	85.336	141.425	214.038
119	E_1	E_1	*B*	110.465	170.918	241.055
120	E_1	E_1	*B*	117.242	178.939	247.846
123	E_1	E_1	E_1	137.571	203.003	269.971
125	E_1	E_1	E_1	151.124	219.045	287.495
130	*E1*	*E1*	*E1*	185.302	260.220	331.304
…	…	…	…	…	…	…
163	*E2*	*E2*	*E1*	517.351	560.975	633.768
165	*E2*	*E2*	*E2*	541.608	584.027	653.447

Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model without Time-to-Build but with utilization-dependent abandonment values. We denote with C the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost X = 100, riskless rate r = 5%, Δt = 1 per period.

Table 2

Sensitivity of option value and optimal operating policy vs. payout yield with four alternative operating modes without Time-to-Build but with utilization-dependent abandonment values

	OPTIMAL INITIAL DECISION			OPTION VALUE		
S	δ= 0%	δ= 10%	δ= 20%	δ= 0%	δ= 10%	δ= 20%
				c= 0.70		
75	*W*	*W*	*W*	50.040	6.873	0.156
…	...	...	...	…	…	…
90	*W*	*W*	*W*	138.703	28.574	1.455
92	*C*	*W*	*W*	153.296	32.902	1.901
95	*C*	*W*	*W*	180.258	39.526	2.768
98	*C*	*C*	*W*	207.620	47.930	3.705
100	*C*	*C*	*W*	226.043	55.544	4.342
104	*B*	*C*	*W*	264.963	71.113	6.603
105	*B*	*C*	*C*	275.779	75.520	7.319
106	*B*	*B*	*C*	286.619	80.519	8.641
109	*B*	*B*	*B*	320.448	97.590	12.846
110	*B*	*B*	*B*	331.921	103.623	14.626
115	*B*	*B*	*B*	390.135	133.789	25.257
120	*B*	*B*	*B*	449.833	164.518	38.026
121	E_1	*B*	*B*	462.950	170.787	40.587
122	E_1	E_1	*B*	476.647	178.253	43.148
123	E_1	E_1	E_1	490.343	186.707	46.353
125	E_1	E_1	E_1	517.735	203.614	55.718
…	...	...	...	…	…	…
163	*E2*	*E2*	*E2*	1061.020	559.954	270.018
				c= 0.30		
75	*W*	*W*	*W*	56.502	8.582	0.276
80	*W*	*W*	*W*	79.390	14.364	0.590
85	*W*	*W*	*W*	109.626	21.801	1.120
90	*W*	*W*	*W*	143.232	31.729	2.253
93	*C*	*W*	*W*	165.671	38.725	3.735
95	*C*	*W*	*W*	183.548	44.335	4.732
98	*C*	*C*	*W*	210.955	53.545	6.343
100	*C*	*C*	*W*	229.995	61.264	7.527
102	*C*	*C*	*C*	249.313	69.622	8.950
103	*B*	*C*	*C*	259.696	74.238	9.974
104	*B*	*B*	*C*	270.330	78.907	10.998
105	*B*	*B*	*B*	280.989	83.933	12.044
110	*B*	*B*	*B*	336.150	109.956	21.985
115	*B*	*B*	*B*	392.938	141.425	41.497
119	E_1	E_1	E_1	440.496	170.918	58.126
120	E_1	E_1	E_1	453.822	178.939	62.808
125	*E1*	*E1*	*E1*	520.456	219.045	86.222
…	...	...	...	…	…	…
163	*E2*	*E2*	*E1*	1061.208	560.975	274.944
165	*E2*	*E2*	*E2*	1092.805	584.027	288.561

Four alternative operating scales (C, B, E_1, E_2), for a (5+1)-stage model without Time-to-Build but with utilization-dependent abandonment values. We denote with C the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost X = 100, riskless rate r = 5%, Δt = 1 per period.

Table 3

Sensitivity of option value and optimal operating policy vs. uncertainty for the shipping, mothballing and scrapping oil tankers application without Time-to-Build but with utilization-dependent switching and abandonment values.

P (in dollars $)	OPTIMAL INITIAL DECISION		OPTION VALUE (in million of dollars $)	
	σ= 15%	σ= 25%	σ= 15%	σ= 25%
	c= 0.70			
165.00	*W*	*W*	0.424	14.660
…	...	...	…	…
185.00	*W*	*W*	11.308	30.754
190.00	*W*	*W*	14.602	36.311
195.00	*W*	*W*	17.897	41.868
196.99	*B*	*W*	19.209	44.080
200.00	*B*	*W*	22.279	47.425
205.00	*B*	*W*	27.379	52.982
210.00	*B*	*W*	32.478	58.539
215.00	*B*	*W*	37.578	64.301
220.00	*B*	*W*	42.677	70.163
225.00	*B*	*W*	47.777	78.766
230.00	*B*	*W*	52.877	88.700
…	...	...	…	…
245.88	*E2*	*W*	69.120	120.576
250.00	*E2*	*W*	82.468	128.865
…	...	...	…	…
287.88	*E2*	*E2*	205.200	205.553
	c= 0.30			
165.00	*W*	*W*	4.662	25.593
…	...	...	…	…
185.00	*W*	*W*	14.068	43.756
190.00	*W*	*W*	17.153	49.143
195.00	*W*	*W*	20.237	54.709
196.41	*B*	*W*	21.109	56.333
200.00	*B*	*W*	24.606	60.470
205.00	*B*	*W*	29.478	66.232
210.00	*B*	*W*	34.363	72.595
215.00	*B*	*W*	39.248	81.640
220.00	*B*	*W*	44.132	90.710
225.00	*B*	*W*	49.119	99.780
230.00	*B*	*W*	54.133	108.850
…	...	...	…	…
247.29	*E2*	*W*	83.160	140.213
…	...	...	…	…
273.54	*E2*	*E2*	164.620	190.086

Shipping, Mothballing and Scrapping Oil Tankers model, for a (5+1)-stage model without Time-to-Build, using utilization-dependent switching and abandonment values. We denote with *C* the small tanker, *B* the medium tanker, E_1 the large tanker, and E_2 the very large crude carrier. Operating cost X = 8.8, riskless rate r = 5%, Δt = 2 per period.