EQUITY RISK OF THE CDB PORTFOLIO

by
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Working Paper 02-10

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Abstract

In this report we analyze different methodologies for assessing the risk of the traded equity component of the portfolio of the Cyprus Development Bank. The approach taken is to fit the Capital Asset Pricing Model (CAPM) to each of the securities. We consider specifications of the CAPM with the CSE General Index as the market index, as well as the use of the relevant sector indexes for each equity. The quality of the fit of the CAPM to the securities is assessed in detail. Once the CAPM has been fit, we examine different methodologies for scenario generation and risk measurement for the CDB portfolio. Detailed results are presented on the effectiveness of using each method for estimating the industry standard risk measure Value-at-Risk (VaR). Methods for dealing with the lack of data and liquidity of the local market are also discussed.

Contents

1 Introduction

As Cyprus moves towards accession to the European Union, the local environment will become increasingly risky and competitive. In this atmosphere of increased uncertainty, financial institutions will be under increased pressure to ensure that they have the tools to properly measure and manage the risks faced by their portfolio. A dramatic example of the need for risk management for equity portfolios in particular was provided by the recent speculative bubble on the Cyprus Stock Exchange (CSE) [1]. This report presents a comprehensive study of the risks facing the equity portfolio of the Cyprus Development Bank. The methodology employed is to fit the Capital Asset Pricing Model (CAPM) to each of the securities, then using this model generate scenarios for the future of the entire portfolio. Based on these future scenarios, various risk measures are calculated, which provide a view on the riskiness of the securities held by the bank. The accuracy of the risk measurement is assessed through a historical backtesting exercise: we examine how each method employed would have performed throughout the entire history of the CSE.

1.1 Risk Management in Emerging Markets

Risk managers in emerging markets face a number of challenges that do not present themselves to their colleagues in more developed economies. The first and most apparent is the often chaotic state of the local economy. The second is the short history of these markets. This has a number of significant consequences. One is the relative novelty of financial markets (both to institutions and households; this can be a major cause of speculative bubbles). Another problem, equally important from a risk measurement perspective, is that there is a startling scarcity of available data. Furthermore, those data that are available are of dubious reliability (for example, the time series of a stock market that has recently undergone a speculative bubble). In many emerging markets it is a daunting, if not impossible, task to estimate a reliable interest rate curve. This is quite different from the situation in advanced markets, where researchers can argue over the theoretical and numerical niceties of different

[1] for an analysis of how different risk management techniques would have performed during the bubble period, see Nerouppos et al.

curve fitting methodologies at the forefront of mathematical research. Emerging markets also typically lack the liquidity and availability of derivative instruments required by the theoretical assumptions of many standard mathematical models. Finally, we observe that in many emerging markets, there is a lack of qualified individuals with the knowledge and experience to develop risk management systems.

1.2 The Cypriot Environment

For a long time, much of the Cypriot economy has been a controlled, stable environment (particularly in the areas of interest rates and foreign exchange). This observation is not true of the stock market, which has recently undergone one of the most dramatic speculative bubbles in history. While liberalizing economic reforms point the need to develop more sophisticated methods for managing interest rate and foreign exchange risk, the need for effective equity risk management systems is readily apparent.

Figure (1) shows the entire history of the CSE General Index, in log-scale. The history of this index, and the performance of different risk management techniques over this period are analyzed in detail in (Nerouppos et al.)

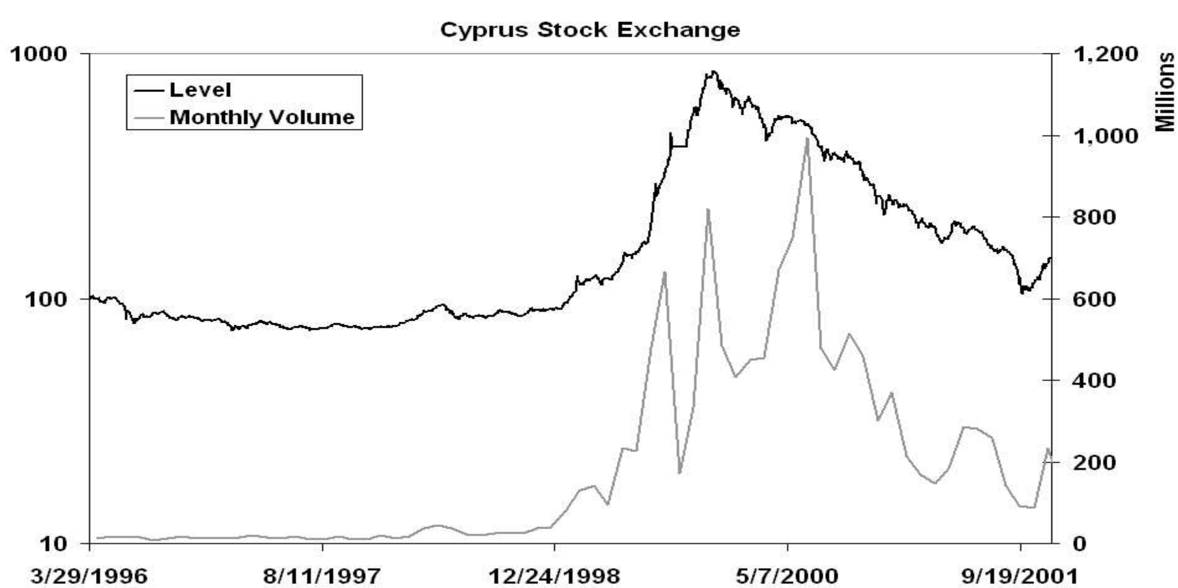

Figure 1: Cyprus Stock Exchange: Level and monthly volumes (period division).

1.3 Risk Measurement for Equity Portfolios

One of the main functions of any risk management operation is to determine the amount of capital that the institution must hold in order to be reasonably sure of covering its potential losses. Methods for determining this capital amount are specified by the local regulators, often following the lead of the Basel Committee of the Bank for International Settlements. Institutions following the Basel guidelines may either calculate capital allocation for market risk using standard formulae provided by the committee, or by calculating their Value-at-Risk based on internal models (if local regulators approve of the use of internal models, and the bank's models are tested and approved).

Let $\Pi_{t+\tau}$ denote the value of a bank's portfolio at the risk horizon $T = t + \tau$, assuming that today is time t. Then the bank's Value-at-Risk at the horizon T and the confidence level α is given by the largest number ξ such that

$$\mathbb{P}[\Pi_t - \Pi_{t+\tau} \geq \xi] = 1 - \alpha \tag{1}$$

we then write $\xi = \mathrm{VaR}_{t,\tau}(\Pi; \alpha)$.

In a regulatory environment where the use of internal models is approved (as is the case in all developed markets, but not the case in many emerging markets, due to the factors discussed above), required regulatory capital is determined as a multiple of the bank's Value-at-Risk. That is, the bank is required to hold an amount C given by the equation:

$$C = \max(\mathrm{VaR}_{t-1,\tau}(\Pi; \alpha), \kappa \cdot \overline{\mathrm{VaR}_{t,\tau}}(\Pi; \alpha)) \tag{2}$$

where Π is the bank's portfolio, $\alpha = 0.99$, τ is 10 days, κ is determined by the performance of the bank's models in a backtesting experiment, and

$$\overline{\mathrm{VaR}_{t,\tau}}(\Pi; \alpha) = \frac{1}{60} \sum_{i=t-60}^{t-1} \mathrm{VaR}_{i,\tau}(\Pi; \alpha). \tag{3}$$

This paper presents results for a number of different VaR methodologies using the back-testing methodology recommended by the Basel Committee of the Bank for International Settlements (2001).

1.4 Structure of the Report

The remainder of this report is structured as follow. The second section introduces the Different methods used for measuring and simulating the performance of equity portfolios. The third section provides an analysis of the returns of the CSE General Index and its various subindexes. The fourth section presents a brief introduction to the Capital Asset Pricing Model, upon which many of the results in the remainder of the report are based. The fifth section presents a detailed analysis of the securities contained in the CDB equity portfolio, including fitting the CAPM for each security to both the General Index and the relevant subindex. The sixth section presents the results of the historical backtests on the measurement of Value-at-Risk for a portfolio composed on the same equities as those held by CDB. The seventh section concludes. Appendices present further backtesting results, as well as useful results on the theory underlying our calculations.

2 Simulation Methods

Two fundamentally different but popular simulation methods will be used in our study [2]. Namely the *historical* and the *Monte-Carlo* methods. The main assumption that underlies the historical method is that history repeats itself, while the main assumption of the Monte-Carlo is that the shocks that drive returns are normally distributed.

2.1 Historical Scenario Generation Method

The significance of this method lies in the fact that we can replicate the actual distribution of daily/weekly/monthly returns (depending on what we are interested in), without any assumptions on parameters. The way it works is the following. At the first step, after choosing the relevant time horizon (T days/weeks/months) we calculate all the observed returns for the historical periods in which we are interested[3]. Then we randomly sample from these returns, after giving weights to each data point, to create future scenarios. We can either apply equal weights or weights depending on an exponential distribution[4].

For example if we want to generate a 4-week scenario (i.e. generate a scenario in which we create one value for each index for four weekly intervals), using only information from a particular period, we read all the weekly returns for this period and then we choose randomly four weekly returns to make up our scenario. This procedure can be repeated several times to construct a scenario set. The size of the scenario set is constrained by the number of historical observations available.

As already mentioned, the main assumption of this method is that history repeats itself. Another strong assumption is that the returns we sample from are independent. This allows us to generate our scenarios by random sampling. As we will see later, this assumption does not hold as there is significant serial correlation in the CSE returns. When we replicate entire periods we relax the assumption as we only require the first shock to be independent of the

[2] These are the same methods used by Nerouppos et al (2002).

[3] This is where the relevance of the subjective information comes into play. We choose the periods we believe are more relevant to the future periods for which scenarios are generated.

[4] With the exponential distribution we assume that the more recent data are more significant.

history. The rest of the scenario will be as it was realized at some point in the past.

2.2 Monte Carlo Scenario Generation Method

The Monte Carlo method simulates the different risk factors according to a particular specification of a process. Here the process we use is the Geometric Brownian Motion (GBM), which is the solution of the stochastic differential equation (SDE):

$$dX_t = M_t X_t dt + \Sigma_t diag(X_t) dW_t \qquad X_0 = x_0 \tag{4}$$

or equivalently the discretized version

$$\Delta X_t = M_t X_t \Delta t + \Sigma_t diag(X_t) \xi_t \sqrt{\Delta_t} \qquad X_0 = x_0 \tag{5}$$

where

X_t is the $n \times 1$ vector of the n risk factors

$diag(X_t)$ is the $n \times n$ matrix with the elements X_t of its diagonal

M_t is a $n \times n$ diagonal matrix with the growth rates as elements of the diagonal

Σ_t is the $n \times n$ variance-covariance matrix

W_t is an $n \times 1$ vector of Wiener processes

ξ_t is an $n \times 1$ vector of i.i.d. $N(0,1)$ random variables.

There are various ways to estimate the variance-covariance matrix depending on which model seems most appropriate. The three estimation models used in our study are the following:

- Simple constant sample estimation model. The volatility is the historical sample standard deviation of all the returns available.

- Exponentially Weighted Moving Average (EWMA) model. This is the RiskMetrics approach and the variance is a weighted average of the past squared returns. The weights depend on a parameter, λ, which is estimated to fit the data.

- General Auto-Regressive Conditional Heteroskedasticity (GARCH) model. This model is a more general case of the above (EWMA) and the spot variance depends on the previous observed squared return, and on the estimated variance.

3 Analysis of the CSE indexes

In this section we analyze the CSE general index and some sector indexes (Banks, Investment, Manufacturing and Finance) which are used for furthermore analysis in the following sections. Following the analysis of Nerouppos et al (2002) we split the history of the Cyprus Stock Exchange into three periods, in order to analyze and better understand the behavior of the estimated CAPM parameter within them, defined as:

- Period 1: 29/03/1996 - 30/06/1999
- Period 2: 01/07/1999 - 31/10/2000
- Period 3: 01/11/2000 - 23/11/2001

The complete history of the CSE General Index is shown in Figure 2, together with the period division and the monthly volumes, while in Figure 3 the level history of some sector indexes is shown.

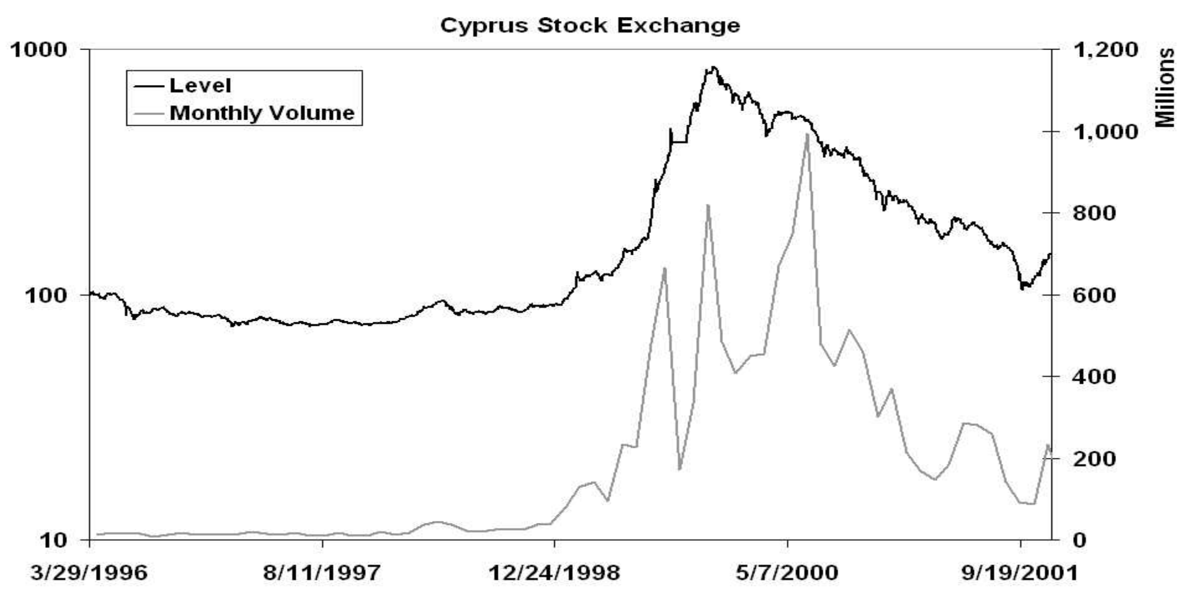

Figure 2: Cyprus Stock Exchange: Level and monthly volumes (period division).

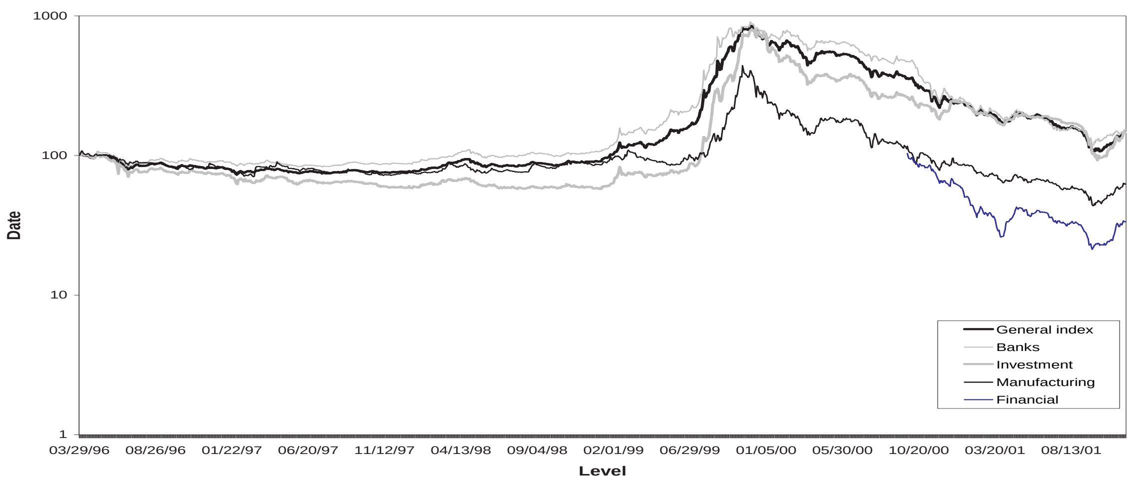

Figure 3: Cyprus Stock Exchange: Level of some Sector Indexes.

3.1 Daily Returns

We first analyze the daily returns of the General Index and the sector indexes. The daily returns are defined as

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{6}$$

where S_t is the closing level of the index or the subindex on day t.

The descriptive statistics of the daily returns for the CSE general index and the subindexes are shown in Table (1). Moreover the normality tests of some sector indexes, which are used for further analysis in the following sections, are shown in Table (2). From these statistics we infer that the returns of almost all indexes[5] are not normally distributed as they exhibit particularly fat tails. The Excess Kurtosis for all indexes is considerably higher than 0, which corresponds to normal distributions. The General Index and the Banks Index are slightly positively skewed, something that is also indicated by the comparison between the two tail extremes, Min and Max. As already mentioned, only the returns of the Financial index seems

[5]Except only the Financial index.

to be normally distributed as indicated by the descriptive statistics and the normality tests, Figure (5). This is somehow expected, because this specific sector started on 28/09/2000, therefore it's operating period coincides with period 3 of the CSE general index. As shown by Nerouppos et al (2002), the returns of that period were close to being accepted as normally distributed.

	Mean	StDev	Skewness	Exc.Kurtosis	Min	Max
General Index	0.0277%	1.945%	1.85	24.40	-10.08%	23.68%
Banks	0.0314%	2.166%	2.92	43.69	-11.53%	31.95%
Approved Investments	0.0275%	2.234%	1.00	9.79	-13.61%	17.09%
Insurance Companies	-0.0430%	2.573%	0.01	7.61	-19.96%	14.16%
Manufacturing Companies	-0.0346%	2.260%	1.17	10.89	-12.09%	18.84%
Trading Companies	0.0168%	2.584%	-0.03	8.69	-20.66%	14.84%
Tourism Companies	-0.0684%	2.834%	0.33	7.96	-21.60%	19.07%
Financial	-0.393%	2.642%	0.31	1.68	-10.78%	9.907%
Other Companies	0.0383%	2.311%	0.94	9.84	-13.08%	20.44%

Table 1: Cyprus Stock Exchange: Descriptive statistics of daily returns, 29/03/1996-23/11/2001.

Note: The following statistics test the null hypothesis of normality. At the 5% level the null hypothesis is rejected when the p-value (in parentheses) is lower than 5%.

Test	CSE G.I.	Banks	Investment	Manufacturing	Finance
Anderson-Darling (A^2)	54.09(0%)	inf (0%)	51.252(0%)	inf (0%)	0.749(12%)
Kolmogorov-Smirnov (D)	0.014($<$ 1%)	0.15 (0%)	0.13 (0%)	0.14($<$ 1%)	0.045(61%)
Chi-squared (χ^2)	1775.53(0%)	850.72(0%)	749.82(0%)	792.49(0%)	25.86(3.9%)

Table 2: Cyprus Stock Exchange: Normality tests for sectors daily returns.

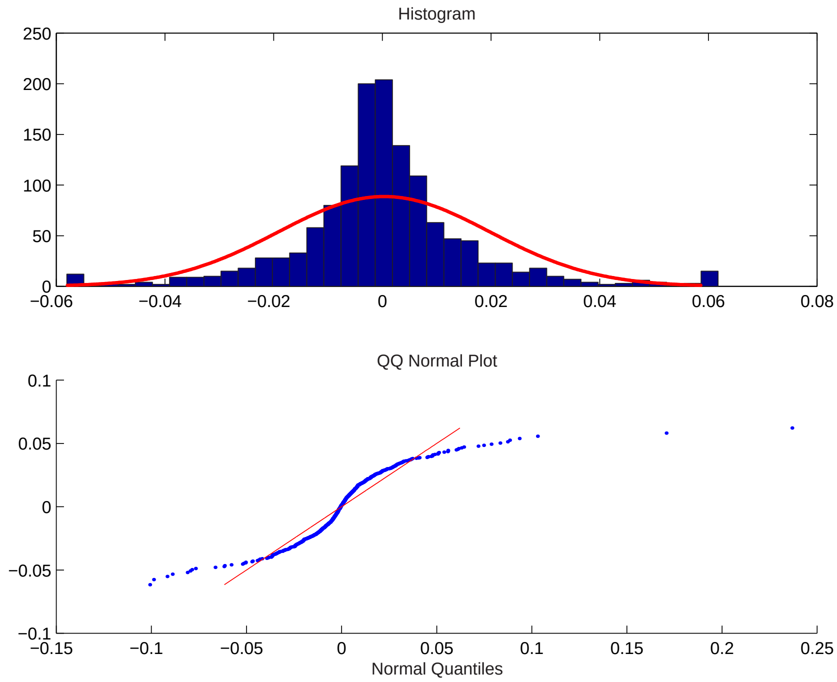

Figure 4: Cyprus Stock Exchange: Histogram and QQ-plot of the General Index daily returns.

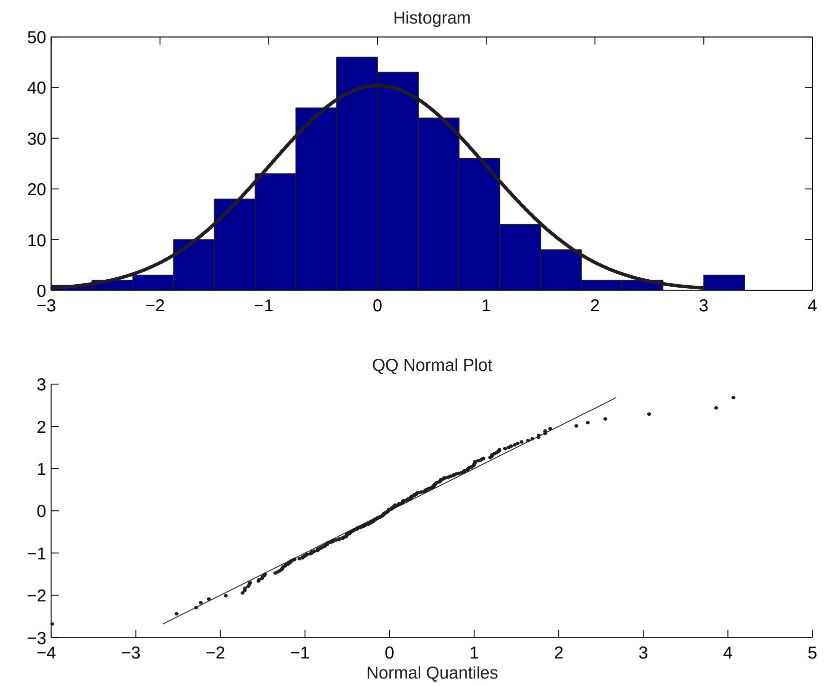

Figure 5: Cyprus Stock Exchange: Histogram and QQ-plot of Finance index daily returns.

3.2 Volatility of Daily Returns

The volatility[6] of a risk factor is very important for risk management as it is one of the parameters required to capture the tail of a distribution. Under the normality assumption the variance is actually the only parameter required. **The assumption that the volatility of the daily returns of the CSE is constant collapses immediately when we look at the running 25-day volatility window during the index's entire history**. This is shown in Figure 6. The volatility at a particular day is estimated as the sample standard deviation of the previous 25 returns, including the current return.

$$\sigma_{25,t} = \sqrt{\frac{1}{24}\sum_{i=0}^{24}(r_{t-i} - \bar{r}_{25,t})^2} \quad (7)$$

$$\text{where } \bar{r}_{25,t} = \frac{1}{25}\sum_{i=0}^{24} r_{t-i} \quad (8)$$

This is one way to estimate the current volatility, namely it is a historical estimation. Of course different amounts of historical data can be used for every estimation, for example using the 10 previous returns instead of 25, in a tradeoff between accuracy and relevance. Even the absolute value of the spot return can be taken to be the volatility, but all of the above are just approximations. They suffice, though, in indicating that the volatility is varying and maybe even stochastic. One other way would be to estimate the Black-Scholes implied volatility [7]. Finally, the volatility at every point in time can be estimated through a model of the daily returns which allows for varying volatility. This will be pursued in Section 3.3.

[6] We take volatility to be the standard deviation of the variable.

[7] In our case this is not possible, since there are no traded derivatives.

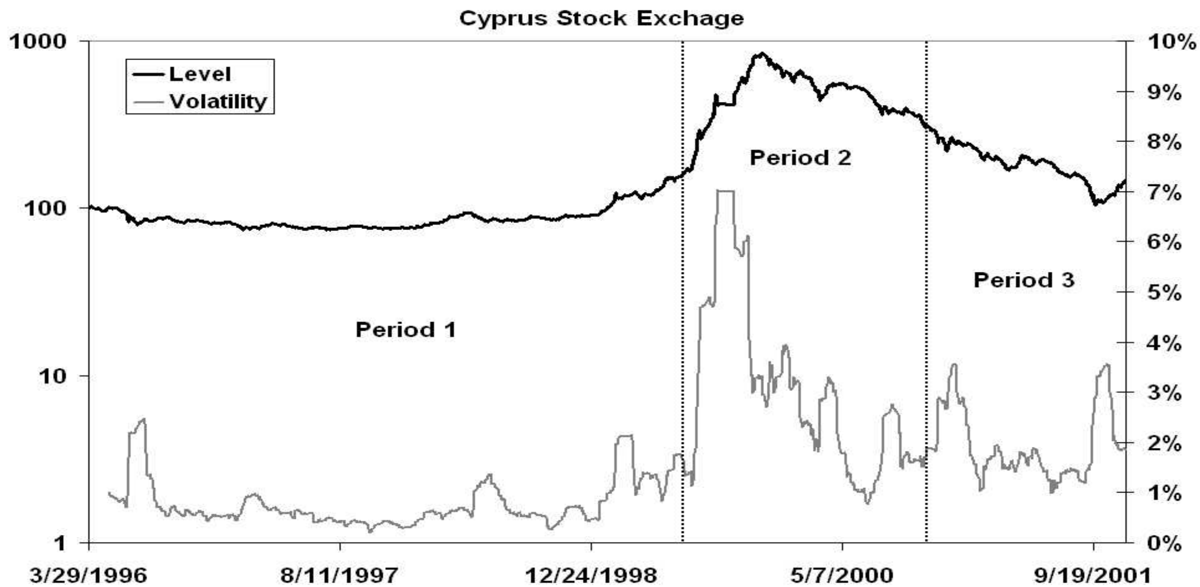

Figure 6: Cyprus Stock Exchange: Level and volatility of daily returns.

3.3 The Basic Model of Daily Returns

We now develop a basic model for estimating daily returns. We take $r_1, r_2, ..., r_t, ..., r_T$ to be our sample of daily returns on particular days, as they are defined by eqn (6) using continuous compounding. A basic model would be

$$r_t = \mu_t + \sigma_t \varepsilon_t \tag{9}$$

where

μ_t is the expected value for the return r_t conditional on information up to time t. The unconditional process can be either deterministic or stochastic.

σ_t is the standard deviation of r_t conditional again on the same information as above. The unconditional process can be either deterministic or stochastic.

ε_t are shocks with zero mean and standard deviation one. They are usually assumed to be independently and identically distributed (*i.i.d.*), as well as normally distributed. This is what we will assume for the rest of this paper as well.

Note that this model is very close to the Monte Carlo simulation model we have seen in Section 2. The latter, in its univariate form, is the solution of the stochastic differential equation

$$\frac{dS_t}{S_t} = \mu_t dt + \sigma_t dW_t \qquad S_0 = s_0 \tag{10}$$

where S_t, μ_t, σ_t and W_t are as previously defined. Now, using Taylor expansion we can show that this equation is analogous to the previous one (9) as

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) = \ln\left(\frac{S_{t-1} + dS_t}{S_{t-1}}\right) = \ln\left(1 + \frac{dS_t}{S_{t-1}}\right) = \frac{dS_t}{S_{t-1}} + O(dS_t^2)$$

so $r_t \rightarrow \dfrac{dS_t}{S_{t-1}}$ as $dS_t \rightarrow 0$

while $dt = 1$, for 1 day and $\varepsilon_t = W_t$

For the conditional process of μ_t (trend) we will use the following:

$$\mu_t = \mu_{m,t} = \begin{cases} \frac{1}{t}\sum_{i=1}^{t} r_{t-i} & \text{, for } t < m \\ \frac{1}{m}\sum_{i=1}^{m} r_{t-i} & \text{, for } t \geq m \end{cases} \tag{11}$$

for all $t \in [0, 1, ..., T]$ and for some constant m

$$\mu_t = 0 \text{ , for all } t \in [0, 1, ..., T] \tag{12}$$

$$\mu_t = \bar{\mu} = \frac{1}{T}\sum_{i=1}^{T} r_i \text{ , for all } t \in [0, 1, ..., T] \tag{13}$$

Equation (11) implies that the trend is taken to be the average of the previous m returns and is used in order to capture the changing trend when the trend is assumed to be varying. For the other two models in (12) and (13) we mention that they may not be consistent with financial theory[8], although they may be of value in risk-management. In order to capture the risk we will have to capture the left tail (downward potential) of the distribution. Any over-estimation of the daily returns' trend causes the under-estimation of the tails and, therefore, the risk[9]. One of course might argue that by setting the mean to zero we may overestimate risk, which is not desirable either. So we will have to test how these models work in practice.

[8] The expected return may be estimated to be lower than the risk free rate. It is not very comfortable to assume that the expected return of a risky asset is lower than the risk-free rate. This would exist only in an economy with risk loving investors.

[9] An overestimation of μ shifts the distribution to the right and therefore the left tail will extend far enough towards negative values, resulting in an underestimate of the risk.

For the conditional standard deviation σ_t, two of the most commonly used models will be employed, the Exponentially Weighted Moving Average (EWMA) and the General Auto-Regressive Conditionally Heteroskedastic (GARCH(1,1)) model.

$$\textit{EWMA:} \quad \sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)(r_{t-1} - \mu_{t-1})^2 \tag{14}$$

$$\textit{GARCH(1,1):} \quad \sigma_t^2 = \omega + \alpha(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2 \tag{15}$$

3.4 Estimation of the EWMA and the GARCH model

As indicated before the returns of the Finance index are considered to be normally distributed and therefore no volatility modelling is needed. In order to estimate the EWMA parameter λ we considered the Maximum Likelihood Estimation (ML)[10] method with conditional mean set to zero, $\mu_t = 0$. The estimated parameter, using the whole history of each sector, is shown in table 3. The daily estimated parameter, shown in Figures (7), (8), (9) and (10), tends for all indexes towards λ=0.9. **This value is somewhat lower than the value of $\lambda = 0.94$ used in the standard RiskMetrics methodology for developed markets.**

	CSE G.I.	Banks	Investment	Manufacturing
λ	0.8816	0.8903	0.8733	0.9034

Table 3: CSE General Index: λ estimates for EWMA model.

Under the assumptions about ε_t, as given in the basic model equation (9), the distribution of the return r_t conditional on the information up to and including time $t-1$, denoted by $\mathcal{F}_{t-1}$, is normal, i.e.,

$$r_t|\mathcal{F}_{t-1} \sim N(\mu_t, \sigma_t^2) \tag{16}$$

where μ_t and σ_t^2 are as given by (11), (12) or (13) and (14) or (15) respectively. Therefore, using the conditional densities we can construct the likelihood function and estimate the parameters. For more details see Appendix A. The validity of the model can be checked by testing the assumptions about the standardized returns ε_t, i.e., independence and normality.

[10]For further analysis information of the two volatility models, please refer to Nerouppos et al (2002).

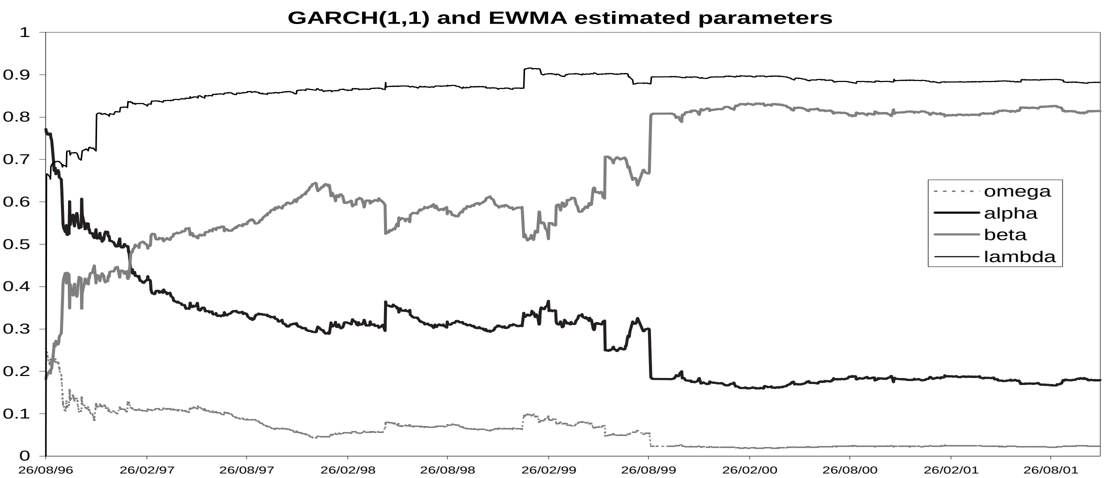

Figure 7: CSE: GARCH(1,1) and EWMA estimated parameters.

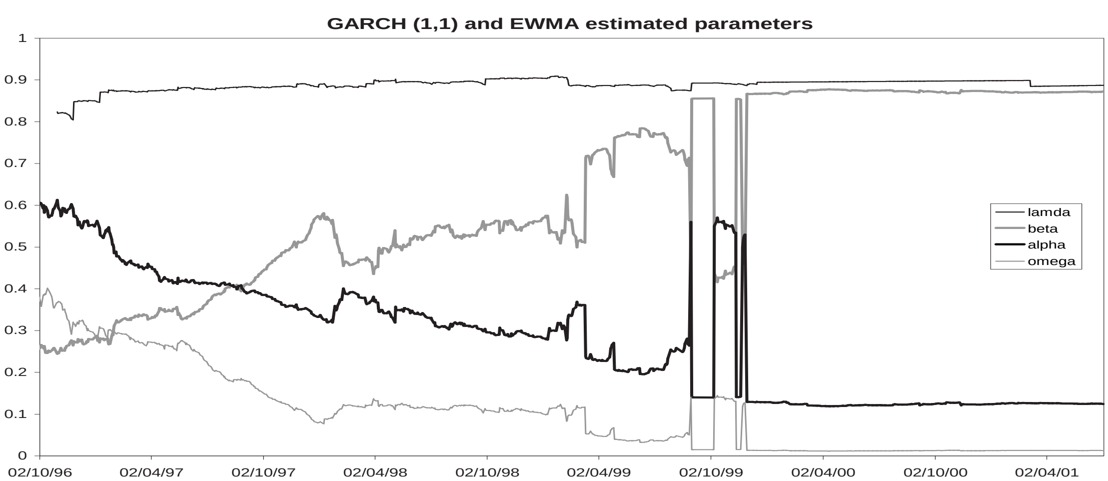

Figure 8: Estimated GARCH(1,1) and EWMA parameters for Banks Index.

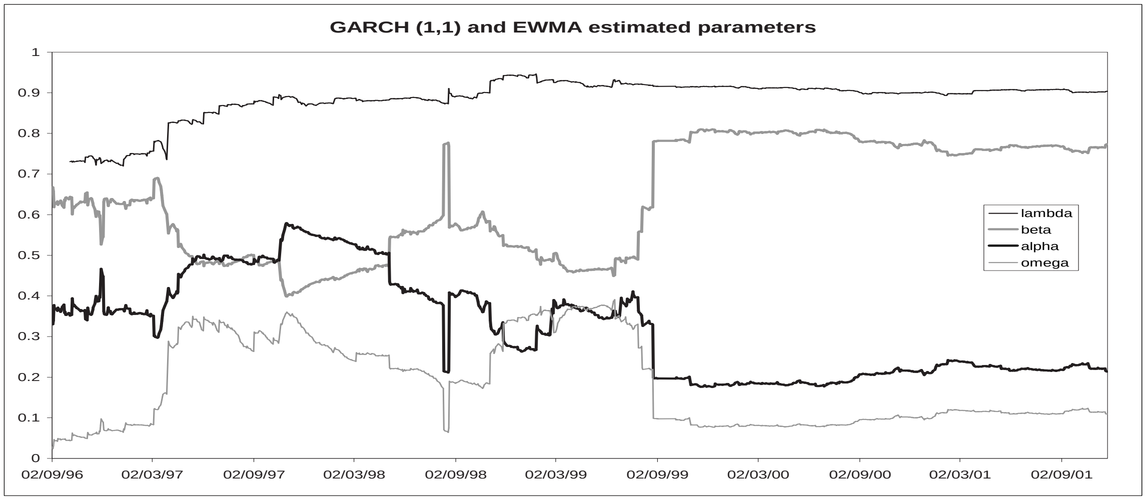

Figure 9: Estimated GARCH(1,1) and EWMA parameters for Manufacturing Index.

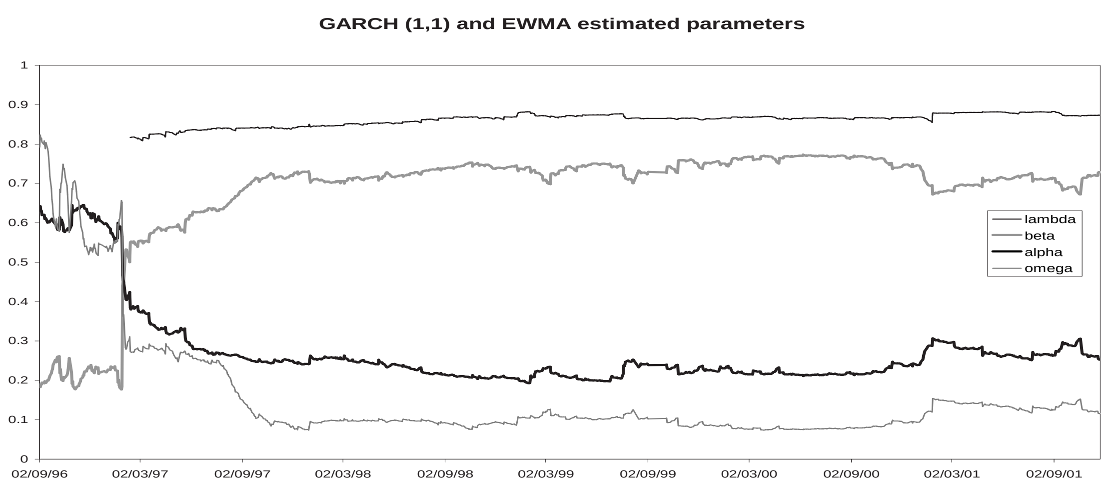

Figure 10: Estimated GARCH(1,1) and EWMA parameters for Investment Index.

In Table 4 we show the descriptive statistics and normality tests of the standardized returns for the CSE genaral index and the three sub indexes estimations with $\mu_t = 0$. The normality tests suggest that no index can be considered as conditionally normal.

$\mu_t = 0$	CSE G.I.	Banks	Investment	Manufacturing
Mean	4.27%	-2.05%	-5.51%	-2.65%
St.Deviation	110.94%	110.62%	110.714	111.36%
Skewness	-0.37	0.31	0.14	1.06
Kurtosis	7.97	6.54	4.87	8.37
Minimum	-925.27%	-603.27%	-740.49%	-433.71%
Maximum	507.77%	687.41%	557.14%	802.18%
And.-Darling (A^2)	6.47 (0%)	11.57(0%)	7.37 (0%)	16.68(0%)
Kolm.-Smirnov (D)	0.053 ($< 1\%$)	0.065($< 1\%$)	0.0482($< 1\%$)	0.079($< 1\%$)
Chi-squared (χ^2)	139.58(0%)	250.18 (0%)	192.65(0%)	258.03($< 1\%$)

Table 4: CSE General Index: EWMA descriptive statistics of standardized returns.

Next we fit the more general GARCH(1,1) process to the time series. The estimation of the parameters is made using ML, as presented in appendix A, and setting $\mu_t = 0$. Initially we estimate the time series of the parameters in order to see whether there is any indication of changes in the behavior of the indexes. As expected such an indication exists. Figure 7 shows the time series of the estimated parameters for the CSE general index. The volatility process is becoming more persistent through time, since β is increasing from 0.2 to 0.8. An analogous behavior is observed for other indexes too, Figures (8), (9) and (10). Moreover the sum of α and β is close (but not equal) to 1.0 which suggests that the volatility process might be integrated[11].

Estimation of the parameters has also been made for the different sub indexes of the CSE as shown by Table 5. For both the CSE G.I. and the Banks index the volatility persistence (indicated by β) is relatively high, $\beta \simeq 0.8$, while for the other 2 indexes it is lower, $\beta \simeq 0.7$.

We note here that every family of estimated parameters implies a long-run volatility (see Appendix A), given by $\sigma_\infty = \sqrt{\frac{\omega}{1-\alpha-\beta}}$. The long-run volatilities of CSE general index and

[11] For a stable GARCH(1,1) process we require $\alpha + \beta < 1$.

the Banks index are very close to each other, $\sigma_\infty \simeq$1.9%, while the long-run volatility of the other sector indexes is higher.

	ω	α	β	Long-Run Volatility	Log-L
CSE G.I.	0.02374	0.17976	0.81373	1.906%	-967.052
	(0.00503)	(0.01774)	(0.01726)		
Banks	0.0135	0.1271	0.8693	1.936%	-1093.68
	(0.0034)	(0.01234)	(0.01227)		
Investment	0.1159	0.2533	0.7284	2.516%	-1289.97
	(0.0210)	(0.02610)	(0.0240)		
Manufacturing	0.1095	0.2128	0.7733	2.806%	-1354.08
	(0.01964)	(0.0228)	(0.0213)		

Table 5: CSE General Index: GARCH(1,1) parameters.

In order to check the validity of the GARCH(1,1) model for the CSE General Index we need to analyze the standardized returns. The null hypothesis is that conditional returns are normally distributed, i.e.,

$$H_0: \quad \frac{r_t}{\sqrt{\sigma_t^2}} | \mathcal{F}_{t-1} \sim N(0,1) \tag{17}$$

As seen in Table 6 the normality test suggests that no index can be considered as conditionally normal.

	CSE G.I.	Banks	Investment	Manufacturing
Mean	-0.0146	0.7929	-4.82	-4.03
St.Deviation	1.0416	105.09	101.58	101.03
Skewness	0.2609	0.286	0.06	0.745
Kurtosis	6.8646	6.892	4.68	7.06
Minimum	-4.8596	566.6	-5.10	-480.8
Maximum	7.1966	736.6	394	711.14
And.-Darling (A^2)	5.843(0%)	11.28(0%)	6.86(0%)	13.56(0%)
Kolm.-Smirnov (D)	0.5362($<$ 1%)	0.056(0%)	0.059($<$ 1%)	0.085(0%)
Chi-squared (χ^2)	143.63($<$ 1%)	326.04(0%)	268.04(0%)	346.55(0%)

Table 6: CSE General Index: GARCH(1,1) descriptive statistics of standardized returns.

We also apply Ljung-Box tests. LB 1 and LB 2 test whether there is any autocorrelation within the normalized and the squared returns respectively, while LB 3 tests whether the GARCH model manages to adequately describe the volatility process. **From the results of Table 7 it follows that the GARCH(1,1) model manages to describe the volatility of each index.** However the models do not eliminate the autocorrelation within the normalized returns.

$LB1:$ *Standardized Returns,* $\left(r_t/\sqrt{\sigma_t^2}\right)$

$LB2:$ *Squared Returns,* $\left(r_t^2\right)$

$LB3:$ *Squared Standardized Returns,* $\left(\frac{r_t^2}{\sigma_t^2}\right)$

	LB 1	LB 2	LB 3
CSE G.I.	260.9(0%)	455.21(0%)	34.96(24%)
Banks	79.76(< 1%)	282.53(0%)	48.23(22%)
Investment	133.85(< 1%)	1335.21(0%)	39.05(12%)
Manufacturing	82.56(< 1%)	1459.1(0%)	31.07(41%)

Table 7: CSE General Index: Ljung-Box statistic (lag 30).

4 Capital Asset Pricing Model

The CAPM gives a pricing formula that relates the excess return of a security or a portfolio to that of the excess return of the market, that is the expected return of a security is given by:

$$E(R_i) = R_{free} + \beta_i(E(R_M) - R_{free}) \tag{18}$$

where

$E(R_i)$ is the expected return of the security

$E(R_M)$ is the expected return of the market

R_{free} is the short-term riskless interest rate

β_i is the security *beta*.

The beta (β_i) measures the relative comovements of security i for which investors should be compensated and is estimated from the regression equation,

$$R_{it} - R_{free} = \alpha_i + b_i((R_{Mt}) - R_{free}) + \varepsilon_{it} \tag{19}$$

where R_{it} and R_{Mt} are the rates of return measured between time $t-1$ and t for security i and an index representing the *market portfolio of risky assets*, respectively. ε_{it} is the residual value, and α_i[12] and b_i are the two regression parameters, with b_i being the statistical estimate of β_i. Merton (1972) has shown that the CAPM can also be derived in a continues time framework, under the assumptions that trades can be executed at any time and that the return-generating process for the stock prices is smooth, with no jumps in prices (i.e. it behaves like a diffusion process).

Taking the variance of equation (18) we have that the total variance of the equity is

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2, \tag{20}$$

[12]The underlying assumption of the CAPM model is that α_i is insignificantly different than zero.

which can be partitioned into the systematic risk[13] $\beta_i^2\sigma_M^2$ and the unsystematic[14], σ_ε^2.

The value of the security is therefore, using equations (6) and (18), given by,

$$V_t = V_{t-1}exp^(R_{free} + \beta_i(E(R_M) - R_{free}) \tag{22}$$

where V_{t-1} is value of the security on day $t-1$.

4.1 The regression model

The generic form of the linear regression model[15] is

$$y_i = f(x_{i1}, x_{i1}, ...x_{ik}) + \varepsilon_i \tag{23}$$

$$= \beta_1 x_{i1} + \beta_2 x_{i2} + ...\beta_k x_{ik} + \varepsilon_i, i = 1, ..., n, \tag{24}$$

or equivalently in matrix form

$$\underline{Y} = \underline{X}\beta + \varepsilon \tag{25}$$

where y is the dependent or explained variable, x_1, x_2,..., x_k are the independent or explanatory variables (*regressors*) and i indices the n sample observations. The term ε is a random shock -*disturbance*, usually drawn from a Normal distribution, $\varepsilon \sim$ N(0,σ^2).

The least square estimator of β and its variance are given by:

$$\widehat{\beta} = (\underline{X}^T\underline{X})^{-1}\underline{X}^T\underline{Y} \tag{26}$$

$$Var(\widehat{\beta}) = \sigma^2(\underline{X}^T\underline{X})^{-1} \tag{27}$$

and asymptotically

$$\widehat{\beta} \sim N(\beta, Var(\widehat{\beta})). \tag{28}$$

[13]The systematic risk is a measure of how the asset covaries with the economy.

[14]The unsystematic risk is independent of the economy as shown.

$$Cov(R_i, R_M) = \sigma_{i,M} = \beta_i\sigma_M^2 + Cov(\varepsilon, R_M) \tag{21}$$

$$= \sigma_{i,M} + Cov(\varepsilon, R_M)$$

which implies that $Cov(\varepsilon, R_M)$=0, only when ε, R_M are normally distributed.

[15]A further analysis of regression models can be found in the classic book of econometric analysis by Green W.H.,"Econometric Analysis", available from Prentice-Hall International, London, 1997.

The classical linear regression model consists of a set of assumptions so that the model is considered suitable for further use. These conditions are:

1. Linear functional form of the relationship.
2. The variance of y is constant for all the different values of x_i, that is the variance of y is independent of the regressors x_i, i=1,...,n and $\sigma_t^2 = \sigma^2$.
3. The random shocks are independent.
4. The random shocks are normally distributed.

4.1.1 Ordinary Least Squares

In an one variable estimation model, (23) becomes

$$y_i = \alpha + \beta x_i + \varepsilon_i, i = 1, ..., n, \tag{29}$$

and therefore the ordinary least squares estimated parameters are taken by minimizing, with respect to α and β, the sum of squared residuals:

$$Q_{OLS}(\alpha, \beta) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \alpha + \beta x_i)^2 \tag{30}$$

The parameters are estimated through

$$\widehat{\alpha} = \overline{y} - \widehat{\beta}\overline{x} \tag{31}$$

$$\widehat{\beta} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \tag{32}$$

We refer to this method of estimation as the ***Ordinary*** method.

4.1.2 Weighted Least Squares

The parameter estimation of a weighted least squares regression is taken by minimizing,

$$Q_{WLS}(\alpha, \beta) = \sum_{i=1}^{n} w_i \varepsilon_i^2 = \sum_{i=1}^{n} w_i (y_i - \alpha - \beta x_i)^2 \tag{33}$$

The parameters are estimated through

$$\widehat{\alpha} = \frac{\sum_{i=1}^{n} w_i y_i - \widehat{\beta} \sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{34}$$

$$\widehat{\beta} = \frac{\sum_{i=1}^{n} w_i x_i y_i - \frac{\sum_{i=1}^{n} w_i x_i \sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}}{\sum_{i=1}^{n} w_i x_i^2 - \frac{(\sum_{i=1}^{n} w_i x_i)^2}{\sum_{i=1}^{n} w_i}} \tag{35}$$

This method is used by RiskMetrics[16] in order to deal with the heteroscedasticity of the returns. The weights are taken to be exponentially decaying, $w_i = \lambda^i$, and λ is the EWMA estimated parameter of the security. This method is hereafter referred as ***Weighted*** method.

4.2 Analysis of Variance

The components of variance (SST, SSR, and SSE), used thoroughly in the analysis of variance, are typically shown in an ANOVA table.

SSR Sum of squares due to regression explains the portion of the total variation due to the linear relationship of y with x.

SSE Sum of squares of errors explains the portion of total variation due to the deviations from the linear regression, or the residual variation left unexplained by the regression line.

SST Total sum of squares is the total variation of y.

MSR Mean square regression is the SSR divided by its degrees of freedom.

MSR Mean square error is the SSE divided by its degrees of freedom.

The coefficient of determination R^2

$$R^2 = \frac{SSR}{SST} \tag{36}$$

is a measure of the fit of the model, and it measures the proportion of the total variation in y that is accounted for by the regressors. While the statistical explanation of R^2 is that it provides a measure of the goodness of fit of the regression, the economic rationale is that it

[16]The ordinary least squares method is just a special case of the weighted where λ equal to 1.

provides a estimate of the proportion of the risk of a firm that can be attributed to market risk; the balance (1-R^2) can be attributed to firm-specific risk.

The standard deviation (St. Dev.) of the estimates (e.g. $\widehat{\beta}$) implies that the true value of β could range over

$$[\widehat{\beta} - 1.96 \text{ St. Dev.}, \widehat{\beta} + 1.96 \text{ St. Dev.}] \tag{37}$$

with 95% confidence. Therefore a small value of standard deviation indicates that the *true* and the *estimated* value of the parameter do not differ significantly.

For a usual regression analysis the null hypotheses

$$H_0^1 : \alpha = 0 \tag{38}$$

$$H_0^2 : \beta = 0 \tag{39}$$

are checked with t-statistics

$$\frac{\widehat{\alpha}}{\sqrt{Var(\widehat{\alpha})}} \sim t_{n-2} \quad , \quad \frac{\widehat{\beta}}{\sqrt{Var(\widehat{\beta})}} \sim t_{n-2}. \tag{40}$$

respectively. The above hypotheses may also be tested by an analysis of variance procedure. The F-test defined by

$$\frac{MSR}{MSE} \sim F_{1,n-2} \tag{41}$$

is also used for testing the models (significance). If the observed significance level for the F-test is small, the hypothesis that there is no linear relationship can be rejected.

Therefore the CAPM model, to be valid, we must accept the H_0^1 and reject the H_0^2, at a confidence level of 95%.

The Durbin-Watson (DW) test checks for serially correlated (or autocorrelated) residuals. One of the assumptions of regression analysis is that the residuals for consecutive observations are uncorrelated. If this is true, the expected value of the Durbin-Watson statistic is 2. Values less than 2 indicate positive autocorrelation while values greater than 2 indicate negative autocorrelation.

For more details about regression analysis, estimation of parameters and analysis of variance see Appendix B.

5 The CDB Portfolio

In this section we firstly describe how the data provided were treated. Then we analyze the securities of the CDB portfolio, estimate the EWMA parameter and then estimate the CAPM parameter β. Finally an analysis of variance is performed.

5.1 The Data

It is very important to mention that Cyprus Stock Exchange is an emerging stock market where not all the securities are traded every day. As a result the data provided for some securities are very problematic. An example is the EURO security where, being an active stock security for 1341 days (02/04/1996-23/11/2001), only 937 days were traded. Moreover within those trading days the number of null returns is large (134), which thereafter influences the estimations.

5.2 Estimation Choices for Beta Estimation

There are three decisions we must take in setting up the regression described before. The first concerns the *length of the estimation period*. Most estimates of betas use five years of data, while Bloomberg uses two years of data. The trade-off is simple: A longer estimation period provides more data, but the firm itself might have changed in its risk characteristics over the time period. In this project the whole history of each equity was under consideration, since the "older" securities have a trading period that coincides with the trading period of the Cyprus stock exchange while the "younger" securities have a trading period less than two years.

The second estimation issue relates to the *return interval*. Return on stocks are available on an annual, monthly, weekly, daily, and on an intra-day basis. Using daily or intra-day returns will increase the number of observations in the regression, but it exposes the estimation process to a significant bias in beta estimation related to non-trading[17]. Using weekly or

[17]The non-trading bias arises because the returns in non-trading periods are zero (even though the market may have moved up or down significantly in those periods). Using these non-trading period returns in the regression will reduce the correlation between stock returns and, ultimately, the beta of the stock.

monthly returns can reduce the non-trading bias significantly. In order to deal with the above problem in the regression estimation, we first estimated the daily returns of each security and then the non-trading days of each security were extracted from the database[18]. Then each of the remaining daily returns was matched to the corresponding market-index daily return, in order to have the same magnitude of observations for both the independent (market index) and the dependent (security) variables. That is, whenever we had an observed daily return for the security then we had the corresponding observed daily return for the corresponding market index. Finally the β used in the backtesting procedure, for the missing days, was the corresponding previous one.

The third estimation issue relates to the choices a *market index* to be used in the regression. The standard practice used by most beta estimation services is to estimate the betas of a company relative to the index of the market in which its stock trades. Both the CSE general index and the corresponding sector indexes for each security, were considered in this project.

Most of the people who use betas obtain them from an estimation service; Merrill Lynch, Barra, Bloomberg are some of the well known services. All these services begin with the regression beta described above and adjust them to reflect what they feel are better estimates of future risk. Although many of these services do not reveal their estimation procedures, Bloomberg is an exception[19].

To the extent that different services use different estimation periods, different market indexes and different beta adjustments, they will often provide different beta estimates for the same firm at the same point in time. While these beta differences are troubling, note that the beta estimates delivered by each services comes with a standard error and it is very likely that all betas reported for a firm fall within the range of standard errors from the regression.

[18]One could instead first fill the missing values of the security by interpolating the data, but with so many missing data this would be misleading in the estimations.

[19]Bloomberg computes the regression beta and also computes what it calls adjusted beta, which is estimated as follows.

$$\text{Adjusted Beta} = \text{Regression Beta } (0.67) + 1.00 \ (0.33)$$

These weights (0.67 and 0.33) do not vary across stocks and this process pushes all estimated betas towards one.

5.3 Analysis of the securities of the portfolio

The securities used for the portfolio are the following

Quote	Name	Start Date	Sector Index
BOC	Bank of Cyprus	29/03/1996	Banks
CLR	CLR Financial Services	03/04/2001	Finance
EURO	EuroInvestment	02/04/1996	Finance
INF	Interfund	25/10/2000	Investment
LPL	Lordos United Plastics	16/05/2001	Manufacturing

Table 8: Portfolio positions.

A portfolio consisted by the above securities, in equal weights of positions[20], is considered for the purpose of backtesting. We note that the Bank of Cyprus, even though is was not a position of CDB, it was included in the backtesting. The BOC was not traded on the following periods

- 16/08/1999 - 27/08/1999
- 15/11/1999 - 26/11/1999
- 16/10/2000 - 20/10/2000

while the EURO[21] was not traded on

- 19/09/2000 - 18/10/2000

and therefore, for those days, the portfolio positions on that securities is set to zero.

The BOC had a split at 16/08/1999 with ratio 2:1 and at 7/12/2000 gave bonus with ratio 6:5, while the EURO had a split at 01/07/1999, ratio 2:1, and at 18/10/2000, ratio 1.5:1. Therefore an adjustment of the securities prices is considered and the following figures shows the adjusted security values[22] through time.

[20]Each security had 1 position.

[21]Note that even though the EURO started trading the same period as the BOC, it has much lower sample size.

[22]When adjusting the security values for splits the returns are not changed in magnitude, except of the day of adjustment. Therefore the return of that specific day is extracted from the calculations.

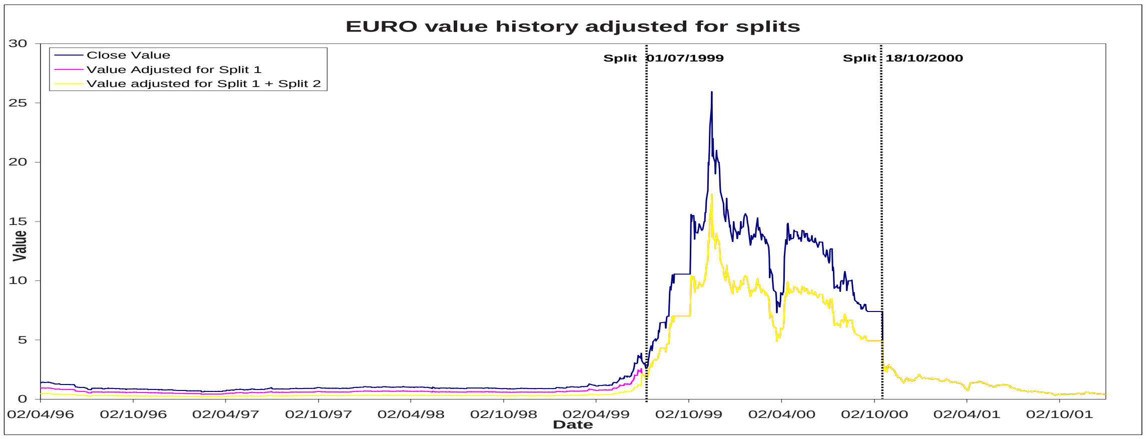

Figure 11: Euroinvestment adjusted value for splits.

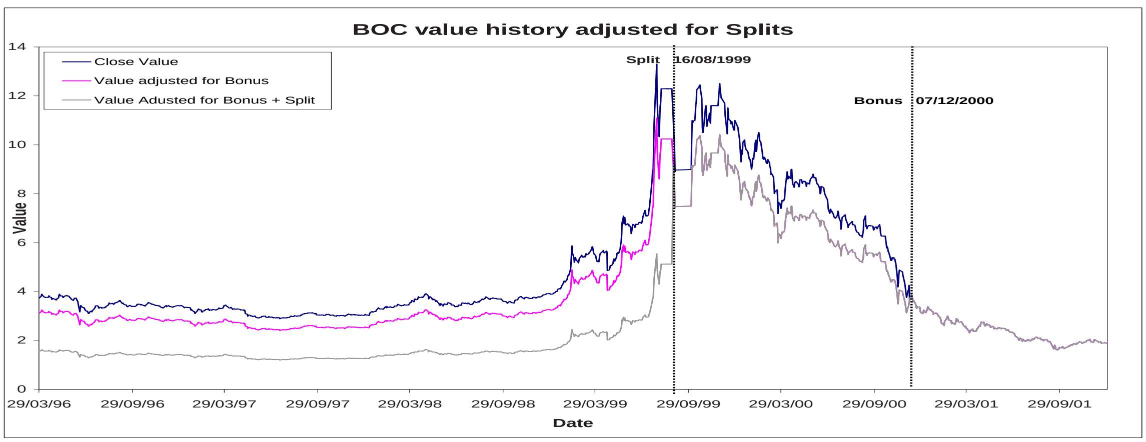

Figure 12: Bank of Cyprus adjusted value for splits.

After appropriately adjusting the security values for splits, the descriptive statistics of the daily returns of each security are shown, for both the sub-periods and the entire history, where available. The high kurtosis indicates that the returns have fat tails and the distributions are peaked. We note that the skewness of EURO is negative in both periods 2 and 3 indicating that there were more and in larger magnitude negative returns as also indicated by comparing the maximum and the minimum. In addition the standard deviation of EURO's returns is significantly high in both periods. Furthermore the CLR, INF and LPL had larger in magnitude minimum than maximum. On the other hand, the returns of LPL (Table 11) are the only ones that can be considered as normal, as indicated by the normality tests.

Note: The following statistics test the null hypothesis of normality. At the 5% level the null hypothesis is rejected when the p-value (in parentheses) is lower than 5%.

	Period 1	Period 2	Period 3	Entire History
Sample	798	280	264	1342
Mean	0.0771%	-0.1988%	-0.2988%	-0.0284%
St.Deviation	1.3137%	3.341%	2.3876%	2.1161%
Skewness	-1.3104	0.829	-0.3147	0.2990
Kurtosis	29.47	6.624	5.41	13.474
Minimum	-14.5%	-11.45%	-11.1%	-14.5%
Maximum	8.3%	20.083%	7.4%	20.083%
And.-Darling (A^2)	30.55(0%)	8.28(0%)	1.39(0%)	∞
Kolm.-Smirnov (D)	0.168(0%)	0.133(0%)	0.191 (0%)	0.067(0%)

Table 9: Bank of Cyprus: Descriptive statistics of daily returns.

	Period 1	Period 2	Period 3	Entire History
Sample	420	257	260	937
Mean	0.2426%	0.0797%	-0.8369%	-0.026%
St.Deviation	2.9768%	6.4874%	6.6484%	5.059%
Skewness	0.5385	-0.881	-0.4543	-0.286
Kurtosis	8.824	8.355	10.8174	9.368
Minimum	-16.8%	-23.6%	-43.3%	-43.3%
Maximum	13.8%	25.15%	28.1%	28.1%
And.-Darling (A^2)	∞	12.74(0%)	6.19(0%)	4.16(0%)
Kolm.-Smirnov (D)	0.114(0%)	0.106(0%)	0.116(0%)	0.102(0%)

Table 10: Euroinvestment: Descriptive statistics of daily returns.

	CLR	INF	LPL
Sample	161	268	120
Mean	0.1009%	-0.1024%	-0.1701%
St.Deviation	2.8395%	4.0484%	5.4151%
Skewness	-1.0604	-0.1543	0.2659
Kurtosis	11.3045	7.1631	3.3997
Minimum	-16.71%	-22.31%	-15.90%
Maximum	10.69%	15.41%	13.90%
And.-Darling (A^2)	6.2496(0%)	4.3852(0%)	0.5571(> 15%)
Kolm.-Smirnov (D)	0.1912(0%)	0.1391(0%)	0.0337(20%)

Table 11: Descriptive statistics of daily returns for CLR, INF and LPL.

5.4 The EWMA decay factor λ for the securities

The estimated parameter λ for the securities for the sub periods is presented in the following table while in Figures (13) - (17) the time-series of the estimated parameter is shown.

The estimated parameter λ for all securities tends to be around 0.9, except for the CLR where $\lambda \simeq 0{,}75$. This implies that CLR has shorter memory, maybe because of the short history of the security and the large number of null returns within its' trading history. **These**

values of λ are somewhat lower than the value of $\lambda = 0.94$ originally used in the standard RiskMetrics methodology for developed markets.

	Period 1	Period 2	Period 3	Entire History
BOC	0.9451	0.8295	0.7469	0.9137
CLR				0.7958
EURO	1	0.7749	0.8035	0.9507
INF				0.8958
LPL				0.8971

Table 12: EWMA : The decay factor λ for each security.

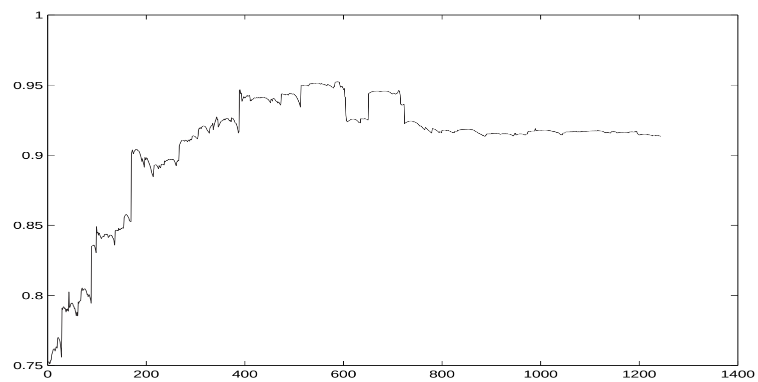

Figure 13: Estimated λ for BOC.

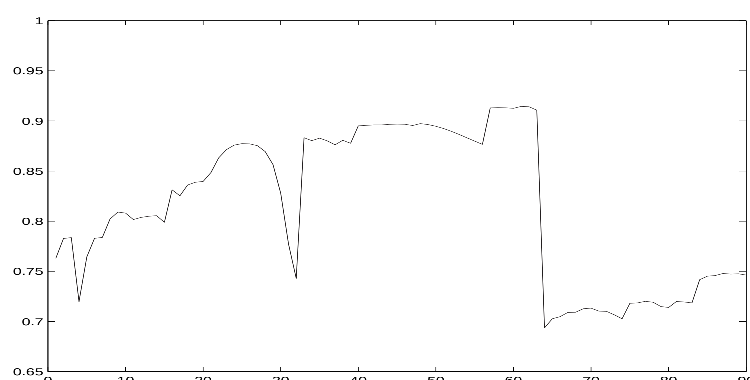

Figure 14: Estimated λ for CLR.

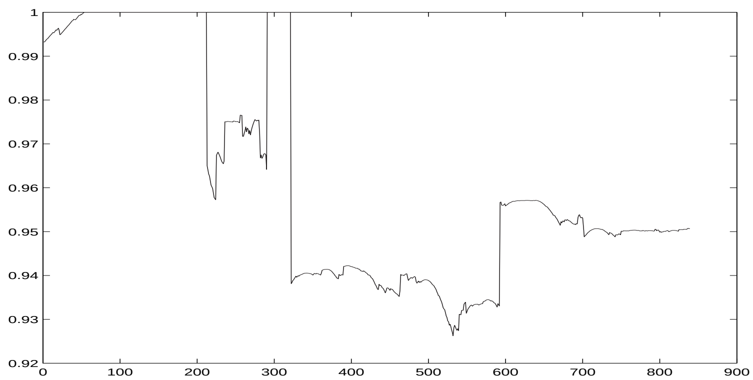

Figure 15: Estimated λ for EURO.

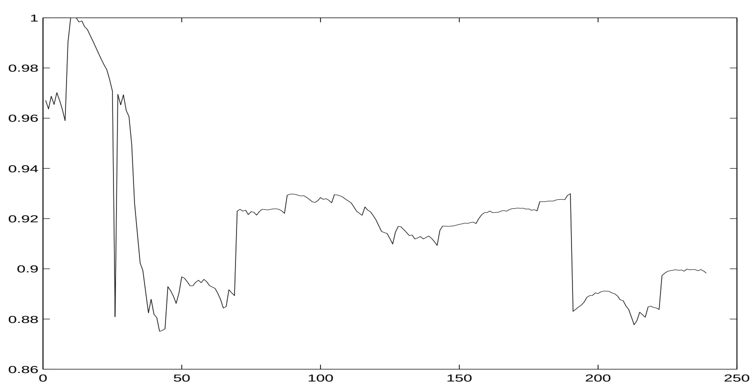

Figure 16: Estimated λ for INF.

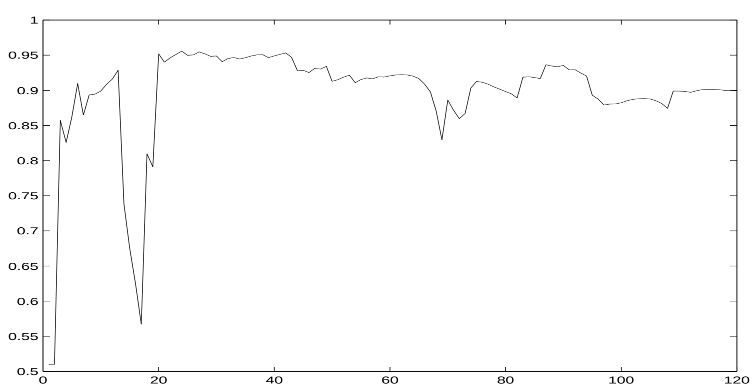

Figure 17: Estimated λ for LPL.

5.5 The estimation of CAPM parameter (*beta* β)

The *beta* estimated for the *Ordinary* model looks stable while, for the *Weighted* model it fluctuates between -1 and +4 as shown by Figure (18). We note that the last estimates (23/11/2001) of β with the CSE general index for the *Ordinary* models are consistently higher than the β with the corresponding indexes. This is natural as the CSE general index is always less volatile than any sector index (no sector index is fully diversified), Table (13).

	General Index	Sector Index
BOC	1.071	0.993
CLR	0.92	0.81
EURO	1.021	0.93
INF	1.321	1.18
LPL	0.909	0.839

Table 13: Estimated β of each security against the CSE General Index and the corresponding sector indexes at 23/11/2001.

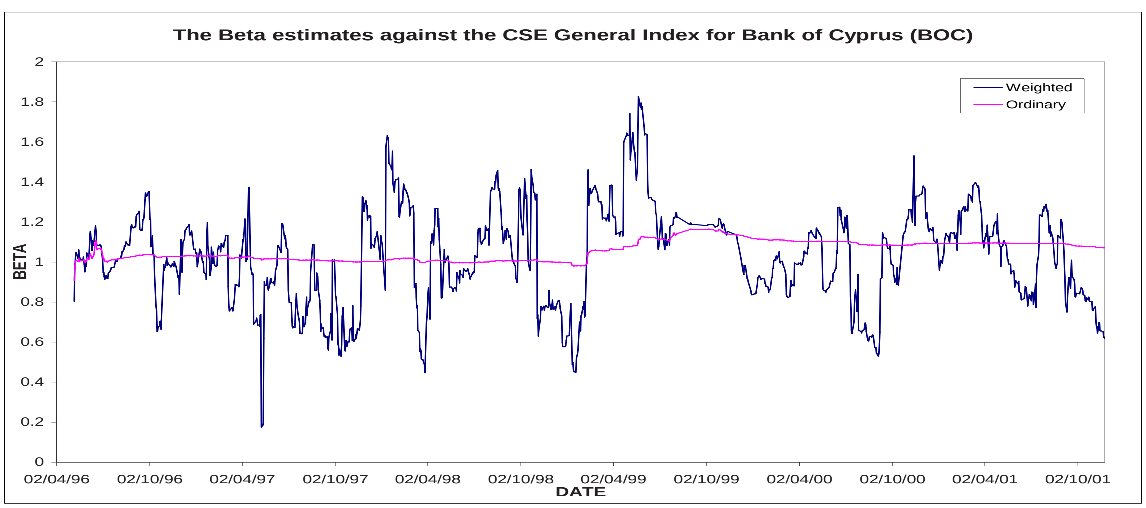

Figure 18: *Beta* estimates of BOC against the CSE General Index.

5.6 Analysis of Variance (ANOVA)

In this section the analysis of variance for the regression models is shown.

We must firstly note that the analysis of variance for the *Weighted* model is only valid for the specific date of estimation, since as already mentioned β is not constant and therefore the weighted least squares estimation model cannot be analyzed (the assumption of an ordinary regression model is that β is a constant).

However we analyze the variance of the *Weighted* model for specific periods. As a result we found that H_0^1 is not always accepted for this method, the significance of the t-statistic of parameter α is zero, Table (14), and therefore a constant term should be added in the model in contrast to the CAPM assumptions. The F-test is indicating that the hypothesis that there is no linear relationship is always rejected, as it should be.

Having fixed the EWMA parameter λ, the *Ordinary* model has consistently greater explanatory power, as indicated by R^2, than the *Weighted* model, Tables (14), (15). The low values of R^2 and the high value of standard deviation, for some securities (e.g. Lordos United Plastics - Table 15), exhibits because of three mainly reasons,

1. The short trading period of the securities,
2. Large number of zero returns and
3. Many outliers occurred during that short period of trading.

A further illustration of the estimated parameter β and further analysis of variance of the securities is presented in Appendix C.

	Period 1	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.004	1	0.004	1238.93	0	0.609
	SSE	0.003	796	3×10^{-6}			
	SST	0.007	797				
Ordinary	SSR	0.09	1	0.09	1572.178	0	0.655
	SSE	0.047	796	6×10^{-5}			
	SST	0.137	797				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-3.4×10^{-3}	0.00049	-6.913	0	1.8876	
	$\widehat{\beta}$	1.23	0.035	35.199	0		
Ordinary	$\widehat{\alpha}$	7.24×10^{-5}	0.0001	0.265	0.791	1.96	
	$\widehat{\beta}$	1.114	0.029	38.887	0		

Table 14: Analysis of variance for BOC against the CSE general index for period 1.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.003	1	0.003	11.793	0.01	0.091
	SSE	0.025	118	0.0002			
	SST	0.028	119				
Ordinary	SSR	0.036	1	0.036	13.687	0	0.104
	SSE	0.313	118	0.03			
	SST	0.349	119				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	2.1×10^{-2}	0.005	4.259	0	2.118	
	$\widehat{\beta}$	0.711	0.207	3.434	0.001		
Ordinary	$\widehat{\alpha}$	-1.32×10^{-3}	0.005	-0.28	0.78	2.374	
	$\widehat{\beta}$	0.846	0.228	3.7	0		

Table 15: Analysis of variance for LPL against the Manufacturing index for the whole history.

6 Risk Management Using VaR Methods

In the previous sections we described the statistics of the CSE and the sector indexes and examined the extent to which the CAPM is a valid model for explaining the variation of some selected stock prices. In this section we firstly present the backtesting framework for assessing the various methodologies. In the second subsection we present the various methods that are used to generate VaR estimates. The backtesting results are presented in the last subsection.

6.1 Backtesting the VaR estimates

The backtest compares VaR measures for one-week, two-week and one-month changes of the portfolio at a 95% or 99% confidence level, against the actual portfolio loss for the corresponding time period. Assuming that the risk factors are correctly modelled and that markets behave accordingly, we expect, on average, the absolute value of actual profit and loss to be greater than the 99% VaR only 2.5 days over the last 250 days.[23] The multiplicative factor κ is determined by the number of times that portfolio losses exceed the corresponding 99% VaR (using a two week time horizon). It usually takes the value of three but it can increase if the number of exceptions is greater than five, and can rise up to four if the number of exceptions reaches ten or more during the period, as shown in Table 16. This multiplier should be viewed as an insurance against model risk or imperfect assessment of specific risks. Another view of this multiplier is as a safety factor against non-normal market moves.

A VaR estimate is generated using the methods described below for a particular time horizon and confidence level. For every VaR figure we keep an overall score that is defined as follows. At some point in time, i.e. a test on a date t, the violation score is defined as

$$V_{p,\tau,m,t} = \begin{cases} 1 & \text{if the actual loss is less than the VaR figure} \\ 0 & \text{otherwise} \end{cases} \tag{42}$$

[23] Indeed, 99 per cent one-tailed confidence level means that we expect losses, but also profits, to be greater than VaR in absolute value 2.5 times per year. Since in this paper we are only interested in downward exceptions (actual net losses greater than VaR) we adjust the number of exceptions to the two year basis-500 days, Table 16.

Number of exceptions in (500) observations	CSE (626)	Multiplier
4 or fewer	6	3.00
5	7	3.40
6	8	3.50
7	9	3.65
8	11	3.75
9	12	3.85
10 or more	13	4.00

Table 16: Multiplier based on the number of exceptions in backtesting.

where p is the one-sided confidence level, τ is the time-horizon and m is the method. If the null hypothesis (H_0) is that the methodology is correct, then for every $V_{p,\tau,m,t}$,

$$H_0 : \quad V_{p,\tau,m,t} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} \tag{43}$$
$$\text{for all } p, \tau, m \text{ and } t \in [1, .., T]$$

The overall score, henceforth called the **violation ratio**[24], for the entire testing period $(1, .., T)$ is defined as

$$V_{p,\tau,m} = \frac{1}{T} \sum_{t=1}^{T} V_{p,\tau,m,t}. \tag{44}$$

If we assume that the violation scores are independent of each other and that T is large, then we can use the Central Limit Theorem to state that under the null hypothesis $V_{p,\tau,m}$ is approximately normally distributed with mean p and variance $\frac{p(1-p)}{T}$. Therefore the null hypothesis for a particular method m, confidence level p, and time horizon τ is defined as

$$H_0 : V_{p,\tau,m} \sim N\left(p, \frac{p(1-p)}{T}\right).^{25}$$

[24]The term violation ratio is somewhat misleading. While we have a violation when the actual loss is greater than the VaR, the violation ratio is greater the greater the number of non-violations we have, since non-violations should occur exactly $\alpha\%$ of the time

[25]Throughout this paper, normally distributed variables are presented as $V \sim N(\mu, \sigma)$

In order to compare VaR figures one should consider the adjusted violation ratio, defined as

$$\widetilde{V}_{p,\tau,m} = \frac{1}{p} V_{p,\tau,m}, \tag{45}$$

with the corresponding null hypothesis

$$H_0^* : \quad \widetilde{V}_{p,\tau,m} \sim N\left(1, \tfrac{(1-p)}{pT}\right). \tag{46}$$

The above distribution can be used to test the null hypothesis by checking whether the estimated violation ratio $V_{p,\tau,m}$ is within the 95% confidence interval specified by the distribution.

It should be noted that while VaR has many desirable properties of a risk measure (it is easy to understand, readily computable, measured in currency), it has some properties that are less desirable. For a general discussion of properties of risk measures, see Artzner et al. (1999).

6.2 The generation of Scenario sets in the CAPM framework

Here we present how the scenarios sets of the different stocks are generated. Using the market information, up to the particular date t, we estimate the β of the security associated with the corresponding *Market Index* (CSE General Index/Sector Index), using one of the different estimation methods (*Ordinary/Weighted*) as described in section 4. In order to generate the scenario sets of the stock we need the scenario sets of the market index. That is, we need to generate scenario sets for the returns of the market index with which the stock is associated. Through those generated scenario sets we derive, by using the estimated β and equation (22), the scenario sets for the value of the stock. Scenario sets for the market index are generated using a number of methods as described in sections 6.2.3-6.2.6 .

6.2.1 The Various Methods

VaR estimates were generated using two different simulation methods, as well as a non-scenario based method that makes use of the RiskMetrics methodology. The different ways for generating the scenario sets and the standard RiskMetrics methodology are described in detail below. The time horizons of interest are 1-week, 2-weeks and 1-month. Each scenario set can generate four VaR figures for each time horizon, as follow:

- *nVaR(95)*: Non-parametric VaR with 95% confidence, i.e., the lower 5% percentile of the empirical distribution of the scenario set.
- *nVaR(99)*: Non-parametric VaR with 99% confidence.
- *pVaR(95)*: Parametric VaR with 95% confidence, i.e., after fitting a normal distribution to the empirical distribution of the returns of the scenario set, we take the lower 5% percentile of the fitted distribution.
- *pVaR(99)*: Parametric VaR with 99% confidence.

Therefore each simulation method generates twelve VaR estimates (four methods for each one of the three time horizons). The RiskMetrics methodology estimates six VaR values at the 95% and 99% confidence level for each of the three time-horizons.

6.2.2 RiskMetrics VaR

Our first approach for VaR estimation is based on the standard methodology pioneered at J.P. Morgan in RiskMetrics. Let the value of a financial instrument be denoted by V, and the difference of today's value from the (random) value at the end of a fixed horizon (T) be denoted by ΔV. Then αVaR, at the one-sided confidence level α, is defined through the relation

$$P(-\Delta V \leq \alpha VaR) = \alpha. \tag{47}$$

Therefore αVaR is the α percentile of the losses. Specifically, the RiskMetrics model assumes that the distribution of the returns is normal with mean zero and variance σ_t^2. It follows that ΔV is a mean-zero normal random variable with single-period variance σ_t^2 ; and the appropriate equation

$$P(-\Delta V \leq \alpha VaR) \approx \alpha \tag{48}$$

implies that

$$\alpha VaR \approx z_{[\alpha]}\sigma_t\sqrt{T-t}. \tag{49}$$

For instance for a 10-day horizon at α=0.95 confidence level we have αVaR=$1.65\sigma_t\sqrt{10}$.

6.2.3 Method 1: Historical Sampling from CSE and other Indexes

With this historical simulation method we generate the scenario sets by sampling the market index historical returns distribution and the distribution of returns of other indexes. In the case of the CSE general index being the market index we arbitrarily give a weight of 82% to the local market (e.g. CSE) information set and 3% to each of the other 6 foreign market information sets[26]. In the case of the market index being the sector indexes the information sets are drawn only from the local market information sets and no foreign market information are used.

As we have already seen the time horizons of interest are 1-week, 2-weeks and 1-month. Therefore we generate the scenarios through sampling from the realized weekly returns to which equal weights were applied. For a particular date t, called the session date, historical scenario sets were generated by sampling from the following periods:

- CSE: 29/03/1996 - t
- ASE: 02/03/1998 - t
- Italy: 03/11/1997 - t
- Mexico: 01/03/1995 - 31/12/1998

[26]Following the same framework of Nerouppos et al (2002) we made use of foreign market information in the historical simulations. The foreign information used was drawn from the stock exchanges of a)Greece b)Italy c)Mexico d)Nasdaq e)Portugal f)Thailand.

- Nasdaq: 01/04/1998 - t
- Portugal: 01/04/1987 - 01/03/1989
- Thailand: 01/10/1991 - 30/10/1998

6.2.4 Method 2: Historical Sampling from CSE and Historical Replication of the Other Indexes

This historical simulation method makes use of the local information set the same way as the previous historical simulation method and the foreign information set by creating *what-if* scenarios. These *what-if* scenarios use the actual evolutions of the other indexes as proxies for possible future evolutions of the local market index. More precisely, when the market index is the CSE general index, a weight of 94% is given to the historical scenarios that are generated by sampling from the local market returns and 1% to each of the 6 *what-if* scenarios. Again when the market indexes are the sectors no foreign information is used and therefore no weight is given to them. The periods that were used for these historical scenarios are the same as the periods of the previous historical simulation method.

6.2.5 Method 3: Monte-Carlo with EWMA Volatility Model

The Monte-Carlo simulation methods use only local market information to produce future scenarios. The process used for the market index is the solution of the driftless SDE (10). That is

$$\frac{dS_t}{S_t} = \sigma_t dW_t \tag{50}$$

with σ_t given by the EWMA model. The discrete approximation of the above (using the Euler discretization) is

$$S_{t+\Delta t} = S_t + S_t \sigma_t \varepsilon_t \sqrt{dt} \tag{51}$$

where ε_t are i.i.d. N(0,1), dt=1 (daily observations) and σ_t is calculated through

$$\sigma_t^2 = \lambda^k r_{t-k}^2 + (1-\lambda) \sum_{i=1}^{k} \lambda^{i-1} r_{t-i}^2 \tag{52}$$

for some k and λ, (we take $k = 74$).

6.2.6 Method 4: Monte-Carlo with GARCH(1,1) Volatility Model

In this Monte-Carlo simulation method we simultaneously simulate both the returns r_t and the volatility σ_t^2 of the market index. It uses, again, the same information set, i.e. only the local market information, so we have that

$$r_t = \sigma_t \varepsilon_t \tag{53}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{54}$$

$$S_t = S_{t-1} e^{(r_t)}. \tag{55}$$

We iteratively simulate this system of equations in order to take into account the effect of volatility upon the tails of the distribution of returns. The difference between this method and the previous one is due to the simultaneous simulation of the volatility together with the return. In this method the future volatility is estimated through the endogenous parameters σ_t^2 [27] and r_t and the exogenous parameter ϵ_t [28], in contrast to the previous method where the future volatility is constant.

[27] The initial value of σ_t^2 is $\sigma_0^2 = \frac{\omega}{1-\alpha-\beta}$ which is the expected volatility as shown in appendix A.

[28] The standard normal shocks are calculated through the Box-Muller algorithm.

6.3 BackTesting Results

In this section we present the results of backtesting with market index being the CSE General Index. The testing period was chosen to start before the bubble of the CSE stock market, 09/02/1999, until the end of the time series, 26/10/2001. Scenario sets using all four simulation methods were generated for each of the dates of the testing period. The VaR figures generated were compared with the actual losses of each security of the portfolio[29], for the dates of the scenario sets. However, for some dates the comparisons were not made. These were the dates for which the forecasting period of the CSE, (1 month ahead), had many days for which there was no trading. More precisely, no testing was made for a date on which there were more than

- Two days of no trading in the one week ahead or
- Five days of no trading in the two weeks ahead or
- Ten days of no trading in the one month ahead

From the 646 trading days in the testing period 20 days were lost because of these restrictions. We need to mention here that, in principle, it does not matter how many trading days there are within a particular time-horizon as what matters is the flow of information which drives the prices. However, the above constraints were applied to avoid any thin-trading effects.

The null hypothesis as defined by equation (46) states that the 95% VaR adjusted violation ratios for a sample of 626 is normally distributed

$$\widetilde{V}_{p,\tau,m} \sim N\left(1, 0.008711\right)^{30}$$

and therefore a 95% symmetric confidence interval [0.9820,1.0180], while for the 99% VaR violation ratios the distribution and 95% symmetric confidence interval are

$$\widetilde{V}_{p,\tau,m} \sim N\left(1, 0.003977\right)$$

[0.9921, 1.0078] respectively.

[29] A similar compare was made for the CSE general index.

[30] The normally distributed variables are presented as $V \sim N\left(\mu, \sigma\right)$

Therefore one way to test each method is to compare its adjusted violation ratio against the appropriate confidence interval. The results are presented in Tables 46-61, Appendix B. The adjusted violation ratios for each method and time horizon are shown.

As the method of generating scenarios of the market index is concerned, regardless of the *beta* estimation method, we seem to have the following results. The best method seem to be the *Parametric VaR 1* method (pVaR 1), while the worst seem to be methods 3 and the *RiskMetrics*, Tables (17) and (18). When the market index is consider to be the sector indexes, then for the best performing methods (pVaR 1, pVaR 2 and PvaR4) the VaR results seems to be better than the corresponding results when the market index is consider to be the CSE general index. Moreover, we see that all parametric methods, using as a market index the corresponding sector indexes seems to perform better than using as a market index the CSE general index.

For both 95 and 99% VaR test results the *parametric* methods are performing better than the *non-parametric* ones, Table 17. The parametric VaR outperforms the non-parametric simply because it always assumes a symmetric distribution for the returns and not because the returns follow a normal distribution. As we have seen in the analysis of the CSE General Index daily returns, they are far from being normally distributed. The scenario sets generated by the historical methods were positively skewed during the period when the level was going up and it took the two methods a while after the turning point[31] to adjust in producing symmetric distributions. It is quite reasonable to assume that the downside potential of the returns is of the same magnitude as the upside potential and thus the parametric method fills in the downside of the distribution, even when no downside has yet been observed. All methods lose power as the time-horizon increases, and this is very natural.

We compare the violation ratios of the securities and the portfolio when estimation of *beta* was made using the *Ordinary* and the *Weighted* method. It seems that the *Ordinary* method performs better than *Weighted* method, Tables (17) and (18).

[31] This is the point when the index stopped going up and started sliding down.

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.930	0.972	0.883	0.955	0.928	0.947	0.928	0.933	0.913
	2-week	0.890	0.950	0.858	0.948	0.913	0.905	0.891	0.908	0.876
	1-month	0.848	0.938	0.805	0.916	0.851	0.851	0.848	0.888	0.827
INF	1-week	0.992	0.999	0.977	0.996	0.953	0.962	0.977	0.982	0.932
	2-week	0.979	0.999	0.977	0.990	0.955	0.953	0.975	0.977	0.920
	1-month	0.967	0.996	0.955	0.980	0.928	0.928	0.950	0.977	0.906
LPL	1-week	0.984	0.992	0.980	0.987	0.980	0.982	0.982	0.984	0.953
	2-week	0.987	0.997	0.985	0.994	0.969	0.965	0.985	0.985	0.935
	1-month	0.974	0.990	0.969	0.990	0.953	0.955	0.965	0.972	0.928
Portfolio	1-week	0.853	0.901	0.819	0.885	0.861	0.868	0.859	0.863	0.841
	2-week	0.809	0.863	0.784	0.859	0.834	0.827	0.817	0.824	0.799
	1-month	0.755	0.822	0.737	0.812	0.770	0.775	0.770	0.795	0.769
BOC	1-week	0.8749	0.9748	0.8851	0.9596	0.8005	0.8039	0.9409	0.9528	0.9240
	2-week	0.8512	0.9562	0.8580	0.9342	0.7666	0.7734	0.9020	0.9156	0.9105
	1-month	0.8039	0.9426	0.8174	0.9172	0.7142	0.7209	0.7700	0.7836	0.8699
INF	1-week	1.0289	1.0408	1.0256	1.0391	0.9223	0.9223	1.0340	1.0340	0.9359
	2-week	1.0188	1.0408	1.0239	1.0391	0.9003	0.9088	1.0154	1.0222	0.9240
	1-month	1.0205	1.0425	1.0137	1.0408	0.9071	0.9139	0.9680	0.9731	0.9206
LPL	1-week	0.9917	1.0052	0.9968	1.0002	0.9579	0.9596	1.0036	1.0036	0.9562
	2-week	1.0019	1.0086	1.0019	1.0086	0.9528	0.9528	1.0036	1.0069	0.9376
	1-month	0.9849	1.0069	0.9883	1.0069	0.9426	0.9426	0.9663	0.9731	0.9172
Portfolio	1-week	0.8123	0.8969	0.8123	0.8919	0.7802	0.7886	0.8665	0.8716	0.8445
	2-week	0.7666	0.8614	0.7666	0.8563	0.7649	0.7717	0.8106	0.8208	0.8039
	1-month	0.7277	0.8208	0.7328	0.8123	0.7057	0.7226	0.6905	0.6989	0.7734

Table 17: Ordinary: Adjusted 95% VaR test results above over the CSE general index and below over the sector indexes (R_{free}=0.00).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.901	0.953	0.861	0.937	0.918	0.932	0.911	0.920	0.901
	2-week	0.868	0.938	0.844	0.928	0.927	0.920	0.879	0.893	0.888
	1-month	0.812	0.895	0.797	0.876	0.866	0.869	0.836	0.861	0.846
INF	1-week	0.970	0.979	0.962	0.977	0.942	0.947	0.960	0.962	0.913
	2-week	0.960	0.977	0.952	0.975	0.942	0.942	0.948	0.953	0.908
	1-month	0.940	0.967	0.938	0.970	0.918	0.925	0.935	0.945	0.905
LPL	1-week	0.990	0.997	0.979	0.990	0.969	0.970	0.977	0.979	0.942
	2-week	0.979	0.987	0.977	0.989	0.970	0.969	0.975	0.975	0.937
	1-month	0.964	0.970	0.955	0.967	0.952	0.952	0.959	0.964	0.922
Portfolio	1-week	0.854	0.896	0.812	0.871	0.859	0.878	0.851	0.853	0.856
	2-week	0.804	0.869	0.789	0.858	0.839	0.834	0.811	0.824	0.802
	1-month	0.760	0.824	0.743	0.805	0.795	0.795	0.770	0.794	0.780
BOC	1-week	0.8749	0.9562	0.8749	0.9511	0.7869	0.7869	0.9274	0.9291	0.9037
	2-week	0.8326	0.9240	0.8377	0.9223	0.7446	0.7514	0.8936	0.8919	0.8902
	1-month	0.7836	0.9274	0.8005	0.9240	0.6955	0.7006	0.7700	0.7717	0.8462
INF	1-week	0.9883	1.0002	0.9799	0.9968	0.8986	0.9003	0.9799	0.9866	0.9088
	2-week	0.9883	1.0069	0.9866	1.0052	0.8868	0.8885	0.9883	0.9900	0.8986
	1-month	0.9799	1.0002	0.9765	1.0036	0.8699	0.8732	0.9139	0.9291	0.8648
LPL	1-week	1.0052	1.0120	1.0002	1.0137	0.9629	0.9629	1.0103	1.0120	0.9562
	2-week	1.0086	1.0103	1.0086	1.0103	0.9646	0.9663	1.0103	1.0120	0.9494
	1-month	1.0002	1.0103	1.0036	1.0103	0.9596	0.9612	0.9748	0.9832	0.9392
Portfolio	1-week	0.8259	0.8851	0.8191	0.8699	0.7954	0.7988	0.8665	0.8732	0.8546
	2-week	0.7751	0.8648	0.7920	0.8665	0.7700	0.7768	0.8191	0.8276	0.8123
	1-month	0.7345	0.8259	0.7497	0.8191	0.7294	0.7429	0.6955	0.7142	0.7886

Table 18: Weighted: Adjusted 95% VaR test results above over the CSE general index and below over the sector indexes (R_{free}=0.00).

We note that since the portfolio is constructed using equal weights for the securities, then each security affects the portfolio VaR according to its price. That is, for example, the proportional VaR, upon the the portfolio VaR, of a security with price equal to 10 will be greater than the proportional VaR of a security with price equal to 0.50. In Table 19 we present the range of values of each security.

	BOC	CLR	EURO	INF	LPL
MIN	1.204	0.439	0.210	0.056	0.162
MAX	10.417	0.920	17.30	0.166	0.369
RANGE	9.213	0.481	17.09	0.110	0.207

Table 19: Range of adjusted for splits prices of each security of the portfolio.

As a result, even thought that each of the *small* securities (CLR,INF,LPL) has a very high violation ratio (around 0.99) we do not have a high violation ratio for the portfolio (around 0.90), Table (20).

In addition we note that the CLR and the EURO securities were not estimated against their sectors, due to the short history of the Financial sector. Instead CLR was excluded of the backtesting and the EURO was included, but, against the CSE General Index again.

The security with the worst performance in the backtesting is EURO. That is because of the large number of missing trading values, the large range of prices and the large variance of the security. **An important result is that some securities have greater VaR violation ratios than the CSE General Index has,** Table (20). However the portfolio is not performing so well, since the contribution of the EURO is very negative. Therefore in order to justify this observation we perform another backtesting for the portfolio where we did not include the EURO as as position. As it is shown in tables (21) and (22) when we compare the performance of the portfolio, using an ordinary method for estimating β and $R_{free} = 0.04$ the difference between the adjusted 95% VaR results is very significant. As an example for 1-week time horizon, the VaR result for the portfolio was 0.9214 when the EURO is excluded from the portfolio, while when the EURO was included the VaR result for the portfolio was 0.8525.

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.930	0.972	0.883	0.955	0.928	0.947	0.928	0.933	0.913
	2-week	0.890	0.950	0.858	0.948	0.913	0.905	0.891	0.908	0.876
	1-month	0.848	0.938	0.805	0.916	0.851	0.851	0.848	0.888	0.827
CLR	1-week	1.021	1.026	1.017	1.021	1.006	1.009	1.016	1.017	0.980
	2-week	1.014	1.017	1.011	1.019	1.001	0.999	1.004	1.006	0.965
	1-month	1.006	1.009	1.004	1.009	0.989	0.989	1.004	1.007	0.959
EURO	1-week	0.748	0.789	0.720	0.784	0.733	0.750	0.725	0.743	0.723
	2-week	0.695	0.755	0.671	0.745	0.733	0.737	0.708	0.720	0.706
	1-month	0.666	0.718	0.647	0.701	0.688	0.693	0.668	0.686	0.671
INF	1-week	0.992	0.999	0.977	0.996	0.953	0.962	0.977	0.982	0.932
	2-week	0.979	0.999	0.977	0.990	0.955	0.953	0.975	0.977	0.920
	1-month	0.967	0.996	0.955	0.980	0.928	0.928	0.950	0.977	0.906
LPL	1-week	0.984	0.992	0.980	0.987	0.980	0.982	0.982	0.984	0.953
	2-week	0.987	0.997	0.985	0.994	0.969	0.965	0.985	0.985	0.935
	1-month	0.974	0.990	0.969	0.990	0.953	0.955	0.965	0.972	0.928
Portfolio	1-week	0.853	0.901	0.819	0.885	0.861	0.868	0.859	0.863	0.841
	2-week	0.809	0.863	0.784	0.859	0.834	0.827	0.817	0.824	0.799
	1-month	0.755	0.822	0.737	0.812	0.770	0.775	0.770	0.795	0.769
CSE G.I.	1-week	0.888	0.947	0.864	0.940	0.900	0.908	0.900	0.910	0.874
	2-week	0.853	0.932	0.827	0.903	0.863	0.859	0.856	0.863	0.827
	1-month	0.770	0.854	0.748	0.839	0.805	0.809	0.784	0.824	0.797

Table 20: General Index-Ordinary: Adjusted 95% VaR test results (R_{free}=0.00).

Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
1-week	0.9214	0.9651	0.8811	0.9584	0.9197	0.9382	0.9231	0.9349	0.9080
2-week	0.8861	0.9450	0.8508	0.9197	0.9063	0.9012	0.8912	0.8962	0.8710
1-month	0.8323	0.9147	0.7886	0.8962	0.8441	0.8458	0.8474	0.8743	0.8189
1-week	0.8525	0.9063	0.8239	0.8878	0.8609	0.8676	0.8659	0.8676	0.8407
2-week	0.8054	0.8659	0.7802	0.8609	0.8340	0.8273	0.8155	0.8239	0.7987
1-month	0.7566	0.8256	0.7264	0.8037	0.7701	0.7751	0.7566	0.7970	0.7684

Table 21: Ordinary: Adjusted 95% VaR test results for the portfolio over the CSE general index. Above in the portfolio the EURO is excluded while below is included (R_{free}=0.04).

Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
1-week	0.8710	0.9651	0.8626	0.9534	0.7819	0.7819	0.9265	0.9366	0.9063
2-week	0.8390	0.9416	0.8390	0.9281	0.7482	0.7550	0.8996	0.9097	0.8962
1-month	0.7903	0.9281	0.7970	0.9029	0.6826	0.6860	0.7583	0.7667	0.8508
1-week	0.8138	0.8861	0.8004	0.8727	0.7751	0.7835	0.8575	0.8609	0.8390
2-week	0.7650	0.8592	0.7701	0.8424	0.7600	0.7667	0.8121	0.8172	0.7987
1-month	0.7180	0.8189	0.7247	0.8071	0.7011	0.7180	0.6894	0.6961	0.7684

Table 22: Ordinary: Adjusted 95% VaR test results for the portfolio over the sector indexes. Above in the portfolio the EURO is excluded while below is included (R_{free}=0.04).

An initial assumption for the phenomenon that some securities performed better that the CSE general index was, due to the value used for the risk-free rate (R_f=0)[32] we had those results. However, when we backtest with R_f=0.04 and compared the results to the ones with R_f=0, no systematic change of the violation ration is appeared[33] , Table (23). For example the violation ratio for BOC using method $nVaR1$, with time horizon 1-week and R_f=0.00 is 0.930 and the corresponding with R_f=0.04 is 0.9248, while for INF with R_f=0.00 is 0.992 and the corresponding with R_f=0.04 is 0.9971. **As a result the existence of an interest term structure is imperative.**

[32]The value of R_f=0 does not however affects the estimation of CAPM *betas*.

[33]That is the violation ratio with R_f=0.04 was sometimes greater and some other time was less the corresponding violation with R_f=0.

R_f=**0.00**	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.930	0.972	0.883	0.955	0.928	0.947	0.928	0.933	0.913
	2-week	0.890	0.950	0.858	0.948	0.913	0.905	0.891	0.908	0.876
	1-month	0.848	0.938	0.805	0.916	0.851	0.851	0.848	0.888	0.827
INF	1-week	0.992	0.999	0.977	0.996	0.953	0.962	0.977	0.982	0.932
	2-week	0.979	0.999	0.977	0.990	0.955	0.953	0.975	0.977	0.920
	1-month	0.967	0.996	0.955	0.980	0.928	0.928	0.950	0.977	0.906
R_f=**0.04**	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9248	0.9719	0.8778	0.9601	0.9282	0.9467	0.9265	0.94	0.9131
	2-week	0.8946	0.9585	0.8508	0.9433	0.9131	0.9047	0.8979	0.9063	0.8761
	1-month	0.8508	0.9332	0.8071	0.9147	0.8508	0.8508	0.8542	0.8828	0.8273
INF	1-week	0.9971	1.0005	0.9753	1.0022	0.9534	0.9618	0.9854	0.9854	0.9316
	2-week	0.9786	1.0039	0.977	1.0005	0.9551	0.9534	0.9719	0.9803	0.9198
	1-month	0.9719	0.9955	0.9433	0.982	0.9282	0.9282	0.9534	0.9736	0.9063

Table 23: General Index-Ordinary: Compare the adjusted 95% VaR test results when R_{free}=0.00 and R_{free}=0.04.

We now compare the VaR violation ratios estimated using only local information to using both local and foreign information. As already mentioned in sections 6.2.3 and 6.2.4, when historical simulation is considered we use additional foreign information when the market index is considered to be the CSE general index. On the other hand no foreign information is used when the market index is considered to be the sector indexes. Therefore we compare the violation ratios of EURO which in both simulations is considering as a market index the CSE general index. As a result we see in Table (24) that the violation ratio for method 1 using foreign information is bigger than the corresponding one where no foreign information is used. On the other hand, method 2 and RiskMetrics are performing better when no foreign information are used. For the RiskMetrics method, that is happening because of the correlation considered between the sector indexes when the covariance matrix is generated.

		nVaR1	pVaR1	nVaR2	pVaR2	RiskMetrics
Foreign	1-week	0.75	0.79	0.72	0.78	0.72
information	2-week	0.70	0.76	0.67	0.75	0.71
used	1-month	0.67	0.72	0.65	0.70	0.67
No foreign	1-week	0.74	0.78	0.73	0.78	0.73
information	2-week	0.69	0.75	0.68	0.74	0.72
used	1-month	0.66	0.71	0.65	0.70	0.70

Table 24: Compare the VaR violation ratios estimated for EURO using only local information to using both local and foreign information : Adjusted 95% VaR test results (R_{free}=0.0).

Finally we present the results for the portfolio when the BOC was excluded from the backtesting. As a result the adjusted violation ratios were not very high (99 % VaR for 1 week is 0.8713) as it should be expected for the reasons described before.

Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
1-week	0.7936	0.8289	0.7516	0.8138	0.7684	0.7835	0.7802	0.7869	0.7566
2-week	0.7398	0.7987	0.7146	0.7936	0.7718	0.7718	0.7348	0.7465	0.7415
1-month	0.7062	0.7516	0.6726	0.7432	0.7146	0.7196	0.7096	0.7280	0.6961
1-week	0.8713	0.8681	0.8213	0.8551	0.8439	0.8390	0.8374	0.8342	0.8132
2-week	0.8261	0.8535	0.7938	0.8406	0.8003	0.8164	0.8261	0.8084	0.7874
1-month	0.7567	0.8261	0.7244	0.8116	0.7696	0.7761	0.7890	0.7825	0.7599

Table 25: General Index-Ordinary: The adjusted 95% (above) and 95% (below) VaR test results for the portfolio when BOC is excluded and R_{free}=0.04.

7 Conclusion

This report has provided a comprehensive study of the equity risk of the portfolio of the Cyprus Development Bank. In particular, we would like to draw attention to the following general conclusions:

1) The short time series of particular variables (CLR, LPL, Financial Index) greatly detracts from the application of quantitative techniques for risk management. This should improve with time. 2) The low liquidity of many of the investments makes reliance solely on the history of their values questionable (e.g. some securities have numerous dates upon which they were not transacted: EURO, CLR). Different methods for adjusting the risk measures for the lack of liquidity should be investigated. 3) The quantitative methods presented in this paper provide a solid foundation for an investigation of the risk of the (traded) equity portfolio of the Cyprus Development Bank. However, they should not be regarded as a panacea. Any quantitative tools for risk management must be supplemented with qualitative tools, such as scenario analysis (reference BIS paper from www.gloriamundi.org) Quantitative tools are not a substitute for good managerial judgement.

The tools presented in this report provide one building block in the development of an effective risk management system for an emerging market. Other components are being developed at RiskLab Cyprus, including models for non-traded equities, credit risk of loan portfolios and interest rate risk. These components together will constitute a comprehensive risk management system, tailored to the needs of Cypriot financial institutions.

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A Volatility Models and Estimation Methods

Here we present the GARCH(1,1) model estimation using Maximum Likelihood. Consider the GARCH(1,1) model with μ_t equal to zero

$$r_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t | \mathcal{F}_{t-1} \sim N(0,1) \tag{56}$$

or equivalently

$$r_t = \varepsilon_t, \qquad \varepsilon_t | \mathcal{F}_{t-1} \sim N(0,\sigma_t^2) \tag{57}$$

where

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \equiv \underline{z}_t^T . \underline{\vartheta} \tag{58}$$

$$\underline{z}_t^T = \begin{bmatrix} 1 & \varepsilon_{t-1}^2 & \sigma_{t-1}^2 \end{bmatrix}, \underline{\vartheta}^T = \begin{bmatrix} \omega & \alpha & \beta \end{bmatrix} \tag{59}$$

The density function of the returns is

$$f(r_t | \mathcal{F}_{t-1}) = \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{1}{2}\frac{r_t^2}{\sigma_t^2}} \tag{60}$$

$$= \frac{1}{\sqrt{2\pi\sigma_t^2}} e^{-\frac{1}{2}\frac{\varepsilon_t^2}{\sigma_t^2}} \tag{61}$$

and therefore the contribution of observation t to the log-likelihood is $\ell_t(\underline{\vartheta})$ and the maximum likelihood estimates $\widehat{\underline{\vartheta}}$ are obtained by the maximization of $\mathcal{L}_T(\vartheta)$.

$$\ell_t(\underline{\vartheta}) = -\frac{1}{2}\left[\ln(2\pi) + \ln(\sigma_t^2) + \frac{\varepsilon_t^2}{2\sigma_t^2}\right] \tag{62}$$

$$\mathcal{L}_T(\underline{\vartheta}) = \sum_{t=1}^{T} \ell_t(\underline{\vartheta}) \tag{63}$$

Differentiating with respect to $\underline{\vartheta}$ yields yields the score function $s_t(\underline{\vartheta})$ of the t^{th} observation.

$$s_t(\underline{\vartheta}) = \frac{\partial \ell_t}{\partial \underline{\vartheta}} = -\frac{1}{2\sigma_t^2}\frac{\partial \sigma_t^2}{\partial \underline{\vartheta}} + \frac{\varepsilon_t^2}{2[\sigma_t^2]^2}\frac{\partial \sigma_t^2}{\partial \underline{\vartheta}} = \frac{1}{2\sigma_t^2}\frac{\partial \sigma_t^2}{\partial \underline{\vartheta}}\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) \tag{64}$$

$$S_T(\underline{\vartheta}) = \sum_{t=1}^{T} s_t(\underline{\vartheta}) \tag{65}$$

where

$$\frac{\partial \sigma_t^2}{\partial \underline{\vartheta}} = \underline{z}_t + \frac{\partial \underline{z}_t^T}{\partial \underline{\vartheta}} \underline{\vartheta} \tag{66}$$

$$\frac{\partial \underline{z}_t^T}{\partial \underline{\vartheta}} = \frac{\partial}{\partial \underline{\vartheta}} \begin{bmatrix} 1 & \varepsilon_{t-1}^2 & \sigma_{t-1}^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\partial \sigma_{t-1}^2}{\partial \underline{\vartheta}} \end{bmatrix} \tag{67}$$

$$\Longrightarrow \frac{\partial \sigma_t^2}{\partial \underline{\vartheta}} = \begin{bmatrix} \frac{\partial \sigma_t^2}{\partial \omega} \\ \frac{\partial \sigma_t^2}{\partial \alpha} \\ \frac{\partial \sigma_t^2}{\partial \beta} \end{bmatrix} = \begin{bmatrix} 1 \\ \varepsilon_{t-1}^2 \\ \sigma_{t-1}^2 \end{bmatrix} + \beta \frac{\partial \sigma_{t-1}^2}{\partial \underline{\vartheta}} \tag{68}$$

The unconditional variance of ε_t is equal to the unconditional expected variance [34].That is $var(\varepsilon_t) = E(\sigma_t^2)$ and is given by

$$\begin{aligned} E(\sigma_t^2) &= E(\omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2) && (69) \\ &= \omega + \alpha E(\varepsilon_{t-1}^2) + \beta E(\sigma_{t-1}^2)) && (70) \\ &= \omega + \alpha E(\sigma_{t-1}^2) + \beta E(\sigma_{t-1}^2)) && (71) \\ &= \omega + (\alpha + \beta)(\omega + \alpha E(\varepsilon_{t-2}^2) + \beta E(\sigma_{t-2}^2)) && (72) \\ &= \omega \sum_{i=0}^{\infty} (\alpha + \beta)^i && (73) \end{aligned}$$

and since $\alpha + \beta < 1 \Longrightarrow$

$$E(\sigma_t^2) = \frac{\omega}{1 - (\alpha + \beta)} \tag{74}$$

Taking the expected volatility to be the initial value of volatility we have

$$\sigma_0^2 = \frac{\omega}{1 - \alpha - \beta} \tag{75}$$

and therefore the initial values of the derivatives with respect to the parameters are

$$\frac{\partial \sigma_0^2}{\partial \omega} = \frac{1}{1 - \alpha - \beta} \tag{76}$$

$$\frac{\partial \sigma_0^2}{\partial \alpha} = \frac{\partial \sigma_0^2}{\partial \beta} = \frac{\omega}{(1 - \alpha - \beta)^2} \tag{77}$$

In order to compute the variance of the parameters one could consider the Hessian matrix, but since this is very time-consuming, one should consider the Outer Product matrix (BHHH[35]) algorithm. The information matrix can be estimated by

$$I_T(\vartheta) = \sum_{t=1}^{T} s_t(\vartheta) s_t^T(\vartheta) \tag{78}$$

[34] or namely long-run volatility.

[35] That is the Berndt, Hall, Hall, Hausman (1974) algorithm.

and the variance of the parameters is estimated by

$$var(\widehat{\vartheta}) = [-I_T(\widehat{\vartheta})]^{-1} \tag{79}$$

Now for the EWMA model we have

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda\sigma_{t-1}^2 \tag{80}$$

therefore the log-likelihood function $\ell_t(\lambda)$ and the corresponding score function $s_t(\lambda)$ are

$$\ell_t(\lambda) = -\frac{1}{2}\left[\ln(2\pi) + \ln(\sigma_t^2) + \frac{\varepsilon_t^2}{2\sigma_t^2}\right] \tag{81}$$

$$s_t(\lambda) = \frac{\partial \ell_t}{\partial \lambda} = -\frac{1}{2\sigma_t^2}\frac{\partial \sigma_t^2}{\partial \lambda} + \frac{\varepsilon_t^2}{2[\sigma_t^2]^2}\frac{\partial \sigma_t^2}{\partial \lambda} = \frac{1}{2\sigma_t^2}\frac{\partial \sigma_t^2}{\partial \lambda}\left(\frac{\varepsilon_t^2}{\sigma_t^2} - 1\right) \tag{82}$$

where

$$\frac{\partial \sigma_t^2}{\partial \lambda} = \varepsilon_{t-1}^2 + \sigma_{t-1}^2 + \lambda\frac{\partial \sigma_{t-1}^2}{\partial \lambda} \tag{83}$$

B Regression Analysis

Assume that the values of a measurement (e.g returns of a stock) can be represented as a linear function of several independent or explanatory variables (e.g. Stock index, inflation, growth rate), then a mathematical representation of the above relationship could be :

$$\begin{aligned} y_i &= f(x_{1i}, x_{2i}, ...x_{pi}) + \varepsilon_i \\ &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_p x_{pi} + \varepsilon_i, i = 1, ..., n. \end{aligned} \tag{84}$$

$$\begin{aligned} \widehat{y_i} &= f(x_{1i}, x_{2i}, ...x_{pi}) \\ &= \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_p x_{pi}, i = 1, ..., n. \end{aligned} \tag{85}$$

$$\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{86}$$

where y_i are the observed values of the measurement, $\widehat{y_i}$ are the calculated ones, dependent on the, related to, values of the p variables $x_1, x_2, ...x_p$, and $\overline{y}$ is the overall mean of the observed y_i. The error term ε is a random shock -or *disturbance*, usually drawn from a Normal distribution, $\varepsilon \sim$ N(0,σ^2), and i indices are the n sample observations.

The classical linear regression model consists of a set of assumptions so that the model is considered suitable for further use. These conditions are:

1. Linear functional form of the relationship.
2. The variance of y is constant for all the different values of x_i, that is the variance of y is independent of the regressors x_i, i=1,...,n and $\sigma_t^2 = \sigma^2$.
3. The random shocks are independent.
4. The random shocks are normally distributed.

The components of variance (SST, SSR, and SSE) used thoroughly in the analysis of variance, are typically shown in an ANOVA table.

SSR Sum of squares due to regression explains the portion of the total variation due to the linear relationship of y with x.

SSE Sum of squares of errors explains the portion of total variation due to the deviations from the linear regression, or the residual variation left unexplained by the regression line.

SST Total sum of squares is the total variation of y.

MSR Mean square regression is the SSR divided by its degrees of freedom, p-1.

MSE Mean square error is the SSE divided by its degrees of freedom, n-p.

where

$$SST = \sum_{i=1}^{n}(y_i - \overline{y})^2 \quad \text{, degrees of freedom n-1,}$$

$$SSR = \sum_{i=1}^{n}(\widehat{y}_i - \overline{y})^2 \quad \text{, degrees of freedom p-1,}$$

$$SSE = \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2 \quad \text{, degrees of freedom n-p.}$$

B.1 First-order model

In a single variable estimation model, like the CAPM model, equation (84) is just a line,

$$y_i = \alpha + \beta x_i + \varepsilon_i, \qquad i = 1, ..., n. \tag{87}$$

The y_i and x_i could be the returns of the stock (*dependent*) and the market index (*independent*) respectively, α and β the unknown parameters of the model (*regression coefficients*) and n is the number of observations. β represents the slope of the regression line and states the change of the mean of the distribution of y for every unit change of x. α represents the point where the regression line intercepts the y-axis.

Once the parameters α and β have been estimated the *estimated regression function* is

$$\widehat{y}_i = \widehat{\alpha} + \widehat{\beta} x_i, \qquad i = 1, ..., n. \tag{88}$$

and the *residuals* $\widehat{\varepsilon}_i$,

$$\widehat{\varepsilon}_i \qquad = y_i - \widehat{y}_i \tag{89}$$

$$= y_i - \widehat{\alpha} - \widehat{\beta} x_i \tag{90}$$

represent the deviation of the observed values of y_i and the corresponding estimated one $\widehat{y}_i$ (*fitted value*).

B.1.1 Ordinary Least Squares

In order to find the best values of α and β we consider the least square estimation (LSE) method, which involves minimizing, with respect to α and β, the sum of squared errors (SSE). The ordinary LSE method consists of minimizing

$$SSE = Q_{OLS}(\alpha,\beta) = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2 = \sum_{i=1}^{n}(y_i - \alpha + \beta x_i)^2 \tag{91}$$

To find the minimum of $Q_{OLS}(\alpha,\beta)$ we differentiate with respect to α and β and set to zero,

$$\frac{dQ_{OLS}(\alpha,\beta)}{d\alpha} = 0 \quad and \quad \frac{dQ_{OLS}(\alpha,\beta)}{d\beta} = 0 \tag{92}$$

leading to the *normal equations* - a system of 2 equations and 2 unknowns.

$$\frac{dQ_{OLS}(\alpha,\beta)}{d\alpha} = -2\sum_{i=1}^{n}(y_i - \alpha - \beta x_i) \quad \underrightarrow{min} \quad 0 \Longrightarrow \sum_{i=1}^{n}(y_i - \widehat{\alpha} - \widehat{\beta} x_i) = 0 \tag{93}$$

$$\frac{dQ_{OLS}(\alpha,\beta)}{d\beta} = -2\sum_{i=1}^{n}x_i(y_i - \alpha - \beta x_i) \quad \underrightarrow{min} \quad 0 \Longrightarrow \sum_{i=1}^{n}x_i(y_i - \widehat{\alpha} - \widehat{\beta} x_i) = 0 \tag{94}$$

$$\begin{aligned} (93) \qquad & \Longrightarrow \sum_{i=1}^{n}(y_i) - n\widehat{\alpha} - \widehat{\beta}\sum_{i=1}^{n}(x_i) = 0 \\ & \Longrightarrow \widehat{\alpha} = \frac{\sum_{i=1}^{n}(y_i)}{n} - \widehat{\beta}\frac{\sum_{i=1}^{n}(x_i)}{n} = \overline{y} - \widehat{\beta}\overline{x} \end{aligned} \tag{95}$$

Substituting equation (95) into (94) leads to

$$\begin{aligned} & \sum_{i=1}^{n} x_i(y_i - (\overline{y} - \widehat{\beta}\overline{x}) - \widehat{\beta}x_i) = 0 \\ \Longrightarrow & \sum_{i=1}^{n} x_i[(y_i - \overline{y}) - \widehat{\beta}(x_i - \overline{x})] = 0 \\ \Longrightarrow & \sum_{i=1}^{n} x_i[(y_i - \overline{y})] - \widehat{\beta}\sum_{i=1}^{n} x_i[(x_i - \overline{x})] = 0 \\ \Longrightarrow & \widehat{\beta} = \frac{\sum_{i=1}^{n} x_i[(y_i - \overline{y})]}{\sum_{i=1}^{n} x_i[(x_i - \overline{x})]} = \frac{\widehat{cov}(X,Y)}{\widehat{var}(X)} \end{aligned} \tag{96}$$

Therefore, equations (95) and (96) give the least squares estimators of α and β respectively.

B.1.2 Weighted Least Squares

Sometimes not all measurements are created equal, for example more recent data are influencing the model greater than older data, and as a consequence we include a weighting factor. Therefore we minimize

$$Q_{WLS}(\alpha,\beta) = \sum_{i=1}^{n} w_i \varepsilon_i^2 = \sum_{i=1}^{n} w_i (y_i - \alpha - \beta x_i)^2 \tag{97}$$

Following the same procedure as in the previous section the weighted least squares estimators of α and β are

$$\widehat{\alpha} = \frac{\sum_{i=1}^{n} w_i y_i - \widehat{\beta} \sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{98}$$

$$\widehat{\beta} = \frac{\sum_{i=1}^{n} w_i x_i y_i - \frac{\sum_{i=1}^{n} w_i x_i \sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}}{\sum_{i=1}^{n} w_i x_i^2 - \frac{(\sum_{i=1}^{n} w_i x_i)^2}{\sum_{i=1}^{n} w_i}} \tag{99}$$

Some choices for w_i could be

1. $w_i = 1$, $Q_{WLS}(\alpha,\beta)$ leads to $Q_{OLS}(\alpha,\beta)$.
2. $w_i = \frac{1}{\sigma_i^2}$, if standard deviation is known.
3. $w_i = \lambda^i$, where λ is the EWMA estimated parameter of the security. This method is used by **RiskMetrics** in order to deal with the heteroscedasticity of the returns.

B.2 Analysis of Variance-ANOVA

Once data has been fitted, the fit must be evaluated. There is a variety of statistical methods to do this. The most commonly used is the analysis of variance of the residuals (ANOVA).

Consider the deviation of observation y_i from the mean $\overline{y}$, which is the main measurement of the variation of the observation:

$$y_i - \overline{y}. \tag{100}$$

This can be written as

$$y_i - \overline{y} = (y_i - \widehat{y}_i) + (\widehat{y}_i - \overline{y}), \tag{101}$$

which means that it is the summation of

1. the deviation of $\widehat{y}_i$ from the mean $\overline{y}$ and

2. the deviation of y_i from $\widehat{y}_i$.

Taking the squares of (100) and summing over all observations leads to

$$\begin{aligned}
\sum_{i=1}^{n}(y_i-\overline{y})^2 \quad & = \sum_{i=1}^{n}(y_i-\overline{y}\pm\widehat{y}_i)^2 \\
& = \sum_{i=1}^{n}(\widehat{y}_i-\overline{y})^2+\sum_{i=1}^{n}(y_i-\widehat{y}_i)^2+2\sum_{i=1}^{n}(\widehat{y}_i-\overline{y})(y_i-\widehat{y}_i) \\
\Longrightarrow (104) \Longrightarrow \quad & = \sum_{i=1}^{n}(\widehat{y}_i-\overline{y})^2+\sum_{i=1}^{n}(y_i-\widehat{y}_i)^2+2\sum_{i=1}^{n}\widehat{\beta}(x_i-\overline{x})(y_i-\widehat{y}_i) \\
\Longrightarrow (102),(103) \Longrightarrow \quad & = \sum_{i=1}^{n}(\widehat{y}_i-\overline{y})^2+\sum_{i=1}^{n}(y_i-\widehat{y}_i)^2+2\widehat{\beta}\sum_{i=1}^{n}x_i(y_i-\widehat{y}_i)-2\widehat{\beta}\overline{x}\sum_{i=1}^{n}(y_i-\widehat{y}_i).
\end{aligned}$$

Therefore rewrite the normal equations (93) and (94)[36] and substitute

$$\sum_{i=1}^{n}(y_i-\overline{y})^2 \quad = \sum_{i=1}^{n}(\widehat{y}_i-\overline{y})^2+\sum_{i=1}^{n}(y_i-\widehat{y}_i)^2 \tag{105}$$

which is

$$\mathbf{SST}=\mathbf{SSR}+\mathbf{SSE}. \tag{106}$$

This relationship states that the total variation of y_i is composed of the deviation due to the model used (SSR) and the variation due to errors (SSE).

The above theory is summarized in the following table, known as an ANOVA table.

[36] Rewrite the normal equations (93),(94),

$$(93) \Longrightarrow \sum_{i=1}^{n}(y_i-\widehat{\alpha}-\widehat{\beta}x_i)=\sum_{i=1}^{n}(y_i-\widehat{y}_i)=0 \tag{102}$$

$$(94) \Longrightarrow \sum_{i=1}^{n}x_i(y_i-\widehat{\alpha}-\widehat{\beta}x_i)=\sum_{i=1}^{n}x_i(y_i-\widehat{y}_i)=0 \tag{103}$$

and also rewrite $$(\widehat{y}_i-\overline{y})=\widehat{\alpha}+\widehat{\beta}x_i-\overline{y}=\overline{y}-\widehat{\beta}\overline{x}+\widehat{\beta}x_i-\overline{y}=\widehat{\beta}(x_i-\overline{x}) \tag{104}$$

	Sum Squares	degrees of freedom	Mean Square
Regression	SSR= $\sum_{i=1}^{n}(\widehat{y}_i - \overline{y})^2$	p-1	MSR= $\frac{SSR}{p-1}$
Error	SSE= $\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$	n-p	MSE= $\frac{SSE}{n-p}$
Total Variation	SST= $\sum_{i=1}^{n}(y_i - \overline{y})^2$	n-1	

Table 26: ANOVA Table.

B.3 Distribution of the estimated parameters

The equivalent matrix formation is

$$\underline{Y} = \underline{X}\underline{\beta} + \underline{\varepsilon} \tag{107}$$

where

$$\underline{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \underline{X} = \begin{bmatrix} 1 & X_{11} & X_{21} & \ldots & X_{k1} \\ 1 & X_{12} & X_{22} & \ldots & X_{k2} \\ \vdots & \vdots & & & \vdots \\ 1 & X_{1n} & X_{2n} & \ldots & X_{kn} \end{bmatrix}, \underline{\beta} = \begin{bmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \underline{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

and the analogous estimation equations of the parameters and the variance of them [37] and the variance of $\underline{\beta}$ are given by

$$\underline{\widehat{\beta}} = (\underline{X}^T \underline{X})^{-1} \underline{X}^T \underline{Y} \tag{108}$$

$$var(\underline{\widehat{\beta}}) = \sigma^2 (\underline{X}^T \underline{X})^{-1} \tag{109}$$

and asymptotically $\underline{\widehat{\beta}}$ is an unbiased estimator of β. In addition $\underline{\widehat{\beta}}$ (if $\varepsilon \sim N()$) is normally distributed,

$$\underline{\widehat{\beta}} \sim N(\underline{\beta}, var(\underline{\beta})), \text{ which implies that } \frac{\underline{\widehat{\beta}} - \underline{\beta}}{\sqrt{var(\underline{\beta})}} \sim N(0,1). \tag{110}$$

Due to the estimation of two parameters ($\alpha, \beta \Longrightarrow$ p=2) the degrees of freedom of SSE are reduced to n-2. Therefore an unbiased estimator for σ^2, referred as MSE, is given by

$$\widehat{var}(\underline{\beta}) = MSE = \frac{SSE}{n-2}. \tag{111}$$

[37] Equation (108) is the general form, in matrix representation, of equations (95) and (96).

B.4 Hypotheses testing

For a usual regression analysis the model is

$$\widehat{y_i} = \widehat{\alpha} + \widehat{\beta} x_i, \qquad i = 1, ..., n. \tag{112}$$

while the null hypotheses and the alternatives are

$$H_0^1 : \alpha = 0 \quad Vs \quad H_1^1 : \alpha \neq 0 \tag{113}$$

$$H_0^2 : \beta = 0 \quad Vs \quad H_1^2 : \beta \neq 0 \tag{114}$$

B.4.1 t-test

The above null hypotheses are checked with the t-statistics

$$t_1 = \frac{\widehat{\alpha}}{\sqrt{\widehat{var}(\widehat{\alpha})}} \quad , \quad t_2 = \frac{\widehat{\beta}}{\sqrt{\widehat{var}(\widehat{\beta})}} \tag{115}$$

respectively. Under the null hypotheses the distributions of the statistics are

$$\frac{\widehat{\alpha}}{\sqrt{\widehat{var}(\alpha)}} \sim t_{n-2} \quad , \quad \frac{\widehat{\beta}}{\sqrt{\widehat{var}(\beta)}} \sim t_{n-2}. \tag{116}$$

following the statements in the above section. The null hypotheses are rejected, at a confidence level $a\%$, if

$$|t_i| > t_{n-2}(\tfrac{a}{2}), \quad i = 1, 2. \tag{117}$$

To do so the p-value of the test is provided, and if the p-value is less than the confidence level then the null hypothesis is rejected

B.4.2 F-test

The alternative hypothesis H_1^2 is equivalent to the hypothesis that there is no linear relationship, and it can also be tested by an analysis of variance procedure. The F-statistic defined by

$$F = \frac{MSR}{MSE} \tag{118}$$

is used for testing the models (significance). Under H_0^2 the distribution of the F-statistic is

$$\frac{MSR}{MSE} \sim F_{1,n-2} \tag{119}$$

and therefore, if the observed significance level for the F-test is small, the hypothesis that there is no linear relationship can be rejected.

B.4.3 Coefficient of Determination R^2

The coefficient of determination R^2, defined as

$$R^2 = \frac{SSR}{SST} \tag{120}$$

is a measure of the fit of the model. It measures the proportion of the total variation in y that is accounted for by the regressors. If we assume that the values of y vary linearly with x, then the total variation is due to the regression and is given by SSR. If that is perfectly true, then R^2 will equal 1. If the data are scattered about the line, the value of R^2 will decrease.

B.4.4 Durbin-Watson (DW) test

One of the assumptions of regression analysis is that the residuals for consecutive observations are uncorrelated. The Durbin-Watson (DW) test checks for serially correlated (or autocorrelated) residuals. The test statistic is calculated from the LSE estimated residuals $\widehat{\varepsilon}_i$ as:

$$DW = \sum_{i=2}^{n} (\widehat{\varepsilon}_i - \widehat{\varepsilon}_{i-1})^2 / \sum_{i=1}^{n} (\widehat{\varepsilon}_i)^2 \tag{121}$$

The DW-statistic has values in the range [0,4]. If the assumption of uncorrelated residuals is true then the expected value of the Durbin-Watson statistic is 2. Values less than 2 indicate positive autocorrelation while values greater than 2 indicate negative autocorrelation.

C Estimated CAPM parameter

In this section we present the estimated β and then a further analysis of variance for the securities of the portfolio is shown.

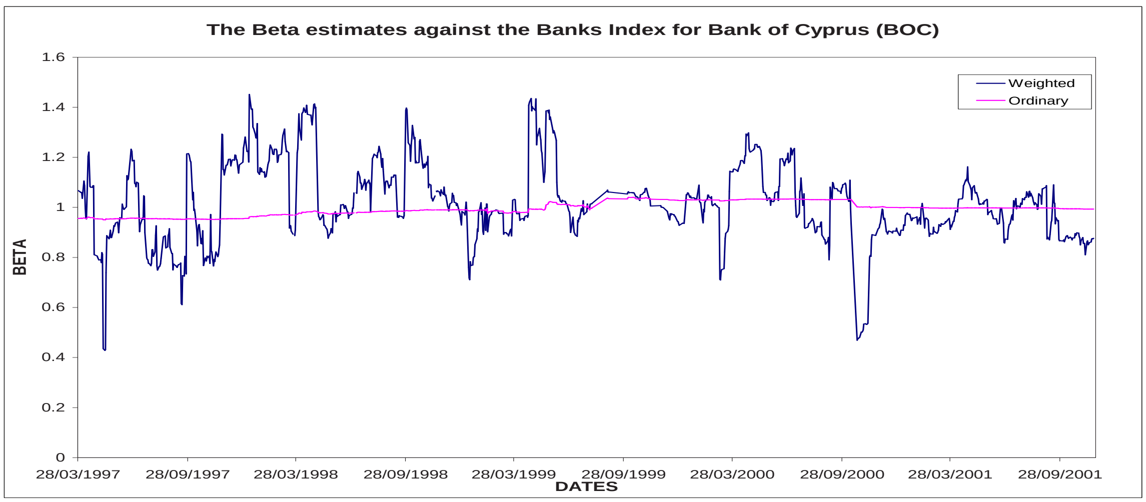

Figure 19: *Beta* estimates of BOC against the Banks Index.

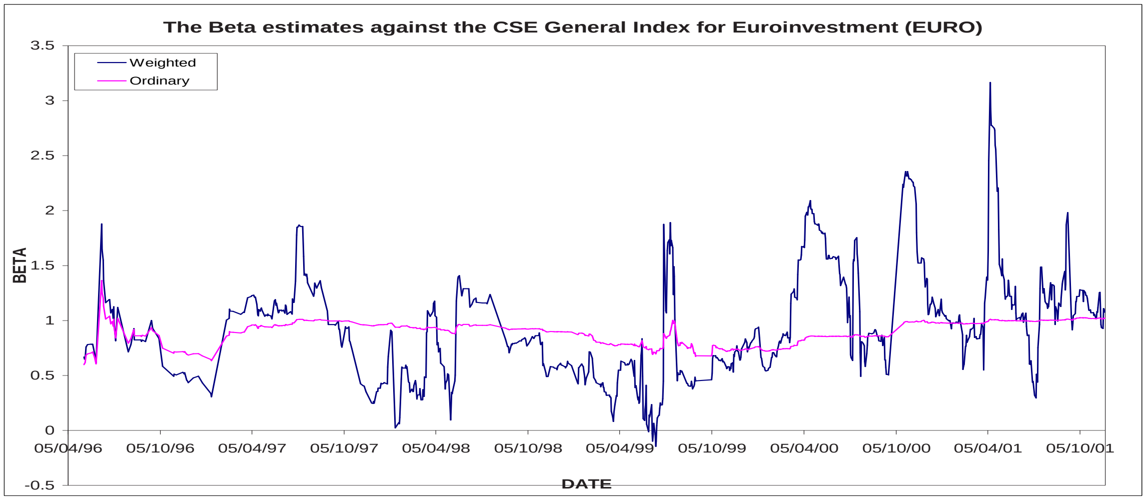

Figure 20: *Beta* estimates of EURO against the CSE General Index.

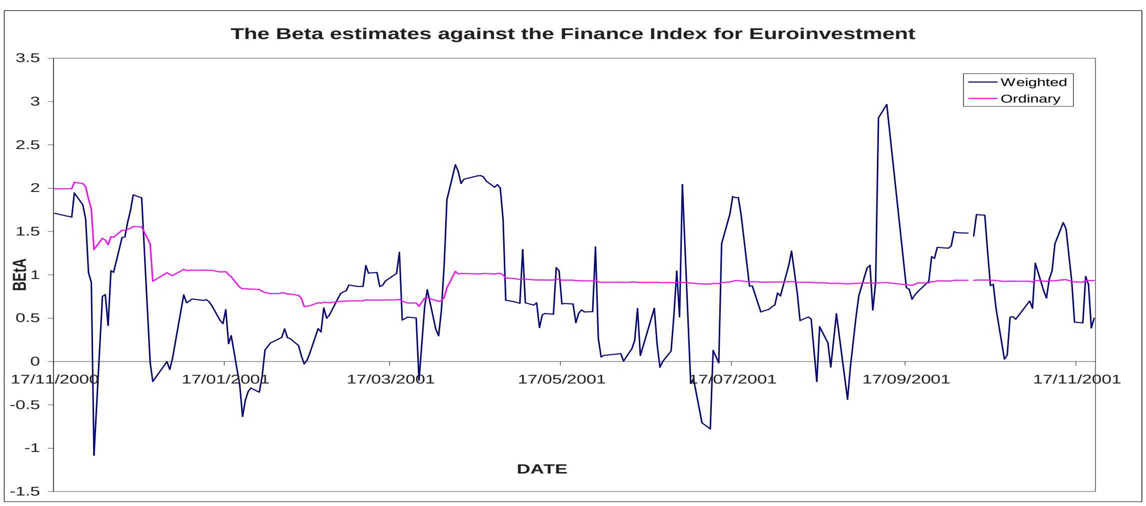

Figure 21: *Beta* estimates of EURO against the Finance Index.

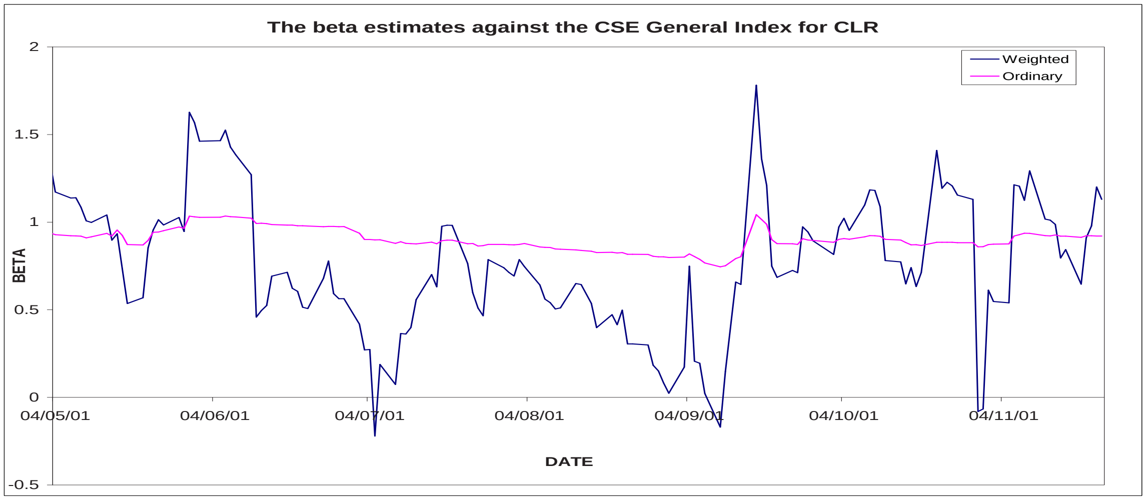

Figure 22: *Beta* estimates of CLR against the CSE General Index.

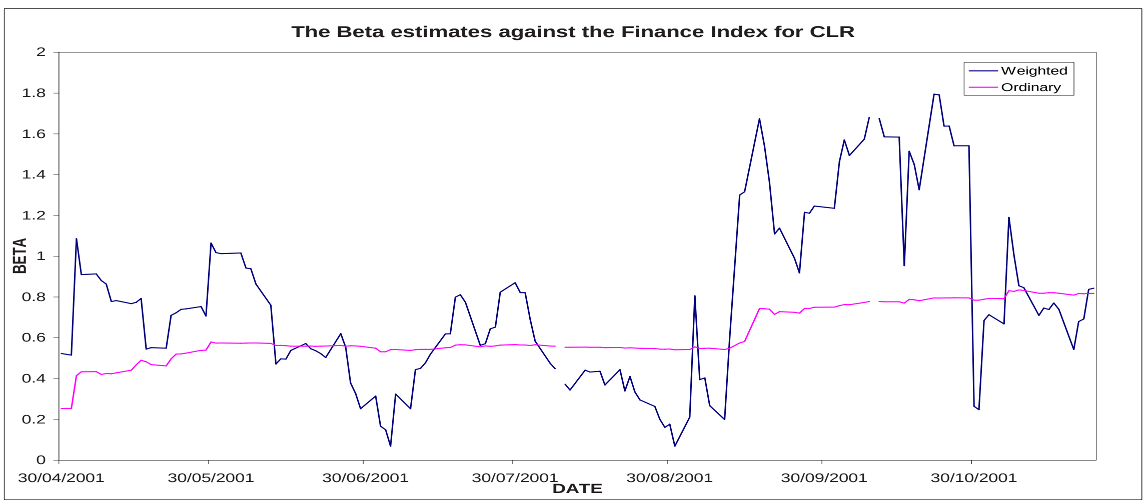

Figure 23: *Beta* estimates of CLR against the Finance Index.

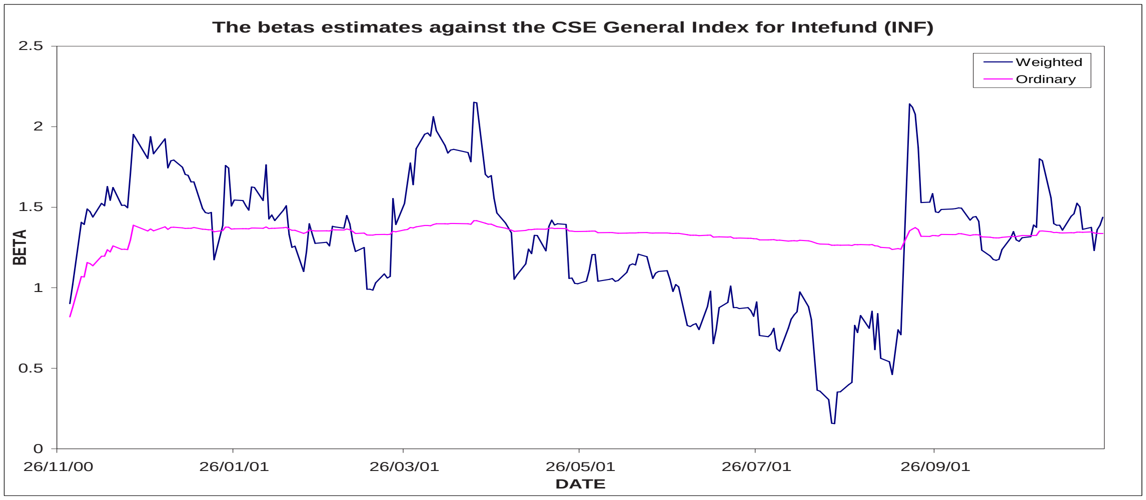

Figure 24: *Beta* estimates of INF against the CSE General Index.

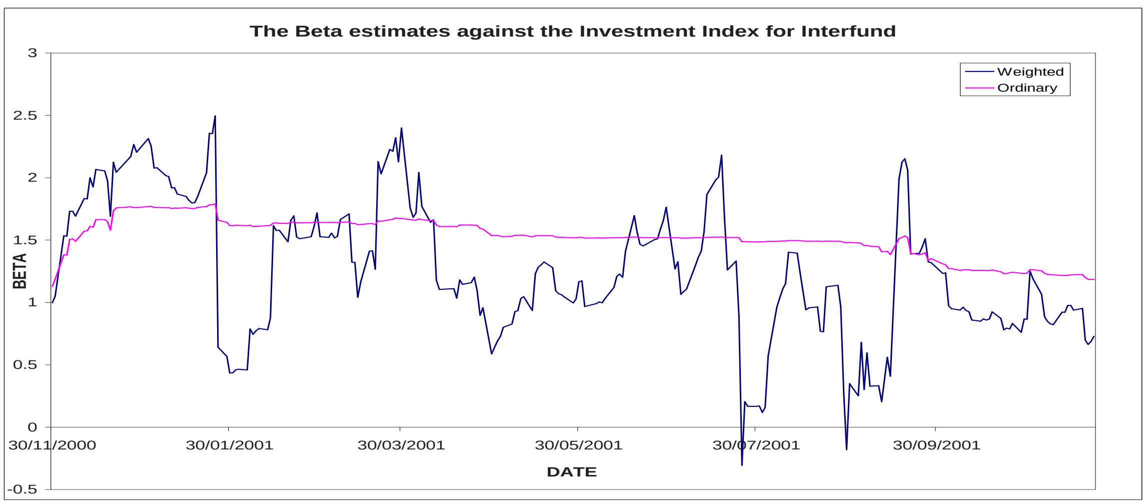

Figure 25: *Beta* estimates of INF against the Investment Index.

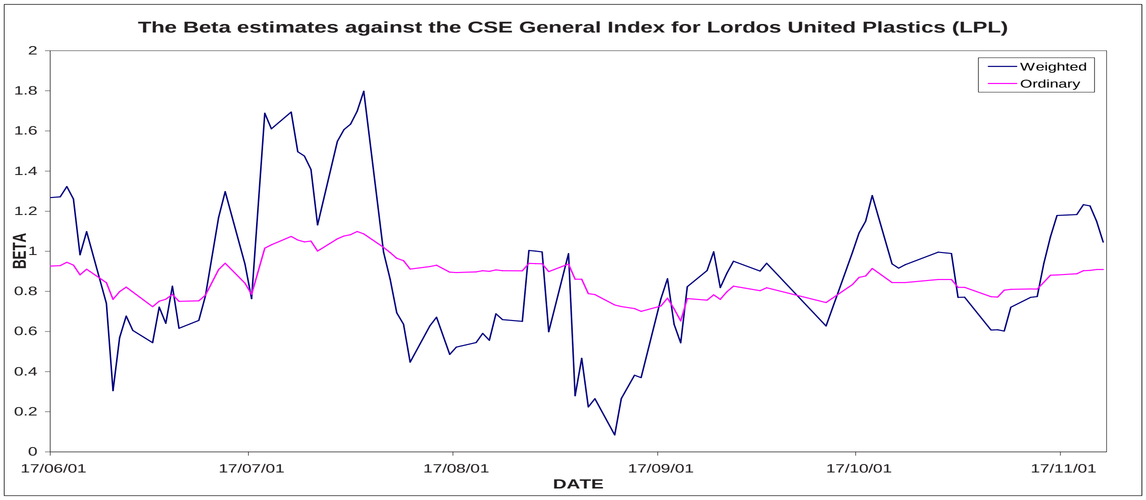

Figure 26: *Beta* estimates of LPL against the CSE General Index.

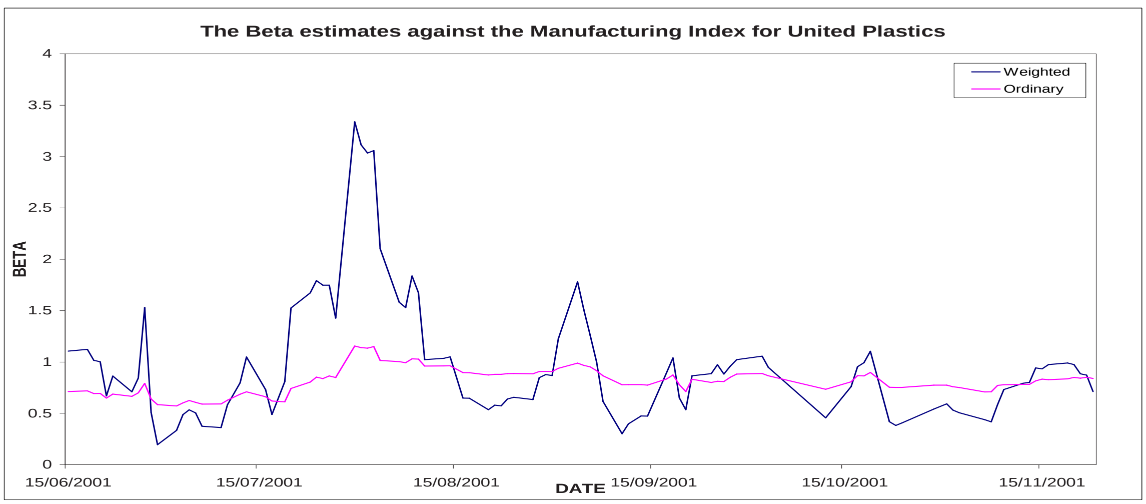

Figure 27: *Beta* estimates of LPL against the Manufacturing Index.

C.1 ANOVA of the securities of the portfolio

BOC : The R^2 for BOC against the CSE General Index is very large (for Period 3 R^2= 0.907, Table (30)) which indicates that the market is strongly related to BOC. Moreover the R^2 is even bigger when it is estimated against the Banks sector, Table (34). The small St. Dev. of $\widehat{\beta}$ indicates that the true beta for BOC ranges in a very small interval.

The Durbin-Watson test against the Banks Index is close to 2 indicating that the residuals are uncorrelated for consecutive observations (lag 1), while against the CSE general index is less than 2 indicating a positive correlation.

CLR : The R^2 is close to 0.5 and therefore the model explains the half of the variation of the dependent security return, Tables (36), (35). However, the small value of St. Dev. indicates that the estimated parameter is a reliable one. In addition DW is close to 2.

EURO : The R^2 is close to 0.25 indicating that the security have more specific risk than market risk, Table (37). The St. Dev. is small for the whole history, against the CSE general index, while against the Finance index is bigger[38]. DW is close to 2.

INF : The R^2 is greater against the CSE General index, (close to 0.5) indicating that the market is explaining better than the sector the variation of this security returns, Tables (42), (43). The small value of St. Dev. (0.09) indicates that the estimated parameter is a reliable one. Again DW is very close to 2.

LPL : The R^2 is small (0.2) and there is a negative correlation within consecutive residual, $DW > 2$, Tables (44), (15). Moreover the high St. Dev. (0.22) implies that the estimated parameter is not so reliable since the true value of $\widehat{\beta}$ ranges from 0.42 to 1.27.

[38]That is because of the short history of the finance index.

	History	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.01	1	0.01	2436.571	0	0.645
	SSE	0.01	1341	5.85×10^{-7}			
	SST	0.02	1342				
Ordinary	SSR	0.797	1	0.797	7605.81	0	0.85
	SSE	0.14	1341	0.000104			
	SST	0.937	1342				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-3.64×10^{-4}	0.00245	-1.483	0.138	2.421	
	$\widehat{\beta}$	0.617	0.013	49.362	0		
Ordinary	$\widehat{\alpha}$	-1.13×10^{-4}	0.0027	-0.405	0.686	1.73	
	$\widehat{\beta}$	1.072	0.012	87.211	0		

Table 27: Analysis of variance for BOC against the CSE general index for the whole history.

	Period 1	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.004	1	0.004	1238.93	0	0.609
	SSE	0.003	796	3×10^{-6}			
	SST	0.007	797				
Ordinary	SSR	0.09	1	0.09	1572.178	0	0.655
	SSE	0.047	796	$6\ \times10^{-5}$			
	SST	0.137	797				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-3.4×10^{-3}	0.00049	-6.913	0	1.8876	
	$\widehat{\beta}$	1.23	0.035	35.199	0		
Ordinary	$\widehat{\alpha}$	7.24×10^{-5}	0.0001	0.265	0.791	1.96	
	$\widehat{\beta}$	1.114	0.029	38.887	0		

Table 28: Analysis of variance for BOC against the CSE general index for period 1.

	Period 2	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.0025	1	0.0002	1564.37	0	0.849
	SSE	0.0005	279	1.58×10^{-6}			
	SST	0.003	280				
Ordinary	SSR	0.586	1	0.586	273155	0	0.907
	SSE	0.06	279	0.00021			
	SST	0.646	280				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-6.47×10^{-4}	0.01	-0.934	0.351	1.8876	
	$\widehat{\beta}$	1.284	0.032	39.522	0		
Ordinary	$\widehat{\alpha}$	-7.1×10^{-4}	0.01	-0.811	0.418	1.623	
	$\widehat{\beta}$	1.083	0.021	52.264	0		

Table 29: Analysis of variance for BOC against the CSE general index for period 2.

	Period 3	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	8.41	1	8.41×10^{-5}	259.863	0	0.499
	SSE	8.45	261	3.23×10^{-7}			
	SST	0.000168	262				
Ordinary	SSR	0.118	1	0.118	953.74	0	0.785
	SSE	0.032	261	0.00012			
	SST	0.15	262				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	3.17×10^{-3}	0.00036	8.797	0	1.96	
	$\widehat{\beta}$	0.469	0.029	16.12	0		
Ordinary	$\widehat{\alpha}$	-3.56×10^{-4}	0.001	-0.517	0.606	1.67	
	$\widehat{\beta}$	1.002	0.032	30.883	0		

Table 30: Analysis of variance for BOC against the CSE general index for period 3.

	History	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.002	1	0.002	9620.17	0	0.878
	SSE	0.0002	1341	2.01×10^{-7}			
	SST	0.0022	1342				
Ordinary	SSR	0.843	1	0.843	11994.2	0	0.899
	SSE	0.094	1341	7×10^{-5}			
	SST	0.937	1342				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	1.2×10^{-4}	0.000139	0.89	0.373	2.829	
	$\widehat{\beta}$	0.876	0.09	98.082	0		
Ordinary	$\widehat{\alpha}$	-1.28×10^{-4}	0.002	-0.559	0.577	2.062	
	$\widehat{\beta}$	0.994	0.009	109.52	0		

Table 31: Analysis of variance for BOC against the Banks index for the whole history.

	Period 1	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.002	1	0.002	2095.9	0	0.725
	SSE	0.001	796	1×10^{-6}			
	SST	0.003	797				
Ordinary	SSR	1.01	1	0.101	2206.6	0	0.735
	SSE	0.036	796	5×10^{-5}			
	SST	0.137	797				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-1.22×10^{-3}	0.0003	-3.829	0	1.863	
	$\widehat{\beta}$	0.959	0.021	45.781	0		
Ordinary	$\widehat{\alpha}$	-1.97×10^{-4}	0.00024	-0.82	0.412	2.065	
	$\widehat{\beta}$	1.01	0.022	46975	0		

Table 32: Analysis of variance for BOC against the Banks index for period 1.

	Period 2	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.002	1	0.002	827.95	0	0.748
	SSE	0.001	279	2.6×10^{-6}			
	SST	0.003	280				
Ordinary	SSR	0.6	1	0.6	3570.68	0	0.928
	SSE	0.047	279	0.00016			
	SST	0.647	280				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-6.53×10^{-3}	0.01	-8.219	0	1.867	
	$\widehat{\beta}$	0.474	0.016	228.774	0		
Ordinary	$\widehat{\alpha}$	-6.93×10^{-5}	0.001	-0.09	0.929	2.021	
	$\widehat{\beta}$	1.001	0.017	59.755	0		

Table 33: Analysis of variance for BOC against the CSE Banks index for period 2.

	Period 3	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.00014	1	0.00014	1997.5	0	0.884
	SSE	1.95	262	7.5×10^{-8}			
	SST	0.00016	263				
Ordinary	SSR	0.139	1	0.139	3452.6	0	0.929
	SSE	0.011	262	4×10^{-5}			
	SST	0.15	263				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-1.89×10^{-3}	0.0002	-8.211	0	2.048	
	$\widehat{\beta}$	1.062	0.0024	44693	0		
Ordinary	$\widehat{\alpha}$	-1.55×10^{-4}	0.0001	-3.92	0.695	2.209	
	$\widehat{\beta}$	0.955	0.016	58.529	0		

Table 34: Analysis of variance for BOC against the Banks index for period3.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.001	1	0.001	100.668	0	0.388
	SSE	0.002	159	1.18×10^{-5}			
	SST	0.003	160				
Ordinary	SSR	0.058	1	0.058	128.33	0	0.447
	SSE	0.071	159	0.00044			
	SST	0.129	160				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-2.33×10^{-3}	0.0002	-1.327	0.187	2.323	
	$\widehat{\beta}$	1.128	0.112	10.033	0		
Ordinary	$\widehat{\alpha}$	1.85×10^{-3}	0.02	1.11	0.269	1.936	
	$\widehat{\beta}$	0.927	0.082	11.328	0		

Table 35: Analysis of variance for CLR against the CSE general index for the whole history.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.002	1	0.002	158.773	0	0.5
	SSE	0.002	159	9.68×10^{-6}			
	SST	0.004	160				
Ordinary	SSR	0.068	1	0.068	174.69	0	0.524
	SSE	0.061	159	0.00038			
	SST	0.129	160				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	1.87×10^{-4}	0.001	0.127	0.899	2.62	
	$\widehat{\beta}$	0.843	0.067	12.601	0		
Ordinary	$\widehat{\alpha}$	-2.4×10^{-4}	0.002	-0.16	0.873	2.082	
	$\widehat{\beta}$	0.823	0.062	13.217	0		

Table 36: Analysis of variance for CLR against the Finance index for the whole history.

	History	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.016	1	0.016	202.84	0	0.178
	SSE	0.074	936	7.9×10^{-5}			
	SST	0.09	937				
Ordinary	SSR	0.573	1	0.573	2257.07	0	0.215
	SSE	2.086	936	0.002			
	SST	2.659	937				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	6.007×10^{-3}	0.002	2.979	0.03	2.505	
	$\widehat{\beta}$	1.089	0.077	14.22	0		
Ordinary	$\widehat{\alpha}$	-1.12×10^{-4}	0.002	-0.73	0.942	1.987	
	$\widehat{\beta}$	1.022	0.064	16.033	0		

Table 37: Analysis of variance for EURO against the CSE general index for the whole history.

	period 1	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.038	1	0.038	47.939	0	0.103
	SSE	0.332	418	0.001			
	SST	0.37	419				
Ordinary	SSR	0.038	1	0.038	47.939	0	0.103
	SSE	0.332	418	0.001			
	SST	0.37	419				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	1.54×10^{-3}	0.001	1.115	0.264	1.989	
	$\widehat{\beta}$	0.765	0.11	6.924	0		
Ordinary	$\widehat{\alpha}$	1.54×10^{-3}	0.001	1.115	0.264	1.989	
	$\widehat{\beta}$	0.765	0.11	6.924	0		

Table 38: Analysis of variance for EURO against the CSE general index for period 1.

	period 2	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.021	1	0.021	145.23	0	0.342
	SSE	0.041	279	0.00041			
	SST	0.062	280				
Ordinary	SSR	0.363	1	0.363	94.325	0	0.254
	SSE	1.073	279	0.004			
	SST	1.435	280				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	2.66×10^{-2}	0.006	4.094	0	1.626	
	$\widehat{\beta}$	2.707	0.225	12.051	0		
Ordinary	$\widehat{\alpha}$	5.11×10^{-4}	0.004	0.138	0.89	1.885	
	$\widehat{\beta}$	1.031	0.106	9.712	0		

Table 39: Analysis of variance for EURO against the CSE general index for period 2.

	period 2	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.002	1	0.002	14.001	0	0.056
	SSE	0.023	235	9.92×10^{-5}			
	SST	0.025	236				
Ordinary	SSR	0.167	1	0.167	58.296	0	0.199
	SSE	0.673	235	0.003			
	SST	0.84	240				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	2.58×10^{-2}	0.005	5.272	0	2.568	
	$\widehat{\beta}$	0.919	0.267	3.742	0		
Ordinary	$\widehat{\alpha}$	-3.06×10^{-3}	0.004	-0.873	0.383	1.929	
	$\widehat{\beta}$	1.097	0.144	7.635	0		

Table 40: Analysis of variance for EURO against the CSE general index for period 3.

	Period 3	Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.01	1	0.01	6.799	0.01	0.027
	SSE	0.24	244	9.92×10^{-5}			
	SST	0.25	245				
Ordinary	SSR	0.2	1	0.2	64.653	0	0.209
	SSE	0.795	244	0.003			
	SST	0.954	245				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	2.9×10^{-2}	0.005	6.366	0	2.481	
	$\widehat{\beta}$	0.501	0.192	2.067	0.01		
Ordinary	$\widehat{\alpha}$	-3.13×10^{-3}	0.004	-0.876	0.382	2.057	
	$\widehat{\beta}$	0.937	0.117	8.041	0		

Table 41: Analysis of variance for EURO against the Finance index for period 3.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.006	1	0.006	211.521	0	0.443
	SSE	0.007	266	0.000029			
	SST	0.013	267				
Ordinary	SSR	0.212	1	0.212	250.473	0	0.485
	SSE	0.225	266	0.001			
	SST	0.438	267				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	-2.83×10^{-3}	0.002	-1.525	0.128	1.351	
	$\widehat{\beta}$	1.437	0.099	14.544	0		
Ordinary	$\widehat{\alpha}$	2.726×10^{-3}	0.002	1.51	1	1.997	
	$\widehat{\beta}$	1.342	0.085	15.826	0		

Table 42: Analysis of variance for INF against the CSE general index for the whole history.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.004	1	0.004	108.567	0	0.29
	SSE	0.009	266	3.43×10^{-5}			
	SST	0.013	267				
Ordinary	SSR	0.174	1	0.174	175.038	0	0.397
	SSE	0.264	266	0.001			
	SST	0.438	267				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	4.6×10^{-4}	0.002	0.227	0.821	0.121	
	$\widehat{\beta}$	0.738	0.001	10.42	0		
Ordinary	$\widehat{\alpha}$	1.127×10^{-3}	0.002	1.583	0.56	1.998	
	$\widehat{\beta}$	1.191	0.09	13.32	0		

Table 43: Analysis of variance for INF against the Investment index for the whole history.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.005	1	0.005	23.2	0	0.164
	SSE	0.023	118	0.000195			
	SST	0.028	119				
Ordinary	SSR	0.049	1	0.049	19.142	0	0.141
	SSE	0.3	118	0.003			
	SST	0.349	119				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	1.78×10^{-2}	0.005	3.68	0	2.151	
	$\widehat{\beta}$	1.047	0.217	4.817	0		
Ordinary	$\widehat{\alpha}$	3.96×10^{-4}	0.005	0.086	0.932	2.393	
	$\widehat{\beta}$	0.917	0.208	4.406	0		

Table 44: Analysis of variance for LPL against the CSE general index for the whole history.

		Sum Squares	d.f.	Mean Square	F-statistic	sign.	R^2
Weighted	SSR	0.003	1	0.003	11.793	0.01	0.091
	SSE	0.025	118	0.0002			
	SST	0.028	119				
Ordinary	SSR	0.036	1	0.036	13.687	0	0.104
	SSE	0.313	118	0.03			
	SST	0.349	119				
		Estimate	St. Dev	t-statistic	sign	Durbin-Watson	
Weighted	$\widehat{\alpha}$	2.1×10^{-2}	0.005	4.259	0	2.118	
	$\widehat{\beta}$	0.711	0.207	3.434	0.001		
Ordinary	$\widehat{\alpha}$	-1.32×10^{-3}	0.005	-0.28	0.78	2.374	
	$\widehat{\beta}$	0.846	0.228	3.7	0		

Table 45: Analysis of variance for LPL against the Manufacturing index for the whole history.

D BackTesting Results

The results using as a market index the CSE General index with R_{free}=0.0 are shown initially and then the corresponding results with R_{free}=0.04.

BOC : For both *Ordinary* and *Weighted* method of estimation of *betas*, the violation ratios are not within the specified confidence level, Tables (46), (47). Thus both methods underestimate the risk.

CLR : The *Ordinary* method is over-estimating the risk using the parametric methods 1 and 2, while using the *Weighted* method the violation ratio is always within the specified C.I.

EURO : Here the *Weighted* method is performing better than the Ordinary method.

INF : The violation ratios are within the C.I. for the *Ordinary* method. For *Weighted* method the violation ratio is under the C.I. .

LPL : The violation ratios for both methods are within the C.I. .

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.930	0.972	0.883	0.955	0.928	0.947	0.928	0.933	0.913
	2-week	0.890	0.950	0.858	0.948	0.913	0.905	0.891	0.908	0.876
	1-month	0.848	0.938	0.805	0.916	0.851	0.851	0.848	0.888	0.827
CLR	1-week	1.021	1.026	1.017	1.021	1.006	1.009	1.016	1.017	0.980
	2-week	1.014	1.017	1.011	1.019	1.001	0.999	1.004	1.006	0.965
	1-month	1.006	1.009	1.004	1.009	0.989	0.989	1.004	1.007	0.959
EURO	1-week	0.748	0.789	0.720	0.784	0.733	0.750	0.725	0.743	0.723
	2-week	0.695	0.755	0.671	0.745	0.733	0.737	0.708	0.720	0.706
	1-month	0.666	0.718	0.647	0.701	0.688	0.693	0.668	0.686	0.671
INF	1-week	0.992	0.999	0.977	0.996	0.953	0.962	0.977	0.982	0.932
	2-week	0.979	0.999	0.977	0.990	0.955	0.953	0.975	0.977	0.920
	1-month	0.967	0.996	0.955	0.980	0.928	0.928	0.950	0.977	0.906
LPL	1-week	0.984	0.992	0.980	0.987	0.980	0.982	0.982	0.984	0.953
	2-week	0.987	0.997	0.985	0.994	0.969	0.965	0.985	0.985	0.935
	1-month	0.974	0.990	0.969	0.990	0.953	0.955	0.965	0.972	0.928
Portfolio	1-week	0.853	0.901	0.819	0.885	0.861	0.868	0.859	0.863	0.841
	2-week	0.809	0.863	0.784	0.859	0.834	0.827	0.817	0.824	0.799
	1-month	0.755	0.822	0.737	0.812	0.770	0.775	0.770	0.795	0.769
CSE G.I.	1-week	0.888	0.947	0.864	0.940	0.900	0.908	0.900	0.910	0.874
	2-week	0.853	0.932	0.827	0.903	0.863	0.859	0.856	0.863	0.827
	1-month	0.770	0.854	0.748	0.839	0.805	0.809	0.784	0.824	0.797

Table 46: General Index-Ordinary: Adjusted 95% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.901	0.953	0.861	0.937	0.918	0.932	0.911	0.920	0.901
	2-week	0.868	0.938	0.844	0.928	0.927	0.920	0.879	0.893	0.888
	1-month	0.812	0.895	0.797	0.876	0.866	0.869	0.836	0.861	0.846
CLR	1-week	1.002	1.006	1.001	1.002	0.996	0.996	1.001	1.004	0.965
	2-week	0.996	0.999	0.990	0.997	0.990	0.990	0.984	0.985	0.957
	1-month	0.980	0.996	0.979	0.992	0.965	0.967	0.975	0.980	0.933
EURO	1-week	0.774	0.811	0.755	0.797	0.785	0.797	0.769	0.774	0.763
	2-week	0.723	0.770	0.705	0.758	0.772	0.767	0.732	0.742	0.742
	1-month	0.703	0.755	0.686	0.750	0.730	0.738	0.703	0.737	0.721
INF	1-week	0.970	0.979	0.962	0.977	0.942	0.947	0.960	0.962	0.913
	2-week	0.960	0.977	0.952	0.975	0.942	0.942	0.948	0.953	0.908
	1-month	0.940	0.967	0.938	0.970	0.918	0.925	0.935	0.945	0.905
LPL	1-week	0.990	0.997	0.979	0.990	0.969	0.970	0.977	0.979	0.942
	2-week	0.979	0.987	0.977	0.989	0.970	0.969	0.975	0.975	0.937
	1-month	0.964	0.970	0.955	0.967	0.952	0.952	0.959	0.964	0.922
Portfolio	1-week	0.854	0.896	0.812	0.871	0.859	0.878	0.851	0.853	0.856
	2-week	0.804	0.869	0.789	0.858	0.839	0.834	0.811	0.824	0.802
	1-month	0.760	0.824	0.743	0.805	0.795	0.795	0.770	0.794	0.780
CSE G.I.	1-week	0.893	0.955	0.861	0.937	0.908	0.916	0.905	0.903	0.886
	2-week	0.858	0.932	0.829	0.908	0.881	0.873	0.861	0.864	0.846
	1-month	0.770	0.842	0.748	0.848	0.814	0.817	0.785	0.814	0.805

Table 47: General Index-Weighted: Adjusted 95% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.973	0.975	0.942	0.971	0.950	0.947	0.967	0.962	0.918
	2-week	0.981	0.991	0.934	0.975	0.931	0.934	0.952	0.957	0.904
	1-month	0.950	0.997	0.902	0.988	0.900	0.904	0.967	0.954	0.881
CLR	1-week	0.991	0.991	0.986	0.988	0.983	0.983	0.988	0.988	0.950
	2-week	0.986	0.988	0.979	0.986	0.968	0.975	0.981	0.983	0.944
	1-month	0.973	0.976	0.970	0.975	0.965	0.967	0.973	0.973	0.939
EURO	1-week	0.836	0.834	0.781	0.818	0.810	0.805	0.802	0.800	0.779
	2-week	0.787	0.815	0.757	0.813	0.765	0.781	0.781	0.765	0.750
	1-month	0.723	0.778	0.686	0.771	0.734	0.739	0.760	0.742	0.720
INF	1-week	0.984	0.984	0.979	0.979	0.962	0.962	0.971	0.971	0.931
	2-week	0.971	0.983	0.965	0.976	0.946	0.952	0.973	0.968	0.921
	1-month	0.971	0.983	0.950	0.976	0.946	0.946	0.970	0.968	0.921
LPL	1-week	0.981	0.983	0.973	0.978	0.954	0.952	0.965	0.963	0.925
	2-week	0.976	0.984	0.968	0.983	0.954	0.957	0.973	0.970	0.925
	1-month	0.965	0.978	0.955	0.981	0.938	0.938	0.965	0.965	0.913
Portfolio	1-week	0.918	0.933	0.879	0.926	0.899	0.892	0.910	0.902	0.867
	2-week	0.904	0.926	0.852	0.917	0.863	0.871	0.881	0.873	0.842
	1-month	0.857	0.907	0.804	0.894	0.834	0.844	0.876	0.862	0.825
CSE G.I.	1-week	0.975	0.983	0.934	0.970	0.931	0.926	0.954	0.947	0.897
	2-week	0.954	0.971	0.907	0.960	0.886	0.899	0.936	0.928	0.873
	1-month	0.883	0.946	0.818	0.929	0.858	0.863	0.918	0.905	0.849

Table 48: General Index-Ordinary: Adjusted 99% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.970	0.968	0.933	0.954	0.955	0.955	0.952	0.941	0.923
	2-week	0.965	0.975	0.907	0.955	0.929	0.939	0.957	0.944	0.908
	1-month	0.938	0.979	0.863	0.971	0.900	0.904	0.944	0.938	0.883
CLR	1-week	0.975	0.975	0.970	0.968	0.967	0.967	0.970	0.970	0.934
	2-week	0.967	0.970	0.962	0.968	0.955	0.955	0.968	0.963	0.923
	1-month	0.963	0.967	0.957	0.963	0.941	0.942	0.965	0.963	0.915
EURO	1-week	0.849	0.852	0.812	0.833	0.844	0.844	0.829	0.813	0.812
	2-week	0.820	0.842	0.771	0.826	0.807	0.815	0.812	0.799	0.787
	1-month	0.770	0.812	0.729	0.799	0.768	0.783	0.794	0.781	0.773
INF	1-week	0.971	0.968	0.960	0.967	0.942	0.939	0.959	0.952	0.908
	2-week	0.952	0.957	0.946	0.952	0.929	0.934	0.947	0.947	0.904
	1-month	0.959	0.970	0.936	0.971	0.936	0.941	0.962	0.960	0.918
LPL	1-week	0.975	0.973	0.965	0.975	0.954	0.952	0.968	0.965	0.923
	2-week	0.968	0.971	0.954	0.970	0.942	0.947	0.962	0.955	0.918
	1-month	0.954	0.973	0.942	0.968	0.923	0.926	0.954	0.957	0.900
Portfolio	1-week	0.921	0.933	0.883	0.905	0.900	0.900	0.904	0.899	0.868
	2-week	0.896	0.917	0.837	0.896	0.879	0.894	0.883	0.870	0.863
	1-month	0.842	0.896	0.789	0.888	0.846	0.855	0.879	0.855	0.844
CSE G.I.	1-week	0.979	0.983	0.934	0.963	0.941	0.931	0.959	0.952	0.905
	2-week	0.949	0.975	0.904	0.970	0.907	0.915	0.944	0.936	0.888
	1-month	0.881	0.950	0.821	0.931	0.875	0.881	0.920	0.905	0.862

Table 49: General Index-Weighted:Adjusted 99% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9248	0.9719	0.8778	0.9601	0.9282	0.9467	0.9265	0.94	0.9131
	2-week	0.8946	0.9585	0.8508	0.9433	0.9131	0.9047	0.8979	0.9063	0.8761
	1-month	0.8508	0.9332	0.8071	0.9147	0.8508	0.8508	0.8542	0.8828	0.8273
CLR	1-week	1.0224	1.024	1.0173	1.0207	1.0055	1.0089	1.0156	1.014	0.9803
	2-week	1.0089	1.0173	1.0106	1.0173	1.0005	0.9988	1.0022	1.0022	0.9652
	1-month	1.0055	1.0089	1.0022	1.0072	0.9887	0.9887	1.0055	1.0072	0.9585
EURO	1-week	0.7432	0.7886	0.713	0.7718	0.7331	0.75	0.7264	0.7432	0.7231
	2-week	0.6945	0.7584	0.6726	0.7432	0.7331	0.7365	0.7113	0.713	0.7062
	1-month	0.6659	0.713	0.639	0.6995	0.6877	0.6928	0.6743	0.6861	0.6709
INF	1-week	0.9971	1.0005	0.9753	1.0022	0.9534	0.9618	0.9854	0.9854	0.9316
	2-week	0.9786	1.0039	0.977	1.0005	0.9551	0.9534	0.9719	0.9803	0.9198
	1-month	0.9719	0.9955	0.9433	0.982	0.9282	0.9282	0.9534	0.9736	0.9063
LPL	1-week	0.9854	0.9921	0.982	0.9938	0.9803	0.982	0.9854	0.982	0.9534
	2-week	0.9871	0.9988	0.9803	0.9904	0.9686	0.9652	0.9837	0.9887	0.9349
	1-month	0.9702	0.9904	0.9618	0.9871	0.9534	0.9551	0.9669	0.977	0.9282
PORTFOLIO	1-week	0.8525	0.9063	0.8239	0.8878	0.8609	0.8677	0.866	0.8677	0.8408
	2-week	0.8054	0.866	0.7802	0.8609	0.834	0.8273	0.8155	0.8239	0.7987
	1-month	0.7567	0.8256	0.7264	0.8038	0.7701	0.7752	0.7567	0.797	0.7685
CSE G.I.	1-week	0.8912	0.9484	0.8626	0.9366	0.8996	0.908	0.9063	0.908	0.8744
	2-week	0.8542	0.9316	0.829	0.908	0.8626	0.8593	0.8525	0.8677	0.8273
	1-month	0.7701	0.8492	0.7449	0.8307	0.8054	0.8088	0.7937	0.8172	0.797

Table 50: General Index-Ordinary: Adjusted 95% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.903	0.9517	0.8508	0.9332	0.9181	0.9316	0.9114	0.9198	0.9013
	2-week	0.8828	0.945	0.8374	0.9299	0.9265	0.9198	0.8727	0.8862	0.8878
	1-month	0.8139	0.8912	0.7802	0.8811	0.866	0.8693	0.8357	0.8542	0.8458
CLR	1-week	1.0022	1.0039	0.9971	1.0055	0.9955	0.9955	1.0005	1.0005	0.9652
	2-week	0.9921	0.9988	0.9854	0.9988	0.9904	0.9904	0.9803	0.9871	0.9568
	1-month	0.9786	0.9955	0.9803	0.9887	0.9652	0.9669	0.9719	0.9786	0.9332
EURO	1-week	0.7668	0.8172	0.7483	0.7903	0.7853	0.797	0.7701	0.7735	0.7634
	2-week	0.7247	0.7735	0.7012	0.7651	0.7718	0.7668	0.7298	0.7348	0.7416
	1-month	0.6978	0.76	0.6861	0.7365	0.7298	0.7382	0.7079	0.7298	0.7214
INF	1-week	0.9686	0.9753	0.9551	0.9803	0.9417	0.9467	0.9517	0.9568	0.9131
	2-week	0.9652	0.9786	0.9534	0.9702	0.9417	0.9417	0.9467	0.9517	0.908
	1-month	0.94	0.9652	0.9316	0.9686	0.9181	0.9248	0.9299	0.9484	0.9047
LPL	1-week	0.9887	0.9971	0.9854	0.9938	0.9686	0.9702	0.977	0.9786	0.9417
	2-week	0.977	0.9871	0.977	0.9837	0.9702	0.9686	0.9736	0.9753	0.9366
	1-month	0.9618	0.9702	0.9635	0.9686	0.9517	0.9517	0.9601	0.9618	0.9215
PORTFOLIO	1-week	0.8492	0.8979	0.8122	0.8794	0.8593	0.8778	0.8458	0.8525	0.8559
	2-week	0.7954	0.866	0.787	0.8559	0.8391	0.834	0.8071	0.8206	0.8021
	1-month	0.76	0.8139	0.7315	0.8054	0.7954	0.7954	0.7735	0.797	0.7802
CSE G.I.	1-week	0.8979	0.9551	0.8492	0.9467	0.908	0.9164	0.8996	0.9063	0.8862
	2-week	0.8542	0.9299	0.8273	0.9181	0.8811	0.8727	0.8441	0.8677	0.8458
	1-month	0.7718	0.8408	0.7466	0.8374	0.8139	0.8172	0.7802	0.8273	0.8054

Table 51: General Index-Weighted: Adjusted 95% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9762	0.9714	0.9359	0.9714	0.9504	0.9472	0.9585	0.9601	0.9181
	2-week	0.9746	0.9891	0.923	0.973	0.931	0.9343	0.9585	0.9536	0.9036
	1-month	0.9585	0.9956	0.8939	0.9891	0.9004	0.9036	0.9649	0.9552	0.881
CLR	1-week	0.9924	0.994	0.9875	0.9924	0.9827	0.9827	0.9907	0.9875	0.9504
	2-week	0.9859	0.9875	0.9827	0.9843	0.9681	0.9746	0.9843	0.9827	0.9439
	1-month	0.9714	0.9762	0.9698	0.9762	0.9649	0.9665	0.973	0.973	0.9391
EURO	1-week	0.8423	0.8407	0.7987	0.8165	0.81	0.8052	0.8197	0.8068	0.7794
	2-week	0.789	0.8197	0.7519	0.8036	0.7648	0.781	0.789	0.7681	0.7503
	1-month	0.731	0.7826	0.6922	0.7713	0.7342	0.739	0.76	0.739	0.7197
INF	1-week	0.9859	0.9859	0.973	0.9827	0.9617	0.9617	0.9746	0.9665	0.931
	2-week	0.973	0.9778	0.9698	0.9811	0.9456	0.952	0.9714	0.9681	0.9214
	1-month	0.9665	0.9827	0.9569	0.9794	0.9456	0.9456	0.9714	0.9714	0.9214
LPL	1-week	0.9794	0.9827	0.9665	0.9811	0.9536	0.952	0.9714	0.9633	0.9246
	2-week	0.9762	0.9811	0.9681	0.9794	0.9536	0.9569	0.973	0.9665	0.9246
	1-month	0.9617	0.9827	0.9504	0.9762	0.9375	0.9375	0.9681	0.9665	0.9133
PORTFOLIO	1-week	0.9294	0.9294	0.8859	0.9278	0.8988	0.8923	0.9165	0.902	0.8665
	2-week	0.9004	0.9246	0.8423	0.9101	0.8633	0.8713	0.8859	0.8697	0.8423
	1-month	0.8552	0.9068	0.7987	0.9004	0.8342	0.8439	0.8665	0.8649	0.8245
CSE G.I.	1-week	0.9746	0.9827	0.9359	0.9714	0.931	0.9262	0.9569	0.9488	0.8972
	2-week	0.9504	0.9746	0.9101	0.9569	0.8859	0.8988	0.9391	0.931	0.8729
	1-month	0.8842	0.9456	0.8116	0.9326	0.8584	0.8633	0.9149	0.902	0.8487

Table 52: General Index-Ordinary: Adjusted 99% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9681	0.9698	0.931	0.9569	0.9552	0.9552	0.9536	0.9456	0.923
	2-week	0.9617	0.9762	0.9181	0.9569	0.9294	0.9391	0.9536	0.9456	0.9084
	1-month	0.9214	0.9843	0.8713	0.973	0.9004	0.9036	0.9407	0.9359	0.8826
CLR	1-week	0.9746	0.9746	0.9714	0.9714	0.9665	0.9665	0.9665	0.9681	0.9343
	2-week	0.9665	0.9681	0.9633	0.9681	0.9552	0.9552	0.9649	0.9617	0.923
	1-month	0.9633	0.9665	0.9552	0.9649	0.9407	0.9423	0.9633	0.9633	0.9149
EURO	1-week	0.8455	0.8487	0.8019	0.8326	0.8439	0.8439	0.8262	0.8132	0.8116
	2-week	0.8132	0.8391	0.7697	0.8262	0.8068	0.8149	0.81	0.7955	0.7874
	1-month	0.7729	0.8181	0.7358	0.8116	0.7681	0.7826	0.7907	0.7777	0.7729
INF	1-week	0.9714	0.9714	0.9601	0.9665	0.9423	0.9391	0.9552	0.9552	0.9084
	2-week	0.9504	0.9536	0.9423	0.9504	0.9294	0.9343	0.9456	0.9472	0.9036
	1-month	0.9633	0.9714	0.9391	0.9698	0.9359	0.9407	0.9601	0.9569	0.9181
LPL	1-week	0.973	0.9714	0.9665	0.9681	0.9536	0.952	0.9649	0.9633	0.923
	2-week	0.9649	0.973	0.9569	0.9649	0.9423	0.9472	0.9617	0.9585	0.9181
	1-month	0.9472	0.9698	0.9407	0.9665	0.923	0.9262	0.9504	0.9488	0.9004
PORTFOLIO	1-week	0.9181	0.9294	0.8746	0.9246	0.9004	0.9004	0.9036	0.8955	0.8681
	2-week	0.8955	0.9149	0.8439	0.8988	0.8794	0.8939	0.881	0.8681	0.8633
	1-month	0.8391	0.8988	0.8003	0.8907	0.8455	0.8552	0.8649	0.852	0.8439
CSE G.I.	1-week	0.9746	0.9827	0.9278	0.9649	0.9407	0.931	0.9633	0.9456	0.9052
	2-week	0.9536	0.9714	0.9052	0.9536	0.9068	0.9149	0.9488	0.9326	0.8875
	1-month	0.8891	0.9472	0.8213	0.9294	0.8746	0.881	0.9165	0.9084	0.8617

Table 53: General Index-Weighted:Adjusted 99% VaR test results (R_{free}=0.04).

The results using as a market index the sector indexes with R_{free}=0.0 are shown initially and then the corresponding results with R_{free}=0.04.

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.8749	0.9748	0.8851	0.9596	0.8005	0.8039	0.9409	0.9528	0.9240
	2-week	0.8512	0.9562	0.8580	0.9342	0.7666	0.7734	0.9020	0.9156	0.9105
	1-month	0.8039	0.9426	0.8174	0.9172	0.7142	0.7209	0.7700	0.7836	0.8699
INF	1-week	1.0289	1.0408	1.0256	1.0391	0.9223	0.9223	1.0340	1.0340	0.9359
	2-week	1.0188	1.0408	1.0239	1.0391	0.9003	0.9088	1.0154	1.0222	0.9240
	1-month	1.0205	1.0425	1.0137	1.0408	0.9071	0.9139	0.9680	0.9731	0.9206
LPL	1-week	0.9917	1.0052	0.9968	1.0002	0.9579	0.9596	1.0036	1.0036	0.9562
	2-week	1.0019	1.0086	1.0019	1.0086	0.9528	0.9528	1.0036	1.0069	0.9376
	1-month	0.9849	1.0069	0.9883	1.0069	0.9426	0.9426	0.9663	0.9731	0.9172
PORTFOLIO	1-week	0.8123	0.8969	0.8123	0.8919	0.7802	0.7886	0.8665	0.8716	0.8445
	2-week	0.7666	0.8614	0.7666	0.8563	0.7649	0.7717	0.8106	0.8208	0.8039
	1-month	0.7277	0.8208	0.7328	0.8123	0.7057	0.7226	0.6905	0.6989	0.7734

Table 54: Sectors-Ordinary: Adjusted 95% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.8744	0.9702	0.8677	0.9551	0.7954	0.7987	0.9349	0.9383	0.9181
	2-week	0.8357	0.9467	0.8475	0.9282	0.7617	0.7685	0.8963	0.9147	0.9047
	1-month	0.797	0.9349	0.8139	0.9181	0.7096	0.7163	0.7701	0.7819	0.8643
INF	1-week	1.0224	1.0341	1.0207	1.0341	0.9164	0.9164	1.024	1.0274	0.9299
	2-week	1.0123	1.0341	1.0156	1.0291	0.8946	0.903	1.014	1.014	0.9181
	1-month	1.014	1.0358	1.0089	1.0358	0.9013	0.908	0.9568	0.9719	0.9147
LPL	1-week	0.9887	1.0022	0.9955	0.9971	0.9517	0.9534	0.9955	0.9988	0.9501
	2-week	0.9921	1.0005	0.9921	1.0005	0.9467	0.9467	1.0005	1.0022	0.9316
	1-month	0.9753	0.9988	0.9803	0.9921	0.9366	0.9366	0.9618	0.9669	0.9114
PORTFOLIO	1-week	0.8139	0.8862	0.8004	0.8727	0.7752	0.7836	0.8576	0.8609	0.8391
	2-week	0.7651	0.8593	0.7701	0.8424	0.76	0.7668	0.8122	0.8172	0.7987
	1-month	0.718	0.8189	0.7247	0.8071	0.7012	0.718	0.6894	0.6961	0.7685

Table 55: Sectors-Ordinary: Adjusted 95% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.8749	0.9562	0.8749	0.9511	0.7869	0.7869	0.9274	0.9291	0.9037
	2-week	0.8326	0.9240	0.8377	0.9223	0.7446	0.7514	0.8936	0.8919	0.8902
	1-month	0.7836	0.9274	0.8005	0.9240	0.6955	0.7006	0.7700	0.7717	0.8462
INF	1-week	0.9883	1.0002	0.9799	0.9968	0.8986	0.9003	0.9799	0.9866	0.9088
	2-week	0.9883	1.0069	0.9866	1.0052	0.8868	0.8885	0.9883	0.9900	0.8986
	1-month	0.9799	1.0002	0.9765	1.0036	0.8699	0.8732	0.9139	0.9291	0.8648
LPL	1-week	1.0052	1.0120	1.0002	1.0137	0.9629	0.9629	1.0103	1.0120	0.9562
	2-week	1.0086	1.0103	1.0086	1.0103	0.9646	0.9663	1.0103	1.0120	0.9494
	1-month	1.0002	1.0103	1.0036	1.0103	0.9596	0.9612	0.9748	0.9832	0.9392
PORTFOLIO	1-week	0.8259	0.8851	0.8191	0.8699	0.7954	0.7988	0.8665	0.8732	0.8546
	2-week	0.7751	0.8648	0.7920	0.8665	0.7700	0.7768	0.8191	0.8276	0.8123
	1-month	0.7345	0.8259	0.7497	0.8191	0.7294	0.7429	0.6955	0.7142	0.7886

Table 56: Sectors-Weighted: Adjusted 95% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.871	0.9585	0.871	0.9433	0.7819	0.7819	0.9198	0.9265	0.8979
	2-week	0.8239	0.9248	0.8307	0.9181	0.7399	0.7466	0.8828	0.8979	0.8845
	1-month	0.787	0.9164	0.8054	0.9147	0.6911	0.6961	0.755	0.7701	0.8408
INF	1-week	0.9837	0.9988	0.977	0.9988	0.8929	0.8946	0.9753	0.9786	0.903
	2-week	0.9803	1.0039	0.9803	1.0055	0.8811	0.8828	0.9786	0.9837	0.8929
	1-month	0.9686	0.9955	0.9702	0.9988	0.8643	0.8677	0.9097	0.9232	0.8593
LPL	1-week	0.9938	1.0055	0.9921	1.0022	0.9568	0.9568	1.0039	1.0055	0.9501
	2-week	1.0005	1.0039	1.0055	1.0055	0.9585	0.9601	1.0039	1.0072	0.9433
	1-month	0.9921	1.0039	0.9938	1.0055	0.9534	0.9551	0.9702	0.977	0.9332
PORTFOLIO	1-week	0.8206	0.8929	0.8054	0.8811	0.7903	0.7937	0.8593	0.8677	0.8492
	2-week	0.7718	0.8593	0.7752	0.8525	0.7651	0.7718	0.8172	0.8307	0.8071
	1-month	0.7382	0.8223	0.7432	0.8038	0.7247	0.7382	0.6978	0.7113	0.7836

Table 57: Sectors-Weighted: Adjusted 95% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9679	0.9776	0.9711	0.9728	0.7892	0.7941	0.9792	0.9711	0.9386
	2-week	0.9581	0.9890	0.9500	0.9760	0.7681	0.7763	0.9679	0.9630	0.9224
	1-month	0.9224	1.0004	0.9224	0.9874	0.7275	0.7340	0.8574	0.8396	0.8997
INF	1-week	1.0004	1.0069	1.0020	1.0069	0.9159	0.9159	1.0020	1.0020	0.9370
	2-week	1.0052	1.0085	1.0020	1.0085	0.8932	0.8964	1.0036	0.9987	0.9257
	1-month	1.0020	1.0101	1.0004	1.0101	0.9029	0.9078	0.9825	0.9809	0.9289
LPL	1-week	0.9825	0.9857	0.9857	0.9792	0.9240	0.9240	0.9890	0.9874	0.9240
	2-week	0.9857	0.9890	0.9890	0.9857	0.9240	0.9289	0.9939	0.9890	0.9208
	1-month	0.9760	0.9906	0.9841	0.9809	0.9175	0.9175	0.9614	0.9646	0.9013
PORTFOLIO	1-week	0.9029	0.9289	0.9289	0.9240	0.8087	0.8055	0.9289	0.9159	0.8704
	2-week	0.8672	0.9192	0.8867	0.9143	0.7876	0.8022	0.8916	0.8753	0.8526
	1-month	0.8217	0.9127	0.8201	0.8980	0.7535	0.7746	0.7584	0.7438	0.8282

Table 58: Sectors-Ordinary: Adjusted 99% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9649	0.9714	0.9649	0.9617	0.7842	0.789	0.9698	0.9698	0.9326
	2-week	0.9456	0.9778	0.9633	0.9714	0.7632	0.7713	0.9665	0.9585	0.9165
	1-month	0.9165	0.994	0.9214	0.9843	0.7229	0.7293	0.852	0.8326	0.8939
INF	1-week	0.9972	1.0004	0.9988	0.9956	0.9101	0.9101	0.9956	0.9956	0.931
	2-week	0.9988	1.0036	0.9972	0.9988	0.8875	0.8907	0.9972	0.9972	0.9197
	1-month	0.9956	1.0036	1.0004	1.0036	0.8972	0.902	0.9762	0.9714	0.923
LPL	1-week	0.9843	0.9811	0.9811	0.9762	0.9181	0.9181	0.9811	0.9811	0.9181
	2-week	0.9778	0.9827	0.9811	0.9811	0.9181	0.923	0.9875	0.9843	0.9149
	1-month	0.973	0.9843	0.9778	0.9811	0.9117	0.9117	0.9601	0.9569	0.8955
PORTFOLIO	1-week	0.8972	0.9262	0.9149	0.923	0.8036	0.8003	0.923	0.9133	0.8649
	2-week	0.8617	0.9165	0.8891	0.9004	0.7826	0.7971	0.8859	0.8794	0.8471
	1-month	0.8213	0.9052	0.8342	0.8923	0.7487	0.7697	0.7681	0.7455	0.8229

Table 59: Sectors-Ordinary: Adjusted 99% VaR test results (R_{free}=0.04).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9549	0.9679	0.9614	0.9614	0.7811	0.7860	0.9630	0.9533	0.9305
	2-week	0.9208	0.9663	0.9305	0.9565	0.7454	0.7584	0.9451	0.9354	0.9127
	1-month	0.9127	0.9809	0.9370	0.9744	0.7097	0.7145	0.8428	0.8266	0.8867
INF	1-week	0.9695	0.9760	0.9760	0.9776	0.8899	0.8883	0.9646	0.9630	0.9045
	2-week	0.9760	0.9906	0.9809	0.9890	0.8672	0.8786	0.9776	0.9744	0.8980
	1-month	0.9614	0.9922	0.9744	0.9874	0.8688	0.8769	0.9484	0.9468	0.8916
LPL	1-week	0.9792	0.9776	0.9825	0.9744	0.9354	0.9338	0.9874	0.9857	0.9289
	2-week	0.9809	0.9906	0.9809	0.9841	0.9370	0.9370	0.9874	0.9890	0.9289
	1-month	0.9809	0.9890	0.9825	0.9874	0.9240	0.9305	0.9711	0.9695	0.9208
PORTFOLIO	1-week	0.9029	0.9322	0.9192	0.9159	0.8104	0.8087	0.9127	0.9094	0.8753
	2-week	0.8639	0.9224	0.8948	0.9143	0.7909	0.8104	0.8899	0.8802	0.8591
	1-month	0.8315	0.9208	0.8591	0.9094	0.7730	0.7909	0.7730	0.7519	0.8380

Table 60: Sectors-Weighted: Adjusted 99% VaR test results (R_{free}=0.0).

	Method	nVaR1	pVaR1	nVaR2	pVaR2	nVaR3	pVaR3	nVaR4	pVaR4	RiskMetrics
BOC	1-week	0.9504	0.9552	0.9472	0.9439	0.7761	0.781	0.9617	0.9504	0.9246
	2-week	0.9165	0.9617	0.9423	0.9488	0.7406	0.7535	0.9359	0.9278	0.9068
	1-month	0.9084	0.9762	0.9214	0.9649	0.7051	0.71	0.8374	0.8262	0.881
INF	1-week	0.9633	0.9714	0.9665	0.9714	0.8842	0.8826	0.9601	0.9569	0.8988
	2-week	0.9746	0.9827	0.9746	0.9827	0.8617	0.8729	0.973	0.9698	0.8923
	1-month	0.9552	0.9843	0.9649	0.9843	0.8633	0.8713	0.9456	0.9456	0.8859
LPL	1-week	0.973	0.973	0.973	0.9762	0.9294	0.9278	0.9762	0.9778	0.923
	2-week	0.9778	0.9843	0.9859	0.9843	0.931	0.931	0.9843	0.9827	0.923
	1-month	0.9746	0.9827	0.9746	0.9811	0.9181	0.9246	0.9665	0.9633	0.9149
PORTFOLIO	1-week	0.8907	0.9278	0.9084	0.9084	0.8052	0.8036	0.9101	0.902	0.8697
	2-week	0.8649	0.9214	0.8859	0.9068	0.7858	0.8052	0.8859	0.8697	0.8536
	1-month	0.8213	0.9149	0.8504	0.902	0.7681	0.7858	0.7713	0.7648	0.8326

Table 61: Sectors-Weighted: Adjusted 99% VaR test results (R_{free}=0.04).

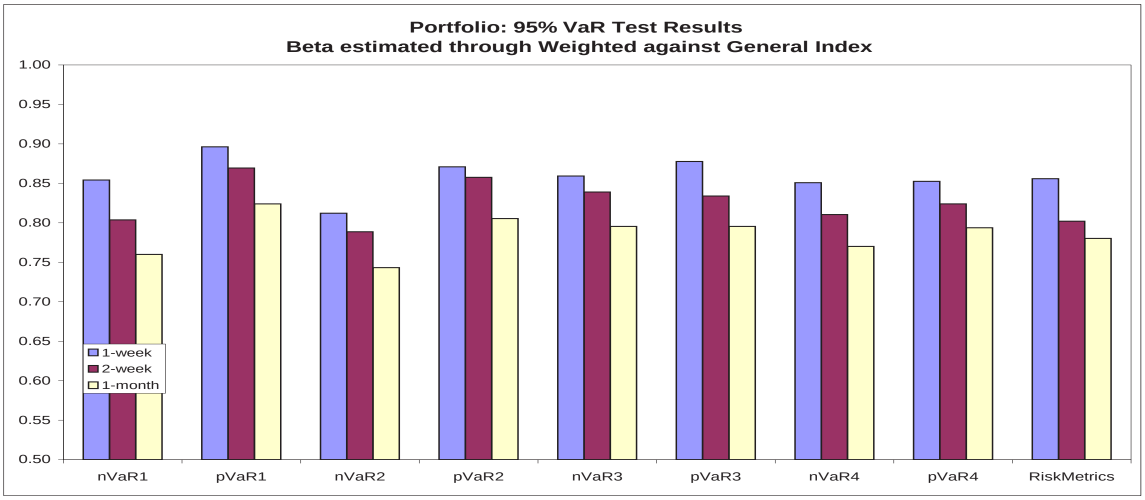

Figure 28: CSE general index-Weighted: Portfolio Adjusted 95% VaR test results.

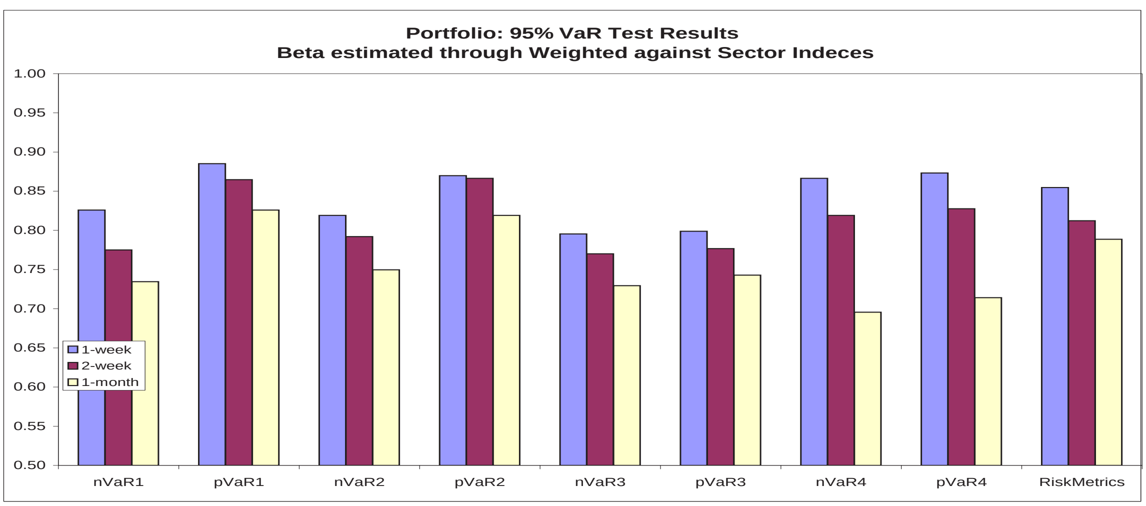

Figure 29: Sector indexes-Weighted : Portfolio Adjusted 95% VaR test results.

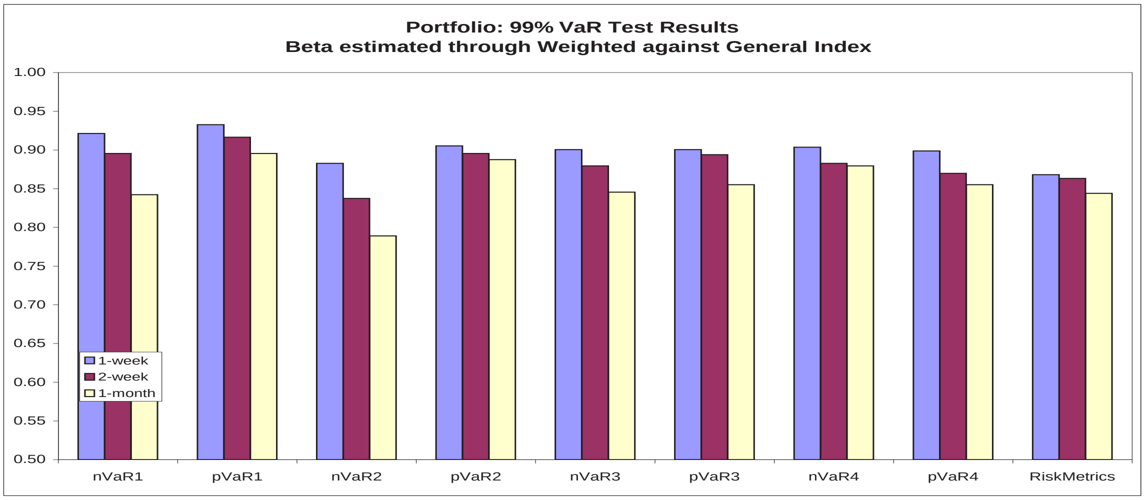

Figure 30: CSE general index-Weighted : Portfolio Adjusted 99% VaR test results.

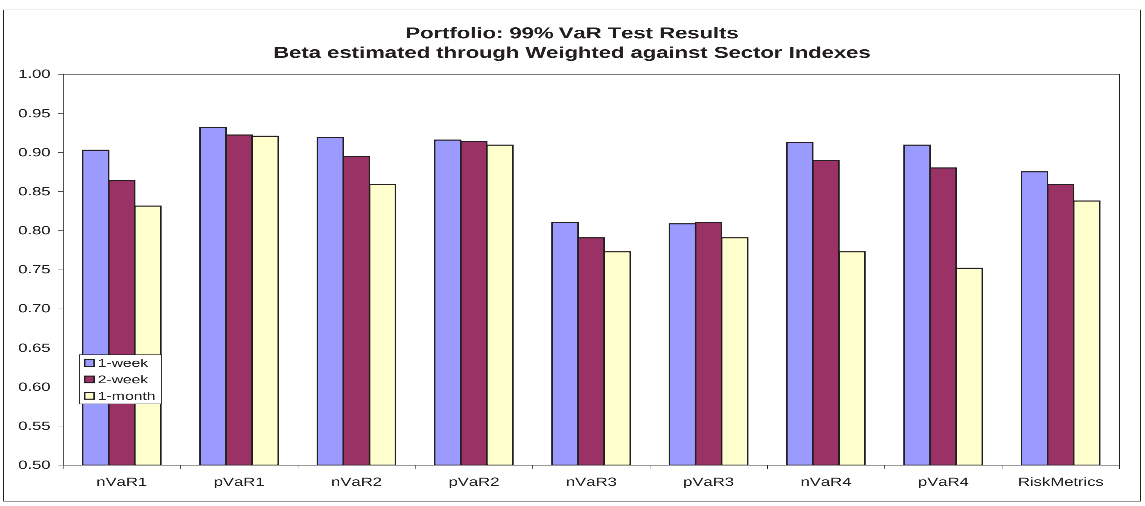

Figure 31: Sector indexes-Weighted : Portfolio Adjusted 99% VaR test results.

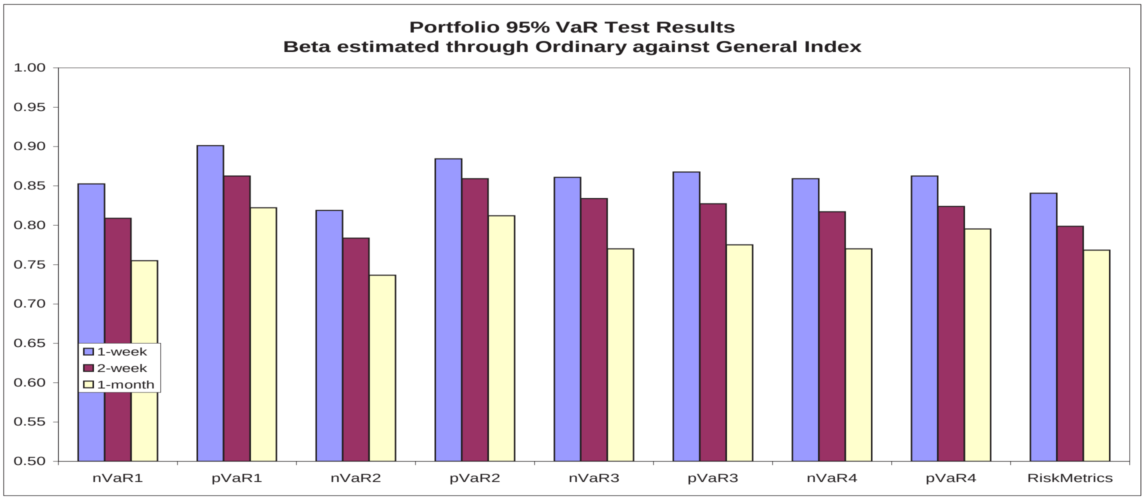

Figure 32: CSE general index-Ordinary : Portfolio Adjusted 95% VaR test results.

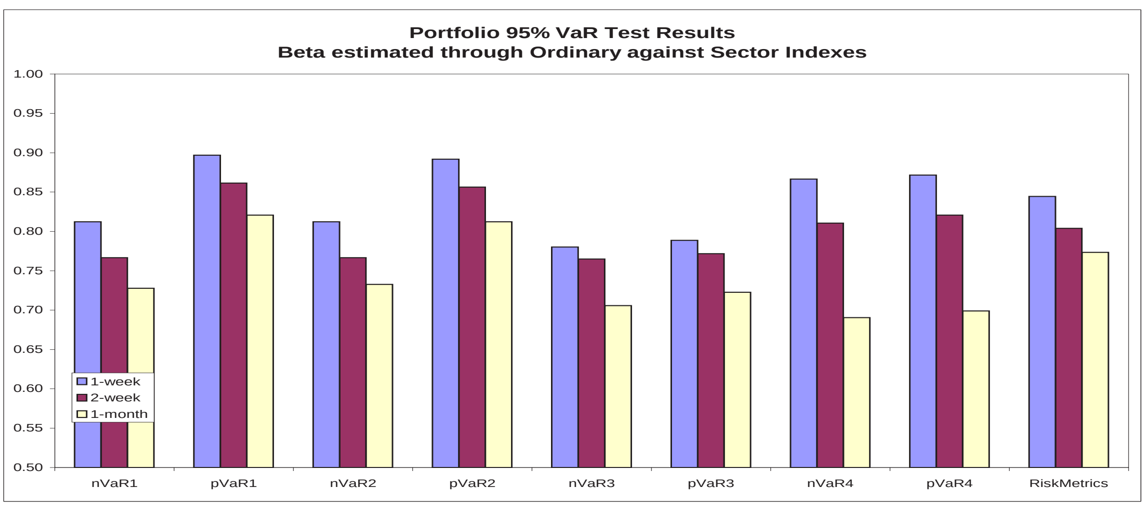

Figure 33: Sector indexes-Ordinary : Portfolio Adjusted 95% VaR test results.

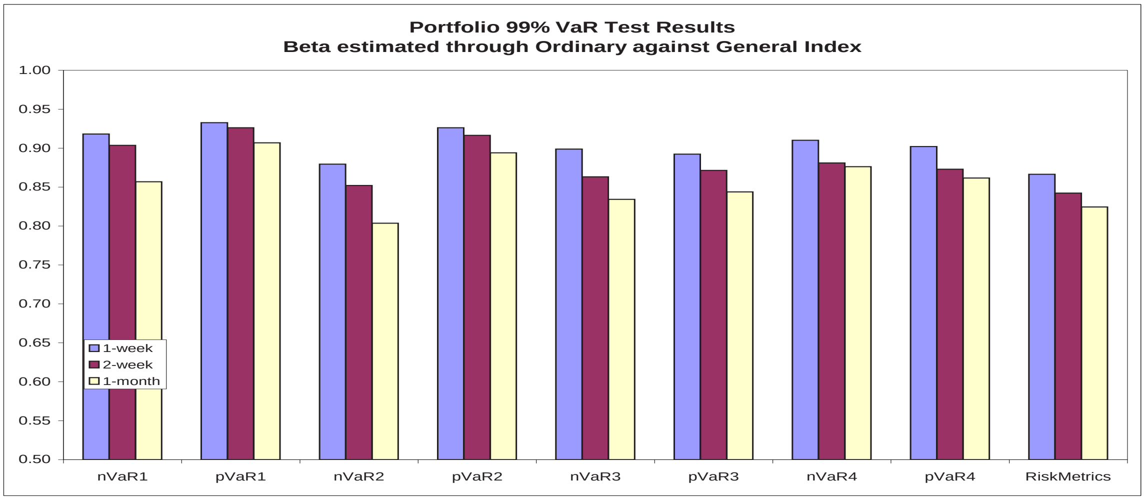

Figure 34: CSE general index-Ordinary : Portfolio Adjusted 99% VaR test results.

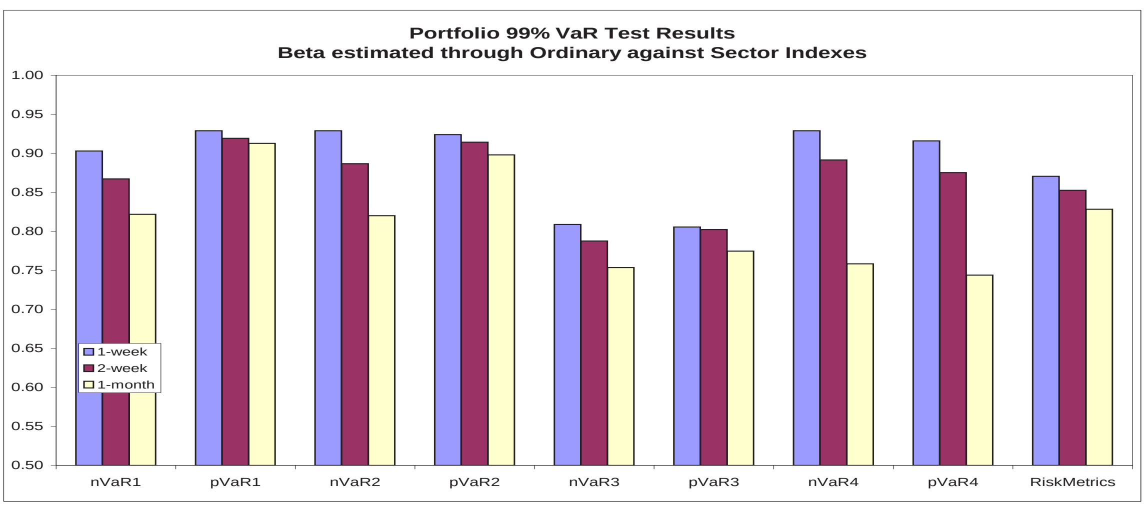

Figure 35: Sector indexes-Ordinary : Portfolio Adjusted 99% VaR test results.

E Residual analysis

Here we present the analysis of regression residuals using as a market index the CSE general index. This indicates that the normality assumption is accepted for the securities BOC for period 3, Table (62) and Figure (39), and EURO for period 3, Table (63), Figure (41).

The Figure (36) shows that the we have homoscedasticity and therefore the *Ordinary* model is suitable for use, while Figure (37) shows heteroscedasticity and therefore the a transformation should be considered (i.e. use *Weighted* method).

	Weighted				Ordinary			
Period	History	1	2	3	History	1	2	3
λ	0.91	0.94	0.82	0.74	0.91	0.94	0.82	0.74
Sample	1343	798	281	264	1343	798	281	264
mean	0.11	0.01%	-6.6%	-10.87%	-1.09%	0.95%	% -4.84%	-2.93%
St.Deviation	99.96	99.93%	99.61%	99.21%	99.95%	99.9%	99.7%	99.76%
Skewness	4.29	-7.3	-1.55	-0.13	-2.96	-7.53	-180	0.24
Kurtosis	110.12	127.77	9.94	6.23	39.4	135.92	13.38	5.20
Min	-975.24%	-1772.7%	-651.99%	-500.8%	-1359.2%	-1799.31%	-731.13%	-398.8%
Max	18.97%	386.60%	237.85%	393.47%	509.73%	405.7%	308.91%	481.03%
A-D	∞	32.51	4.19	1.55	43.11	37.17	4.85	0.62
kolmogorov	0.1502	0.12	0.08	0.06	0.12	0.13	0.10	0.048
X-square	748.18	257.77	172.1	55.11	447.6	293.1	134.8	19.1

Table 62: Descriptive statistics of the regression residual of BOC against the CSE General Index.

	Weighted				Ordinary			
Period	History	1	2	3	History	1	2	3
λ	0.95	1	0.77	0.80	0.95	1	0.77	0.80
Sample	938	420	281	237	938	420	281	237
mean	-0.28%	5.41%	-3.41%	-6.25%	-0.23%	5.46%	0.82%	-5.69%
St.Deviation	99.94%	99.73%	99.76%	99.59%	99.94%	99.73%	99.81%	99.62%
Skewness	1.26	0.76	-0.51	0.81	1.27	0.76	1.39	0.78
Kurtosis	19.49	8.35	12.02	6.54	1.27	8.37	18.88	6.43
Min	-562.9%	-534.49%	-597.2%	-266.54%	-584.83%	-535.3%	-443%	-269.07%
Max	1046.8%	478.19%	547.27%	488.99%	1049.8%	478.34%	799.1%	486.2%
A-D	∞	10.41	6.44	2.128	∞	10.48	7.19	2.05
kolmogorov	0.105	0.12	0.107	0.069	10.82	0.123	0.114	0.07
X-square	310.72	207.4	111.1	38.97	310.1	209.37	99.62	34.55

Table 63: Descriptive statistics of the regression residual of EURO against the CSE General Index.

	Weighted			Ordinary		
Security	CLR	INF	LPL	CLR	INF	LPL
λ	0.79	0.89	0.89	0.79	0.89	0.89
Sample	161	268	120	161	268	120
mean	9.42%	10.22%	1.37%	8.73%	9.33%	0.78%
St.Deviation	99.23%	99.28%	99.56%	99.3%	99.37%	99.57%
Skewness	-0.25	0.46	0.14	-0.51	0.39	0.17
Kurtosis	5.64	59.60	3.92	6.66	6.16	3.93
Min	-380.13%	-311.05%	-341.3%	-385.86%	-342.36%	-338.77%
Max	321.79%	492.8%	275%	343.52%	496.63%	275.44%
A-D	1.7	2.76	0.602	2.266	3.02	0.65
kolmogorov	0.082	0.07	0.076	0.082	0.07	0.083
X-square	118.7	84.93	61.1	88.7	87.01	65.76

Table 64: Descriptive statistics of the regression residual of the securities against the CSE General Index.

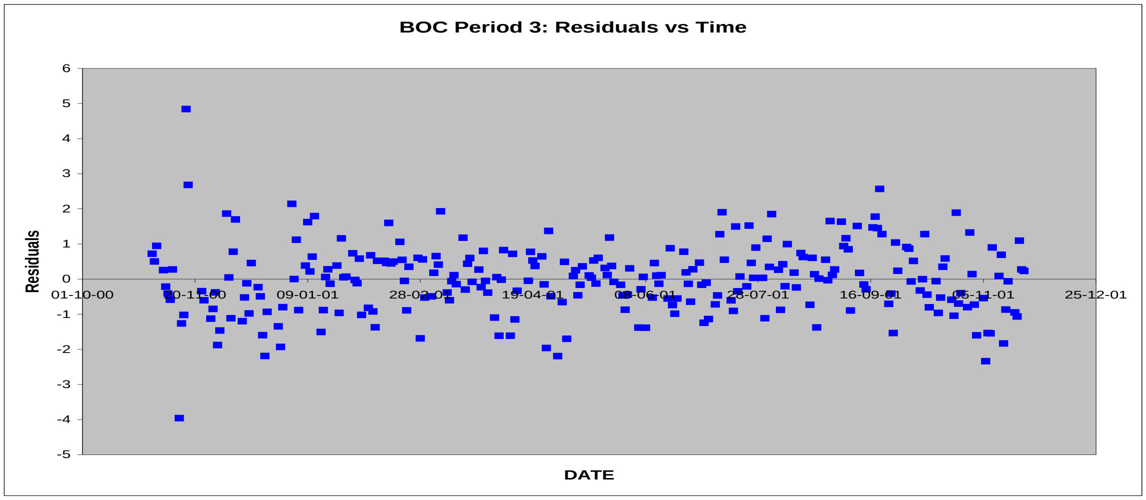

Figure 36: Residual vs Time

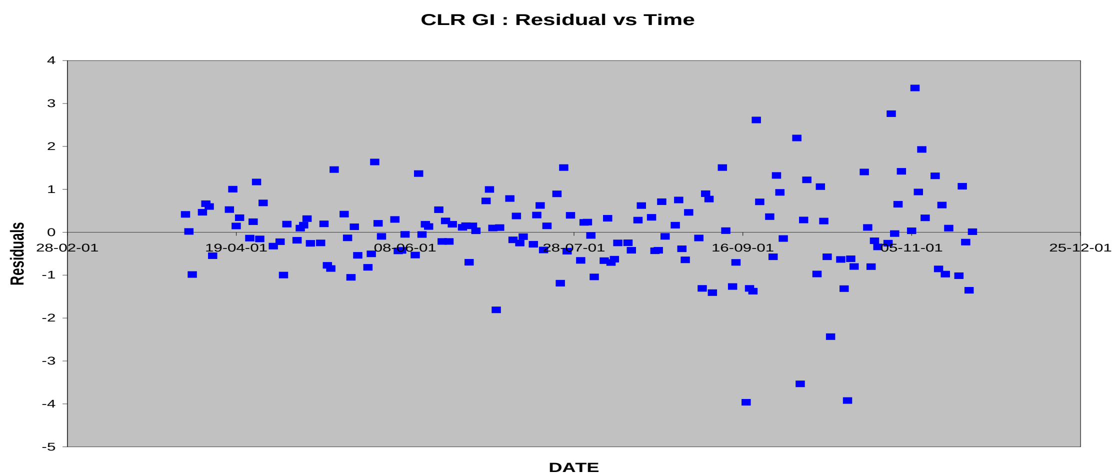

Figure 37: Residual vs Time

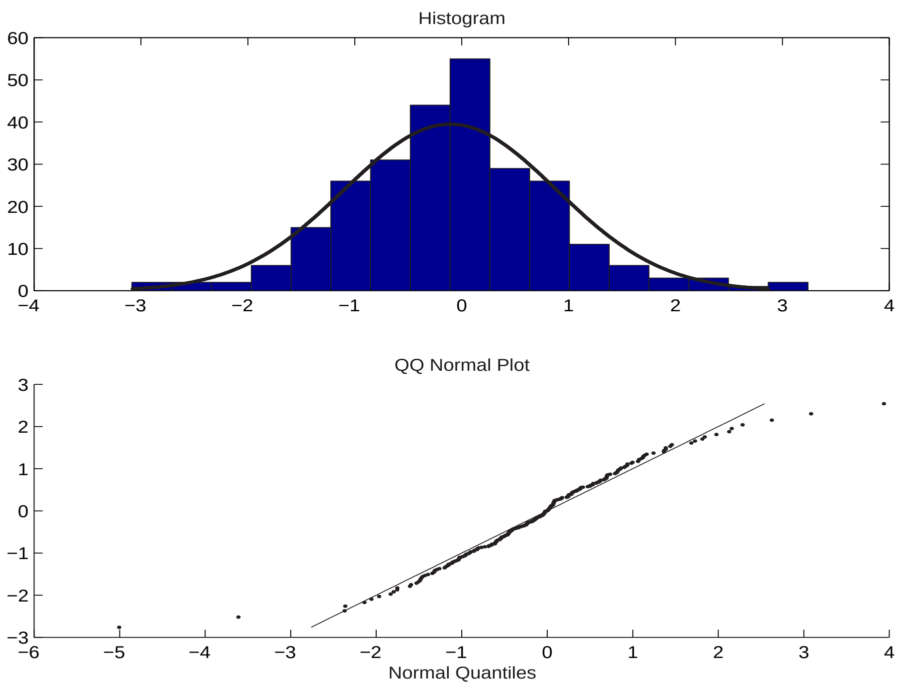

Figure 38: Histogram and QQ-plot of the *Weighted* residuals of BOC against the CSE General Index for period 3.

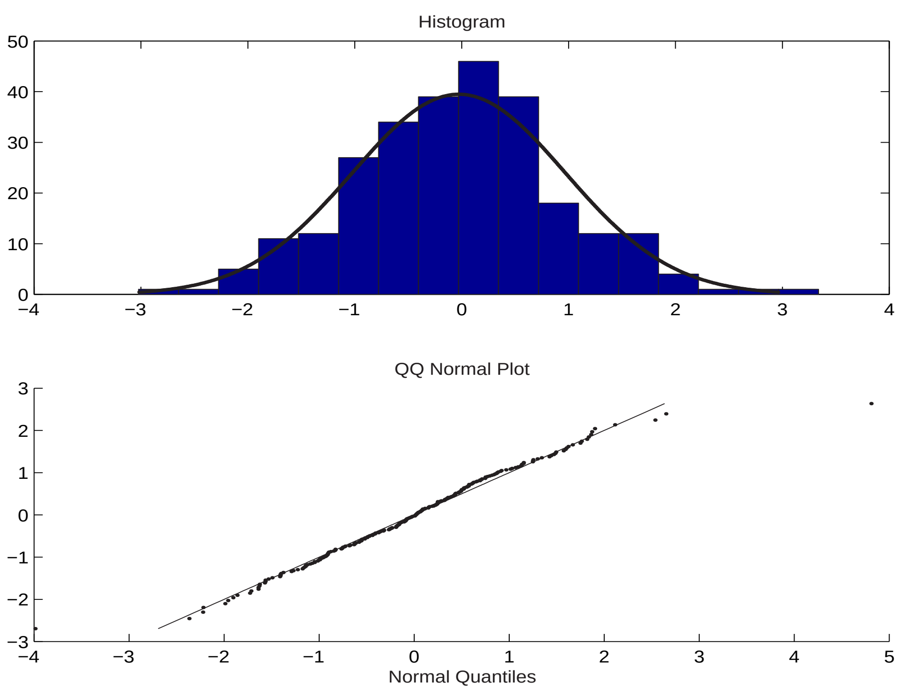

Figure 39: Histogram and QQ-plot of the *Ordinary* residuals of BOC against the CSE General Index for period 3.

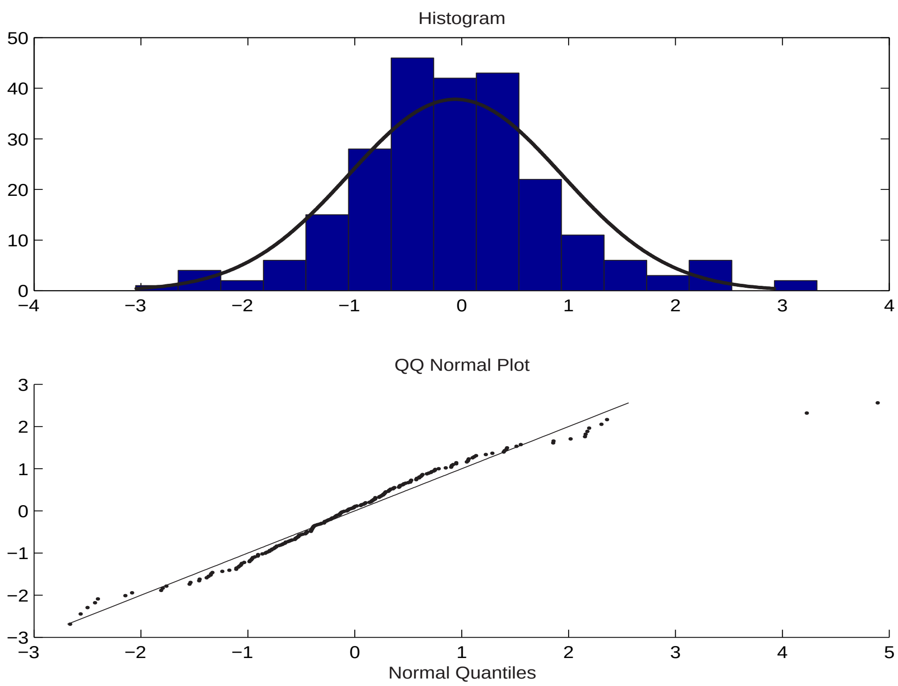

Figure 40: Histogram and QQ-plot of the *Weighted* residuals of EURO against the CSE General Index for period 3.

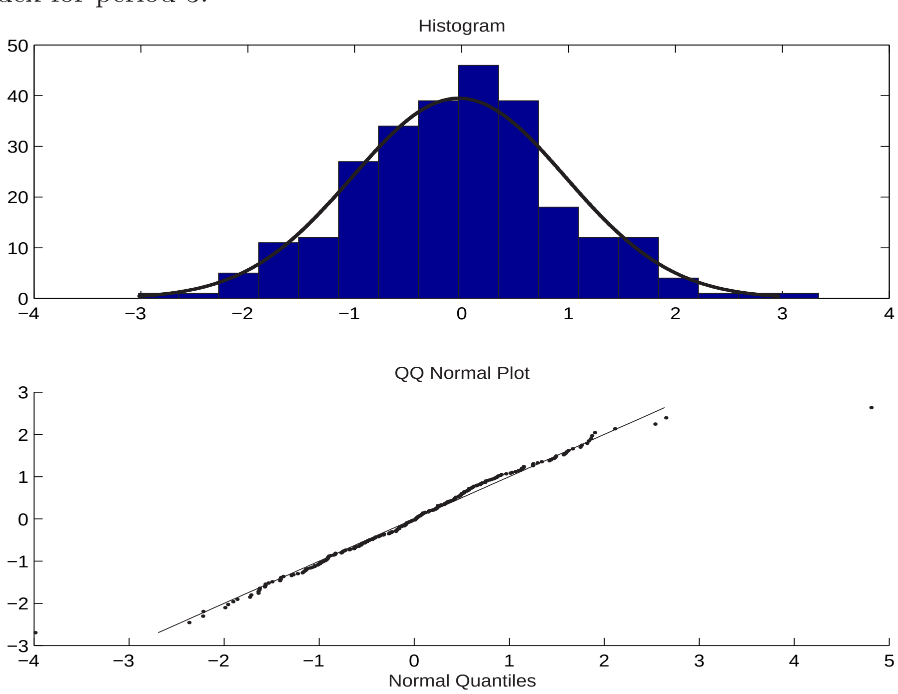

Figure 41: Histogram and QQ-plot of the *Ordinary* residuals of EURO against the CSE General Index for period 3.

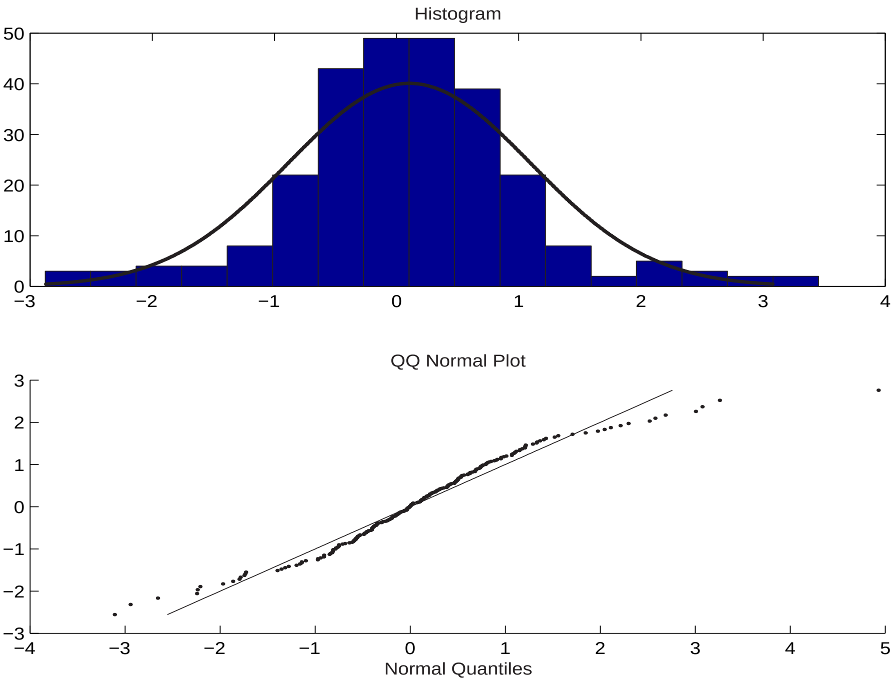

Figure 42: Histogram and QQ-plot of the *Weighted* residuals of INF against the CSE General Index for the whole history.

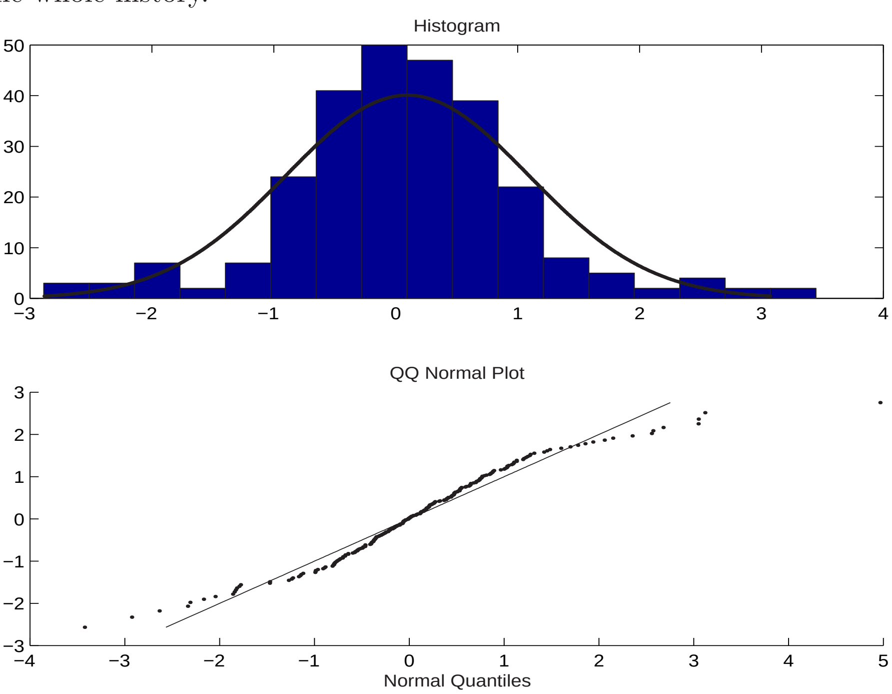

Figure 43: Histogram and QQ-plot of the *Ordinary* residuals of INF against the CSE General Index for the whole history.

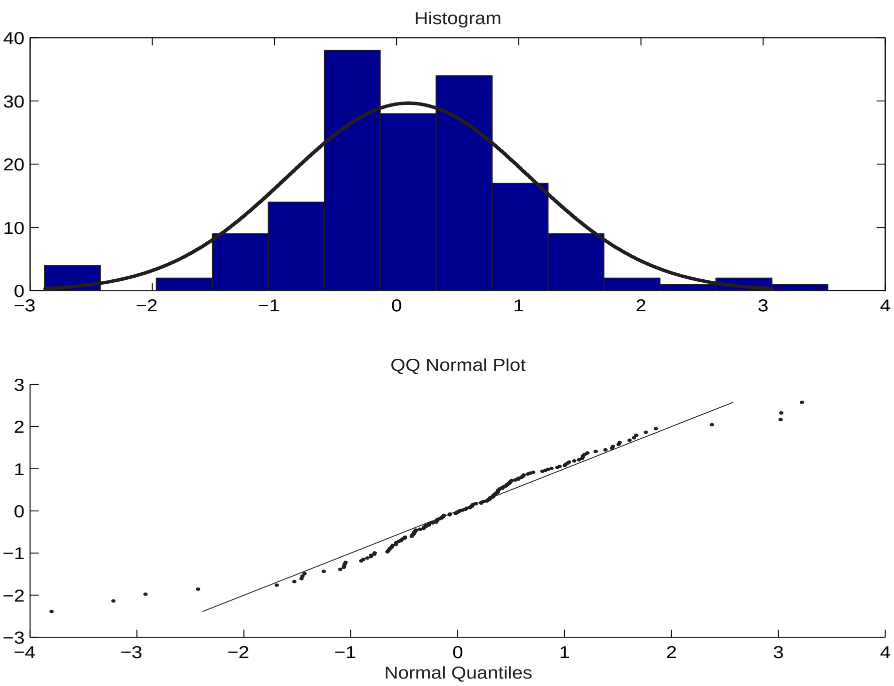

Figure 44: Histogram and QQ-plot of the *Weighted* residuals of CLR against the CSE General Index for the whole history.

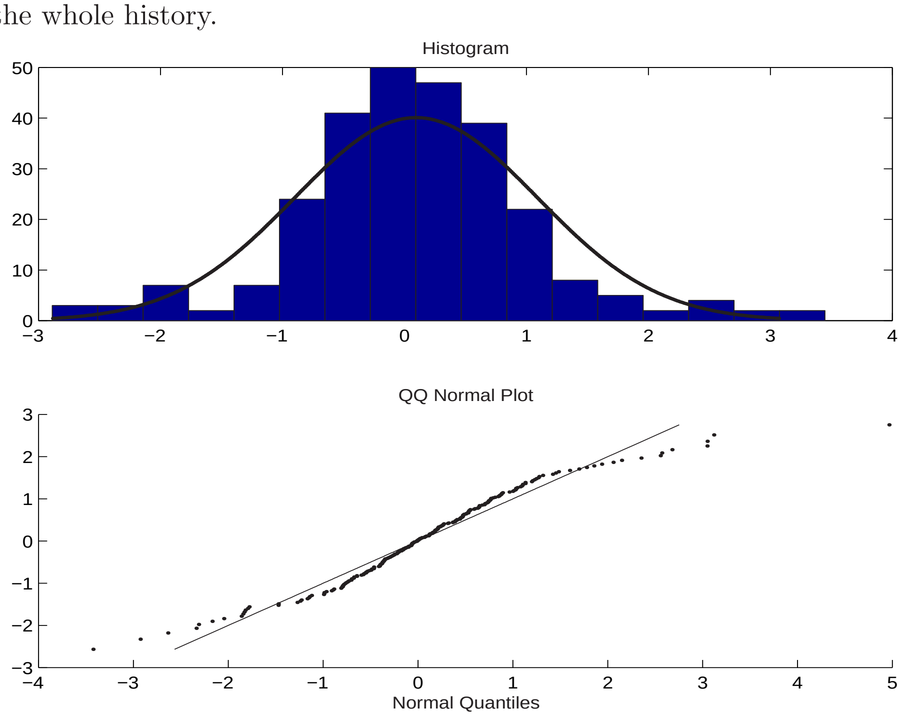

Figure 45: Histogram and QQ-plot of the *Ordinary* residuals of CLR against the CSE General Index for the whole history.

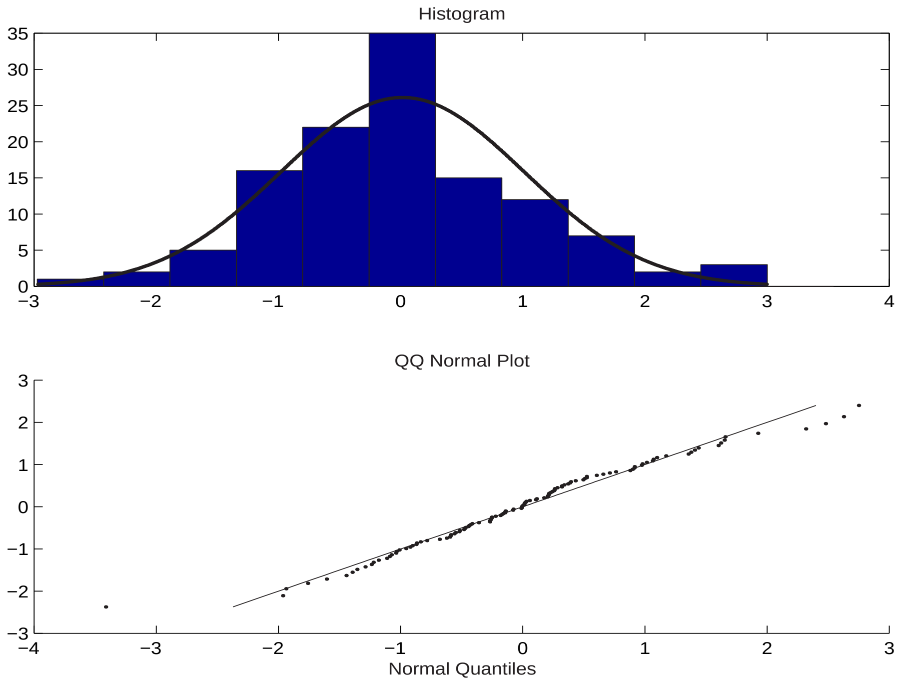

Figure 46: Histogram and QQ-plot of the *Weighted* residuals of LPL against the CSE General Index for the whole history.

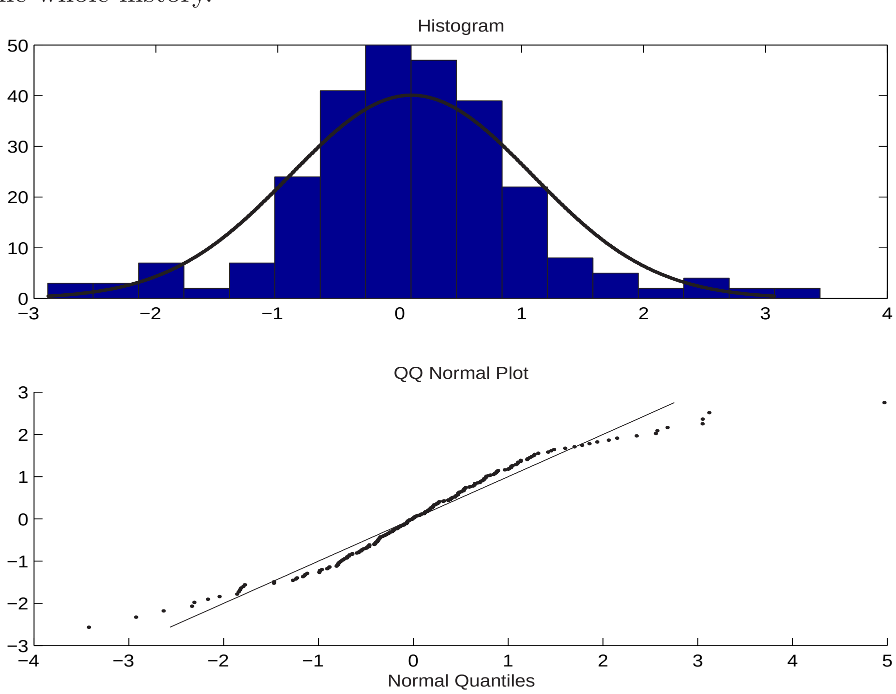

Figure 47: Histogram and QQ-plot of the *Ordinary* residuals of LPL against the CSE General Index for the whole history.

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