



*LIFE-CYCLE ASSET ALLOCATION:  
A MODEL WITH BORROWING CONSTRAINTS,  
UNINSURABLE LABOR INCOME RISK AND  
STOCK-MARKET PARTICIPATION COSTS*

*by  
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*Working Paper 02-03*



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# Life-Cycle Asset Allocation: A Model with Borrowing Constraints, Uninsurable Labor Income Risk and Stock-Market Participation Costs\*

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January 2002

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\*We would like to thank Lorenzo Forni, Luigi Guiso, Michael Haliassos, Pascal Maenhout, Monica Paiella, Raman Uppal, Tan Wang and seminar participants at the May 2001 Asset and Liability Management Conference at the University of Cyprus, the Ente-Einaudi Center, Leicester University and the LSE for helpful comments and suggestions. The second author acknowledges funding from a University of Cyprus Research grant and European Union funding through the HERMES Center. We are responsible for any remaining errors or omissions.

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## **Abstract**

We study life-cycle asset allocation in the presence of liquidity constraints and uninsurable labor income risk. We find that small stock-market entry costs can explain the zero-median stockholding puzzle. Households start saving in the riskless asset and enter the stock market later in life. Moreover, the presence of human capital rationalizes the asset allocation puzzle of Canner, Mankiw and Weil (1997). Nevertheless, conditional on participation, the portfolio allocation remains heavily skewed towards equities. Extensions that generate a larger co-existence of cash and stocks imply a counterfactually high stock market participation rate. These results suggest that simultaneously matching stock market participation and asset allocation remains a challenge.

JEL Classification: G11.

Key Words: Life-Cycle Asset Allocation, Liquidity Constraints, Stock Market Participation Costs, Uninsurable Labor Income Risk.

# 1 Introduction

According to the most recent empirical evidence on life-cycle asset allocation, at least 50% of the population (in any country) does not own equities. Stock-market participation rates are very low among young households, and they increase significantly during working life. Furthermore, even those households that do own equities, still invest a large fraction of their financial wealth in alternative assets.<sup>1</sup> These facts are at odds with the predictions of existing life-cycle asset allocation models.

A seminal reference addressing the problem of portfolio choice over the life-cycle is Samuelson (1969). Under the assumptions of independently and identically distributed returns, homothetic preferences, and the absence of labor income, he concludes that the optimal asset allocation should remain constant over time. However, in practice, labor income (or human capital) constitutes a crucial asset for most households. To the extent that the level and risk of the labor income stream change over the life-cycle, this can provide a rationale for age-varying investment strategies. If markets are complete so that labor income risk can be insured, its role is well understood from the seminal work by Merton (1971). However, the incomplete markets case remained unsolved for many years.

Following Deaton (1991) and Carroll (1992), a large successful literature on buffer stock saving with undiversifiable labor income risk has emerged.<sup>2</sup> More recently these models have been extended to include an asset allocation decision, both in an infinite-horizon setting (see Haliassos and Michaelides (2002), Viceira (2001), Michaelides (2001), Heaton and Lucas (2000, 1997 and 1996), Koo (1998), Lucas (1994) or Telmer (1993)) and in a finite-horizon life-cycle setting (see Campbell, Cocco, Gomes and Maenhout (2001), Storesletten, Telmer and Yaron (2001), Dammon, Spatt and Zhang (2001), Cocco, Gomes and Maenhout (1999), Hochguertel (1999), or Gakidis (1998)).<sup>3</sup>

Consistent with the intuition in Jagannathan and Kocherlakota (1996), and with the

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<sup>1</sup>This evidence is discussed in more detail below.

<sup>2</sup>See, for instance Cagetti (2001), Gourinchas and Parker (2001), Ludvigson and Michaelides (2001), Carroll (1997), Carroll and Samwick (1997), or Hubbard, Skinner and Zeldes (1995).

<sup>3</sup>Constantinides, Donaldson and Mehra (2001) and Bertaut and Haliassos (1997) analyze three period models where each period amounts to 20 years.

recommendations offered by financial advisors (see for example, Malkiel, 1999), these models predict that the optimal share of wealth invested in equities is roughly decreasing during (working) life.<sup>4</sup> Nevertheless, as mentioned above, several important predictions of the model are still at odds with empirical regularities. First, the zero median stock holding puzzle (Mankiw and Zeldes (1991)) persists. The latest Survey of Consumer Finances reports that only around 49 percent of US households hold stocks either directly or indirectly (through pension funds, for instance). Yet, the frictionless portfolio choice model predicts that, given the equity premium, all households should participate in the stock market as soon as saving takes place. Second, in the model, households first enter the stock market and only later decrease their exposure to stocks. This is a prediction that runs counter both to casual observation about stockholding patterns and to the evidence reported by Poterba and Samwick (1999) and Ameriks and Zeldes (2001), who find that equity ownership increases with age, at least until households reach retirement. Third, given the equity premium, households invest almost all of their wealth in stocks, and this result is particularly stronger for young households. Again, this is contrary to both casual empirical observation and more formal evidence (see Poterba and Samwick (1999) or Ameriks and Zeldes (2001), for instance).

In this paper we attempt to improve the performance of the existing life-cycle asset allocation models with respect to these puzzles. We start by examining the potential for one-time, fixed, stock market entry costs to generate significant non-participation. A large literature has concluded that some level of fixed costs seems to be necessary to improve the empirical performance of asset pricing models (see, among others, Brav, Constantinides, Geczy (2002), Polkovnichenko (2000), Basak and Cuoco (1998), Vayanos (1998), Luttmer (1996, 1999), Heaton and Lucas (1996), He and Modest (1995), Aiyagari and Gertler (1991) or Constantinides (1986)).<sup>5</sup> With respect to asset allocation, empirical work by Vissing-Jorgensen

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<sup>4</sup>This intuition can also be obtained from Merton (1971), in a complete markets setting.

<sup>5</sup>In this paper we take a partial equilibrium approach thereby eliminating our ability to explicitly say anything about asset pricing. However, the excessive demand for equities predicted by asset allocation models, is the portfolio-demand manifestation of the equity premium puzzle. Therefore, to the extent that a partial equilibrium model generates a more balanced portfolio which matches more closely the microeconomic data on portfolio demands, we move closer towards building a general equilibrium model that could eventually jointly match portfolio demands and asset returns.

(2002) and Paiella (1999) shows that small entry costs can rationalize the observed low stock market participation rates<sup>6</sup>, while Attanasio, Banks and Tanner (2002) argue that the implications of the Consumption Capital Asset Pricing Model cannot be rejected for households predicted to hold both stocks and bonds (“shareholders”). However, Campbell, Cocco, Gomes and Maenhout (2001) introduce such a fixed-cost in the life-cycle asset allocation model, and find that it has a very small impact on the participation decision. On the other hand, Haliassos and Michaelides (2002) argue that, in an infinite horizons model with impatient households (as defined in Deaton (1991)), a small one-time entry cost can generate stock market non-participation. In the infinite horizons model, the consumer/investor can smooth idiosyncratic labor income risk by accumulating a small buffer of assets (Deaton, 1991), and therefore, the benefit from participating in the stock market is much lower.

Contrary to the results in Campbell et al. (2001), we find that small one-time stock market participation costs can generate stock market entry later in life. The difference in the results arises from changing (in plausible directions) the structural parameters that control the strength of the precautionary savings motive. In particular, we reduce the coefficient of relative risk aversion, thus weakening the desire to accumulate precautionary wealth balances. As a result, a small entry cost becomes important enough to generate stock market participation only late in life, when sufficient wealth has been accumulated, making the benefit strong enough to warrant paying the entry cost. More specifically, stock market participation typically peaks at around ages 45 – 65, depending on the parameters affecting the precautionary savings motive and impatience. Given that surveys usually sample the population between ages 25 and 75, we view this result as moving the model in the right direction for explaining this puzzle. On the negative side, the model still predicts that, by age 65, all households will have paid the stock market cost and will have become stockholders.

Conditional on participation, the model still predicts (counterfactually) large equity holdings. Motivated by this result, we consider three different extensions of the baseline model. First, we increase background risk by allowing for a positive probability of a disastrous labor income shock. In this case, the agent saves more, generating significant riskless asset

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<sup>6</sup>DeGeorge, Jenter, Moel and Tufano (2002), estimate such a fixed-cost in the context of a specific even study and obtain relatively modest values.

holdings, particularly for high values of risk aversion. However, the higher level of savings induces a stronger incentive to enter the stock market, and therefore stock market participation takes place much earlier. The same result is obtained by increasing risk aversion, and the intuition is again the same. From this analysis we conclude that other extensions along the same lines, namely introducing consumption risk (taste shocks or housing, for instance) or allowing for uncertainty about the parameters of the labor income process, will lead to the same results: greater co-existence of cash and stocks in the portfolio but a counterfactually high stock market participation rate over the life cycle on account of the higher level of savings.

In our second extension we try to avoid this trade-off, by considering changes in the risk of the stock-return process itself. In particular, we allow for large positive correlation between stock returns and permanent earnings shocks, since the existing literature shows that this will have a direct impact on the optimal portfolio allocation without significantly affecting the consumption decision (Viceira (1999)), and therefore without increasing precautionary saving. The model can generate large riskless asset holdings, but only for moderately large levels of risk aversion, the ones for which we also obtain (counterfactually) large participation rates. For low values of risk aversion the portfolio rule is only marginally changed, and therefore the results are virtually the same as in the baseline case.

Finally we introduce a third financial asset in model, which we calibrate using data on long-term government bonds. We find that, immediately after paying the fixed cost, households invest almost all of their wealth in stocks. The desire to pay the fixed is motivated by the willingness to hold stocks, not long-term bonds. As a result, the participation rate (defined as percentage of households which have paid the fixed cost) is virtually the same as in the 2-asset model. Nevertheless, for the households that have incurred the fixed cost, long-term bonds become a substitute for cash. Specifically, the share of wealth invested in stocks is not affected by the presence of this new asset, cash is replaced by bonds, and the share of financial wealth allocated to cash drops almost to zero. These results rationalize the asset allocation puzzle identified by Canner, Mankiw and Weil (1997): popular financial advisors recommend that more risk-averse investors should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. This is inconsistent with the predictions of the static



and frictionless Capital Asset Pricing Model which implies that all investors should hold the same combination of risky assets; risk aversion only determines the size of the investment to the risky assets as a whole. Brennan and Xia (2002 and 2000), Campbell, Chan and Viceira (2002) and Campbell and Viceira (2001) show how this result can be derived in the context of an intertemporal asset allocation model with time-varying expected returns. In this paper we show that the presence of undiversifiable human capital also rationalizes this puzzle.

Moreover, Canner et al. (1997) restate this puzzle as implying that the ratio of bonds to stocks should increase when the ratio of stocks to total financial wealth falls. In the context of our model, the ratio of stocks to total financial wealth falls at mid-life, as the ratio of human capital to financial wealth falls and the investor is less willing to take risks. The decrease in stock holdings is compensated by higher bond holdings, thus leading to an increase in the ratio of bonds to stocks, as recommended by financial advisors.

The paper is organized as follows. Section 2 summarizes results from the existing empirical literature on life-cycle asset allocation. Section 3 lays out the microeconomic consumption and portfolio choice model, describes the process of wealth accumulation and outlines the numerical solution method, while the corresponding results are shown and discussed in Section 4. Section 5 provides some robustness/sensitivity analysis by varying some structural parameters of interest, while Section 6 considers extensions to the baseline model. Section 7 concludes.

## **2 Empirical Evidence on Life-Cycle Asset Allocation and Stock Market Participation**

In most industrialized countries, stock market participation rates have increased substantially during the last decade.<sup>7</sup> Nevertheless, the majority of the population still does not own any stocks (either directly or indirectly through pension funds), and the participation decision occurs gradually over the life cycle. The empirical evidence on life-cycle asset allocation is

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<sup>7</sup>See Hong, Kubik and Stein (2001) for a model of time-varying participation that rationalizes this behavior.

significantly constrained by data limitations and important identification problems.<sup>8</sup> Figures 1.1 and 1.2 summarize evidence reported in Ameriks and Zeldes (2001). Figure 1.1 plots the estimated average equity holdings (as a share of total financial wealth) over the life cycle, based on the 1989, 1992 and 1995 waves of the Survey of Consumer Finances (SCF).<sup>9</sup> With respect to the life-cycle profile, the results are particularly sensitive to the inclusion of time dummies versus the inclusion of cohort dummies. In both cases, the average stock holdings are quite low. Figure 1.2 plots the corresponding stock market rate participation rate, based on the same data.<sup>10</sup> These results are less sensitive to the choice of time versus cohort dummies. As expected, a very large fraction of the population does not own equities. In both cases the participation rate gradually increases until approximately age 50. When including cohort dummies, the profile is flat after age 50, while with time dummies it is decreasing. Ameriks and Zeldes (2001) obtain exactly these same results using TIAA-CREF data from 1987-1996, and so do Poterba and Samwick (1999), using SCF data. The next two figures (1.3 and 1.4) report evidence from Guiso, Haliassos and Japelli (2001), using cross-sectional information for five different countries (U.S.A., U.K., Netherlands, Germany and Italy). Figure 1.3 plots equity holdings as a fraction of total financial wealth, conditional on stock market participation. We observe an increasing pattern for 4 countries (the U.K. is the exception), and again a very low level of stock holdings.<sup>11</sup> Figure 1.4 plots the participation rates for the different countries. Virtually all of them show an increasing participation rate until age 60.<sup>12</sup> After that, 4 out of 5 countries have a decreasing participation rate, which could be due to cohort effects.

Although several other papers have contributed to this research, our objective here is to briefly report the main results, rather than provide a detailed literature survey.<sup>13</sup> We can

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<sup>8</sup>Ameriks and Zeldes (2001) provide a good discussion and illustration of the problems associated with identifying time, cohort and age effects in the context of life-cycle asset allocation.

<sup>9</sup>The profiles were estimated by running an OLS regression.

<sup>10</sup>The profiles were obtained by running a Probit regression.

<sup>11</sup>Note that, even ignoring time and cohort effects, Figures 1a and 1c are not directly comparable because Figure 1c also conditions on stock market participation, which explains the higher level of stockholdings.

<sup>12</sup>For all of them the participation rate is higher for the age bracket 50-60 than for the age bracket 20-30.

<sup>13</sup>Other important papers on this topic include Heaton and Lucas (2000) and Guiso, Jappelli and Terlizzese (1996) (which focus mostly on the impact of background risk on asset allocation), Bertaut and Haliassos

summarize the existing evidence as follows. First, at least 50% of the population does not own equities. Second, participation rates increase significantly during working life and there is some evidence suggesting that they might decrease during retirement although this might be due to cohort effects. Third, conditional on stock market participation, households invest a large fraction of their financial wealth in alternative assets but there is no clear pattern of equity holdings over the life-cycle.<sup>14</sup>

### 3 The Model

This section presents the model of individual consumption behavior. The framework extends a life-cycle model of buffer stock saving (Gourinchas and Parker (2001)) to include portfolio choice between a riskless and risky asset (Cocco, Gomes and Maenhout (1999) and Gakidis (1998)) in the presence of a one-time, fixed, stock-market-entry cost.

#### 3.1 Preferences

Time is discrete and  $t$  denotes adult age.<sup>15</sup> Households have Epstein-Zin-Weil utility functions (Epstein and Zin (1989) and Weil (1989)) defined over one single non-durable consumption good:

$$U_t = \{(1 - \beta)C_t^{1-1/\psi} + \beta(E_t[U_{t+1}^{1-\rho}])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \quad (1)$$

where  $\rho$  is the coefficient of relative risk aversion,  $\psi$  is the elasticity of intertemporal substitution,  $\beta$  is the discount factor and  $C_t$  is the consumption level at time  $t$ .<sup>16</sup>

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(1997) and King and Leape (1998).

<sup>14</sup>These conclusions remain valid if we consider alternative classifications of risky assets, for example by including long-term government bonds or even business income (see Ameriks and Zeldes (2001), Guiso, Haliassos and Japelli (2001), Heaton and Lucas (2000), Poterba and Samwick (1999), or Bertaut and Haliassos(1997)).

<sup>15</sup>Following the typical convention in this literature, adult age corresponds to effective age minus 19.

<sup>16</sup>For  $\rho = 1/\psi$  we have the standard, power (CRRA) utility specification.

## 3.2 Retirement and Bequests

The agent lives for a maximum of  $T$  periods, and retirement occurs at time  $K$ ,  $K < T$ . For simplicity  $K$  is assumed to be exogenous and deterministic. We allow for uncertainty in  $T$  in the manner of Hubbard, Skinner and Zeldes (1995). The probability that a consumer/investor is alive at time  $(t + 1)$  conditional on being alive at time  $t$  is denoted by  $p_t$  ( $p_0 = 1$ ). Although the numerical solution can easily accommodate a bequest motive, we set it to zero in this model.

## 3.3 Labor Income Process

The labor income process is the same as the one used by Gourinchas and Parker (2001), or Cocco, Gomes and Maenhout (1999). Before retirement, the exogenous stochastic process for individual labor income is given by

$$Y_{it} = P_{it}U_{it} \tag{2}$$

$$P_{it} = \exp(f(t, Z_{it}))P_{it-1}N_{it} \tag{3}$$

where  $f(t, Z_{it})$  is a deterministic function of age and household characteristics  $Z_{it}$ ,  $P_{it}$  is a “permanent” component, and  $U_{it}$  a transitory component. We assume that the  $\ln U_{it}$ , and  $\ln N_{it}$  are each independent and identically distributed with mean  $\{-.5 * \sigma_u^2, -.5 * \sigma_n^2\}$ <sup>17</sup>, and variances  $\sigma_u^2$ , and  $\sigma_n^2$ , respectively. The log of  $P_{it}$ , evolves as a random walk with a deterministic drift,  $f(t, Z_{it})$ .

Given these assumptions, the growth in individual labor income follows

$$\Delta \ln Y_{it} = f(t, Z_{it}) + \ln N_{it} + \ln U_{it} - \ln U_{it-1}, \tag{4}$$

and its unconditional variance equals  $(\sigma_n^2 + 2\sigma_u^2)$ . This process has a single Wold representation that is equivalent to the MA(1) process for individual earnings growth estimated using household level data (MaCurdy (1982), Abowd and Card (1989), and Pischke (1995)).<sup>18</sup>

<sup>17</sup>With this specification the mean of the level of the log random variables equals 1.

<sup>18</sup>Although these studies generally suggest that individual income changes follow a MA(2), the MA(1) is found to be a close approximation.

In the benchmark specification we do not allow for correlation between labor income innovations and shocks to the return process since the empirical evidence on this hypothesis is mixed. Davis and Willen (2001) find evidence for positive correlation but Campbell et al. (2001) and Heaton and Lucas (2000) only find such evidence when considering small subgroups of the population.<sup>19</sup>

Earnings in retirement ( $t > K$ ) follow:  $Y_{it} = \lambda P_{iK}$ , where  $\lambda$  is the replacement ratio (a scalar between zero and one). Although oversimplified, this specification considerably facilitates the solution of the model, as it does not require the introduction of an additional state variable (see subsection 3.6).

### 3.4 Assets and wealth accumulation

There are two financial assets, one riskless asset (treasury bills or cash) and one risky investment opportunity (stock), and the investment opportunity set is constant.<sup>20</sup> The riskless asset yields a constant gross after tax real return,  $R_f$ , while the gross real return on the risky asset is denoted by  $\widetilde{R}$ .

$$\widetilde{R}_{t+1} - R_f = \mu + \varepsilon_{t+1} \quad (5)$$

The household must allocate her cash-on-hand ( $X_{it}$ ) between consumption expenditures ( $C_{it}$ ) and savings, so that

$$X_{it} = C_{it} + B_{it} + S_{it} \quad (6)$$

where  $S_{it}$  and  $B_{it}$  denote stock holdings and riskless asset holdings (cash) at time  $t$ , respectively.

An agent who has never participated in the stock market prior to date  $t$  enters that period with wealth invested in riskless assets ( $B_{it-1}$ ) and receives  $Y_{it}$  of labor income. The investor then decides whether to pay a fixed lump sum to enter the stock market. This entry fee represents both the explicit transaction cost from opening a brokerage account and

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<sup>19</sup>In Section 6 we investigate the effects of introducing positive correlation between permanent earnings shocks and stock return innovations, as one of the extensions to our baseline model.

<sup>20</sup>Later on will consider a set-up with three different financial assets but, as we will see, the results and the intuition are not significantly affected.

the cost of acquiring information about the stock market. We define a dummy variable  $I_P$  which is equal to one when the cost is incurred for the first time and zero otherwise. The fixed cost ( $F$ ) is scaled by the level of permanent component of labor income ( $P_{it}$ ) because this will simplify the computational method. This assumption is motivated by interpreting the fixed cost as an opportunity cost, and therefore likely to be an increasing function of earnings. Following Deaton (1991) we denote cash on hand as the liquid resources available for consumption and saving. When the stock market entry fee has not been paid yet, cash on hand is given by<sup>21</sup>

$$X_{it+1} = S_{it}\tilde{R}_{t+1} + B_{it}R_f + Y_{it+1} - FI_P P_{it+1} \quad (7)$$

If the fixed cost was already paid in the past, we have

$$X_{it+1} = S_{it}\tilde{R}_{t+1} + B_{it}R_f + Y_{it+1} \quad (8)$$

Following Deaton (1991), we prevent households from borrowing against their future labor income. More specifically we impose the following restrictions:

$$B_{it} \geq 0 \quad (9)$$

$$S_{it} \geq 0 \quad (10)$$

### 3.5 The optimization problem

The complete optimization problem can now be written as<sup>22</sup>

$$MAX_{\{S_{it}, B_{it}\}_{t=1}^T} E \sum_{t=1}^T \beta^{t-1} \{\Pi_{j=0}^{t-1} p_j\} U_t, \quad (11)$$

where

$$U_t = \{(1 - \beta)C_t^{1-1/\psi} + \beta(E_t[U_{t+1}^{1-\rho}])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \quad (12)$$

subject to

$$X_{it+1} = S_{it}R_{t+1} + B_{it}R_f + Y_{it+1} - FI_P P_{it+1} \quad (13)$$

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<sup>21</sup>Note that, if the household decides not to pay the fixed cost in this period, then  $S_{it} = 0$ .

<sup>22</sup>Where  $E$  is the expectation conditional on information available at time 1, and  $\beta$  is the constant discount factor.

$$X_{it} = S_{it} + B_{it} + C_{it} \quad (14)$$

where  $I_P$  is a dummy variable that equals one when the household enters the stock market for the very first time.

$$B_{it} \geq 0 \quad (15)$$

$$S_{it} \geq 0 \quad (16)$$

$$\tilde{R}_{t+1} - R_f = \mu + \varepsilon_{t+1} \quad (17)$$

where  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ .

If  $t \leq K$

$$Y_{it} = P_{it}U_{it} \quad (18)$$

$$P_{it} = \exp(f(t, Z_{it}))P_{it-1}N_{it} \quad (19)$$

while if  $t > K$

$$Y_{it} = \lambda P_{iK} \quad (20)$$

The share of liquid wealth invested in the stock market at age  $t$  is denoted by  $\alpha_{it}$ .

### 3.6 Solution Method

Analytical solutions to this problem do not exist. We therefore use a numerical solution method based on the maximization of the value function to derive optimal policy functions for total savings and the share of wealth invested in the stock market. The details are given in appendix A, and here we just present the main idea.

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income ( $P_{it}$ ). The equations of motion and the value function can then be rewritten as normalized variables and we use lower case letters to denote them (for instance,  $x_{it} \equiv \frac{X_{it}}{P_{it}}$ ). This allows us to reduce the number of state variables to three: age ( $t$ ), normalized cash-on-hand ( $x_{it}$ ) and participation status (whether the fixed cost has already been paid or not). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to

compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards. We optimize over the different choices using grid search.

### 3.7 Parameter Calibration

We begin by solving the model under a set of “baseline” parameter assumptions at an annual frequency but we also investigate how our results vary when these parameter values are changed. For the return process we consider a mean equity premium ( $\mu$ ) equal to 6 percent, a standard deviation of the risky investment ( $\sigma_\varepsilon$ ) equal to 18 percent, and a constant real interest rate ( $r$ ) of 2 percent. The benchmark preference parameters we use are a coefficient of relative risk aversion ( $\rho$ ) equal to 2, an elasticity of intertemporal substitution ( $\psi$ ) equal to 0.5 and a discount factor ( $\beta$ ) equal to 0.95. With these preference parameters the Epstein-Zin-Weil utility function becomes the standard power utility with a coefficient of relative risk aversion of 2.

Carroll (1992) estimates the variances of the idiosyncratic shocks using data from the *Panel Study of Income Dynamics*, and our baseline simulations use values close to those: 0.1 percent per year for  $\sigma_u$  and 0.08 percent per year for  $\sigma_n$ . The deterministic labor income profile reflects the hump shape of earnings over the life-cycle, and the corresponding parameter values, just like the retirement transfers ( $\lambda$ ), are taken from Cocco, Gomes and Maenhout (1999).<sup>23</sup> It is common practice to estimate different labor income profiles for different education groups (college graduates, high-school graduates, households without a high-school degree). For most of the paper we will report the results obtained with the parameters estimated for the sub-sample of high-school graduates as the results for the other two groups are very similar. With respect to the fixed cost of participation we consider two cases; one where the cost is zero and one where it equals 0.1, corresponding to 10% of the household’s expected annual income.

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<sup>23</sup>Cocco, Gomes and Maenhout (1999) report their estimated parameter values and their results are consistent with other studies (for instance, Hubbard, Skinner and Zeldes (1995), Carroll (1997), Gourinchas and Parker (2001), or Storesletten, Telmer and Yaron (2001)).



## 4 Portfolio Choice and Stock Market Participation over the Life-Cycle

In this section we discuss the implications of the model with respect to consumption, portfolio holdings and stock market participation over the life-cycle using computed transition distributions of cash on hand from one age to the next.

### 4.1 Computing Transition Distributions

It is common practice for researchers to simulate a model over the life-cycle for a large number of individuals (say 10000) to compute the statistics of interest (mean wealth holdings, for instance) for any given age. We propose an alternative method of computing various statistics that is based on explicitly calculating the transition distribution of cash on hand from one age to the next.

The numerical details are delegated to Appendix A but the idea is quite simple. Given that normalized cash on hand is a stationary process, it becomes easy to compute the transition probability from one cash on hand state to another from period to period once the optimal policy functions have been computed. Given an initial distribution of cash on hand, the new distribution will be the product of the transition and initial distribution; this will then become the initial distribution for the next period.

There are a number of important advantages in using the transition distributions rather than simulation in this analysis. First, computing the transition distributions is computationally efficient since it avoids the simulation of a large number of individual life histories. Second, there is no need to use an interpolation procedure at any stage (see appendix B), thereby avoiding potential numerical approximation errors that might arise from interpolation. Third, the distributions provide an explicit metric of inequality for different variables over time and therefore can potentially be more suitable in matching the model to the data.

## 4.2 Results without Fixed Participation Cost

When we consider the case where the fixed participation cost is set equal to zero the results are similar to the ones previously obtained in this literature.<sup>24</sup> Figures 2.1 and 2.2 plot some of the distributions of normalized cash on hand for different ages during working life and during retirement, respectively. The distributions can best be thought of as the distributions of cash on hand that would result if a large number of individuals were born at the same time (that is, the evolution of the distribution for cash on hand for a particular cohort). Figure 2.1 shows how wealth accumulation affects the distribution of wealth; as wealth accumulation begins to take place with aging, there is a shift of mean cash on hand to a higher level with age while at the same time the cash on hand distribution widens.<sup>25</sup> Figure 2.2 illustrates the wealth decumulation that should occur during retirement; mean cash on hand now is reduced towards mean pensions as the individual ages, while the distribution of cash on hand is now compressed with age. As the terminal age 100 approaches, the distribution collapses to a single point (the pension level).

These distributions can now be used to compute the relevant statistics of interest for the different variables in the model. For instance, mean consumption for this cohort at age 50, denoted by  $\bar{c}_{50}$ , can be computed as<sup>26</sup>

$$\bar{c}_{50} = \sum_{j=1}^J \pi_{50,j} * c(x_j, 50) \quad (21)$$

where  $c(x_j, 50)$  is the consumption policy rule as a function of the two state variables (normalized cash on hand and age) and  $\pi_{50,j}$  is the probability mass of households aged 50 at state  $x_j$ . Using this method, mean normalized consumption and mean normalized wealth are computed and plotted in Figure 2.3, while Figure 2.4 graphs the mean equity allocation.<sup>27</sup>

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<sup>24</sup>In this section we only report results using the labor income process estimated for high school graduates. The results for the other groups are discussed below.

<sup>25</sup>This process continues until retirement when the maximum mean cash on hand is attained. We did not plot those distributions here to avoid cluttering.

<sup>26</sup>Where  $J$  is the number of grid points used in the discretization of normalized cash on hand.

<sup>27</sup>We compute the mean equity allocation across households as total equity holdings divided by total wealth, and therefore individuals that do not save are automatically excluded. Naturally for these households the share of wealth in stocks is undefined.

The qualitative results are similar to the ones previously obtained in this literature. Even though earnings risk is uninsurable, labor income is a closer substitute for the risk-free asset than for equities. As a result, the presence of future labor income against which the individual cannot borrow, increases the demand for stocks and the share invested in equities is a decreasing function of cash-on-hand. As the investor approaches retirement, discounted future labor income is falling and accumulated financial wealth is rising fast. Therefore the implicit risk-free asset holdings represented by the labor income process become a less important component of total financial wealth and risk free assets are replenished by increasing the holdings of the riskless financial asset. Once retired, the household increases the share of wealth allocated to stocks because labor income uncertainty is eliminated.<sup>28</sup>

Given our combination of parameter values (less labor income risk and lower prudence) the typical household accumulates less wealth than in Cocco et. al. (1999) or Campbell et. al. (2001). As a result, we expect that the investor will allocate a larger fraction of savings in the risky asset since the optimal share invested in equities is a decreasing function of cash-on-hand. Furthermore we have a smaller degree of risk aversion and less background risk which should magnify this portfolio choice effect. Consistent with this intuition, Figure 2.4 identifies a very strong preference for stocks, reflecting the equity premium puzzle from a portfolio demand perspective.

### 4.3 Results with Fixed Participation Cost

We next introduce a positive fixed cost of stock market participation. We set this cost equal to 10% of the household's expected annual income.

#### 4.3.1 Transition Distributions with the Participation Cost.

Appendix B explains how the initial saving decision generates a distribution for cash on hand. At the beginning of life ( $t = 1$ ), the participation rate (defined as the proportion of

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<sup>28</sup>See Heaton and Lucas (2000) for the negative effects of background risk on equity investment from a theoretical and empirical perspective, and Guiso, Japelli and Terlizzese (1996) for further empirical evidence on the temperance effect of earnings risk on risky asset holdings.

households that have incurred the fixed cost at any point in the past) is zero. Let  $\Pi_t^O$  denote the mass of agents at each cash on hand state and each age ( $t$ ), which have not yet paid the fixed cost yet and therefore are out of the stock market. Given the new distribution for cash on hand for those out of the market, we can compute the proportion of households who would be willing to incur the fixed cost; typically, households with high labor income realizations or high accumulated liquid wealth will find it more beneficial to enter the stock market. Specifically, the percentage of households that optimally choose to incur the fixed cost and participate in the stock market can be found by computing the sum of the probabilities in  $\Pi_t^O$  for which  $x > x^*$ , where  $x^*$  is the trigger point that causes participation (given by the optimal participation policy rule). The participation rate can then be computed by adding to the previous period participation rate, the percentage of households that choose to pay the fixed cost times the percentage of households who have never paid the cost in the past. We then compute the distributions for both participants and non-participants, and use the participation rate to weight each of these distributions in computing the unconditional distribution of wealth in the economy. Appendix B describes the specific computations in detail.

### 4.3.2 Benchmark Case

We first plot the evolution for the distributions of cash on hand for the two types of agents: participants and non-participants in the stock market. Figure 3.1 illustrates some of the results by plotting the distributions of normalized cash on hand for individuals aged 30 who have incurred the fixed cost and those who have not. These are conditional distributions and the participation rate can be used as a probability weight to generate the unconditional distribution of cash on hand in the cohort. One recurring feature of the figure is that the distribution of cash on hand conditional on participation in the stock market has a higher variance than the wealth distribution for the households which have not participated in the stock market. Figure 3.2 plots the distributions of cash on hand for ages 50 for both types of agents. There is now a pronounced spike at around the normalized cash on hand level of 1.42; beyond that level of cash on hand, stock market participation becomes optimal and

the two distributions only overlap for a very small interval representing the incurrence of the fixed entry cost. The distribution is now made up almost completely by rich stockholders and poor non-stockholders.

Figure 3.3 plots the unconditional mean wealth and consumption over the life-cycle. The distributions for participants and non-participants are used to compute the respective means in the two types and then the proportion of stockholders (the participation rate) is used to take the unconditional average over the two groups. Specifically, let  $\theta_t$  denote the stock market participation rate in the cohort at time (age)  $t$ . The unconditional mean consumption for age  $t$  can then be computed as<sup>29</sup>

$$\bar{c}_t = \theta_t \left\{ \sum_{j=1}^J \pi_{t,j}^I * c^I(x_j, t) \right\} + (1 - \theta_t) \left\{ \sum_{j=1}^J \pi_{t,j}^O * c^O(x_j, t) \right\} \quad (22)$$

Figure 3.3 illustrates that the wealth accumulation profile has the same shape as in the model without the fixed cost (Figure 2.3). In particular, it is useful to note that wealth accumulation starts to take place at around age 40, and rises quite fast from age 45 onwards, as households begin to complement their precautionary saving with retirement saving, generating a substantial wealth accumulation, thus increasing the incentive to enter the stock market.<sup>30</sup>

Figure 3.4 plots the unconditional portfolio allocation over the life-cycle<sup>31</sup> and the portfolio allocation conditional on participating in the stock market.<sup>32</sup> Even though the share of

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<sup>29</sup>Superscript  $I$  denotes a variable for households participating in the stock market while superscript  $O$  denotes households out of the stock market.

<sup>30</sup>This is broadly consistent with the results of Cagetti (2001) and Gourinchas and Parker (2001), who study a model without portfolio choice, and find that saving for retirement takes place at around ages 45–50 and 40–45, respectively.

<sup>31</sup>For a given age  $t$ , the unconditional portfolio allocation is computed as:

$$\frac{\theta_t * \left\{ \sum_{j=1}^J \pi_{t,j}^I * \alpha^I(x_j, t) * (x_j - c^I(x_j, t)) \right\}}{\theta_t * \sum_{j=1}^J [\pi_{t,j}^I * (x_j - c^I(x_j, t))] + (1 - \theta_t) * \sum_{j=1}^J [\pi_{t,j}^O * (x_j - c^O(x_j, t))]}$$

<sup>32</sup>For a given age  $t$ , the portfolio allocation conditional on participation is computed as:

$$\frac{\left\{ \sum_{j=1}^J \pi_{t,j}^I * \alpha^I(x_j, t) * (x_j - c^I(x_j, t)) \right\}}{\left\{ \sum_{j=1}^J \pi_{t,j}^I * (x_j - c^I(x_j, t)) \right\}}$$

equity in the portfolio as a function of age is strongly increasing for the unconditional distribution, this finding is driven entirely by the low participation rate observed during the early working life period. Figure 3.5 addresses the participation puzzle by plotting the proportion of agents participating on the stock market over the life-cycle.<sup>33</sup> The typical household stays out of the stock market up to age 30, when enough wealth has been accumulated to warrant paying the participation fee. Beyond age 45, the retirement savings motive becomes very strong and wealth accumulation takes place at a more rapid pace, as is visible in figure 3.3. Given the higher benefit from entering the stock market when a higher level of wealth has been accumulated, and given the rapid pace of wealth accumulation during this period, the number of households willing to pay the cost and participate in the stock market rises very quickly; by age 50 roughly half of the population has paid the fixed cost and by age 60 ninety five percent has done so.

Figure 3.6 compares the wealth accumulation in the fixed-cost case with the profile in figure 2.3 ( $F = 0$ ). In the absence of the fixed participation cost, wealth grows much faster and peaks at a higher level. When  $F = 0$  all households can invest in the stock market as of age 20 and, as a result, financial wealth grows faster. In addition, when  $F > 0$ , the fixed cost of learning about stock market opportunities represents a waste that decreases wealth accumulation.

## 5 Sensitivity Analysis

We next provide some sensitivity analysis by varying some of the key parameters of the model.

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<sup>33</sup>Appendix A details the computation of the participation rate. For a given age  $t$ , the participation rate is computed as:

$$\theta_t = \theta_{t-1} + (1 - \theta_{t-1}) * \sum_{x_j > x^*} \pi_{t,j}^O$$

## 5.1 Varying Risk Aversion and the Elasticity of Intertemporal Substitution

### 5.1.1 CRRA utility function

We start by increasing the coefficient of relative risk aversion ( $\rho$ ) to 5 while reducing the elasticity of intertemporal substitution ( $\psi$ ) to 0.2, thus maintaining the CRRA assumption (this corresponds to the parameter values used in Campbell et. al. (2001)). A higher  $\rho$  generates both more prudence and more risk aversion. Increasing prudence leads to higher wealth accumulation over the life-cycle, and consequently increases the demand for stocks (in levels). On the other hand, higher risk aversion leads to a reduction in the share of wealth invested in equities. The balance between these effects will, in general, determine the optimal time at which the fixed entry cost is paid.

Figure 4.1 illustrates that, since wealth accumulation is substantially higher for  $\rho = 5$  than for  $\rho = 2$ , the household finds it optimal to enter the stock market at very young ages: the benefit from utilizing the stock market as a saving vehicle is now higher. As a result, by age 30, around 50 percent of investors have become participants, while the rate of new entrants is so fast that by age 35 almost the whole population has entered the stock market (figure 4.1). The increased rate of stock market participation beyond age 30 is directly associated with the very fast rate with which wealth accumulation takes place beyond that age.

Figure 4.2 plots, for comparison purposes, the unconditional share of wealth invested in the equity market when  $\rho = 2$  and  $\rho = 5$ . Given that stock market participation takes place earlier in life when  $\rho = 5$ , the share of wealth in stocks rises beyond that entry point and there is a complete portfolio allocation to stocks by age 35. As retirement approaches, the implicit risk free asset in the form of labor income is reduced and there is a shift towards greater holdings of the riskless asset in the financial portfolio. During retirement, background labor income risk is eliminated, and the propensity to hold stocks increases.

Figure 4.3 plots the share of wealth invested in stocks conditional on participation. Even though stronger risk aversion generally reduces the share of wealth invested in equities at mid-life, the model still predicts excessive stock holding conditional on participation.

### 5.1.2 Non-CRRA utility function

Motivated by the previous results we now deviate from the CRRA framework. By increasing risk aversion and decreasing the EIS we can reduce the willingness to hold stocks without increasing saving. Our goal is to increase the coefficient of relative risk aversion above our benchmark level of  $\rho = 2$ , thereby reducing the share of wealth allocated in stocks due to stronger risk aversion. By simultaneously reducing the elasticity of intertemporal substitution, we decrease the level of savings (from the weaker motive to smooth consumption intertemporally), and therefore prevent the participation rate from increasing. It is important to remember however that with a CRRA utility function, a higher risk aversion coefficient already implies a low value of the EIS. In particular, for  $\rho = 5$  ( $\psi = 0.2$ ), we cannot reduce  $\psi$  substantially given that  $\psi$  must be non-negative.

Tables 1.1 through 1.4 show, respectively, the mean consumption, mean wealth, mean portfolio allocation in stocks and the participation rate for different values of  $\rho$  and  $\psi$ , for different age groups. Changing  $\psi$  has a very small impact on the wealth accumulation of the young. These households are liquidity constrained, and they only save for precautionary motives. Consistent with this finding, wealth accumulation increases substantially if we raise  $\rho$ . As a result,  $\psi$  has a very small impact on the participation rate which, as seen before, is an increasing function of  $\rho$ . At mid-life, a lower EIS leads to a lower consumption level, as households want to have a smoother transition to retirement. As a result there is more wealth accumulation and the portfolio allocation to stocks is reduced. However, as expected, the improvement is quite marginal and overall the results are very similar to the ones obtained before.

## 5.2 Varying the Discount Factor

The previous results show that the participation rate is essentially driven by wealth accumulation. This suggests that if we decrease the discount factor ( $\beta$ ) and increase the coefficient of relative risk aversion, we can simultaneously match participation rates (by reducing wealth accumulation) and match asset allocation conditional on participation. Figure 4.4 plots the results of such an attempt. We find that, with a high coefficient of relative risk aversion



( $\rho = 10$ ), we need a very low discount factor (0.7) to match the participation rates obtained in the baseline case. Furthermore, the asset allocation conditional on participation is still extremely high, despite the high coefficient of risk aversion. By reducing wealth accumulation we also reduce the ratio of current financial wealth to future labor income, and this induces the household to follow a more aggressive investment strategy, offsetting the high risk aversion effect.

The low discount factor can be viewed as a short-cut representation for tastes shocks such as housing consumption that requires households to increase their expenditures early in life and at mid-life. Our results show that such representation does a good job of matching wealth accumulation and consequently of helping the model to match stock market participation. However this comes at a cost, as the model's predictions with respect to asset allocation are now even more at odds with the data. By reducing financial wealth accumulation over the life-cycle model we make the portfolio choice puzzles even harder to solve.<sup>34</sup>

### 5.3 Other Education Groups

Figure 4.5 shows the participation rate and the mean portfolio allocation when considering the labor income process of the other two education groups: college graduates and high-school drop-outs. For the sample of the less educated households (no high-school degree) stock market participation starts picking up between ages 40 and 50, but it never reaches 100%. The mean allocation to stocks follows a similar pattern and it is actually very close to 1 from ages 55 through 75. The profiles for college graduates are very similar. For this group we start the simulation at age 22 since the estimated labor income profile is only valid from that age onwards. Furthermore, this group has a very steep labor income profile early in life (see, for example, Cocco et. al. (1999)) and as such they will accumulate less wealth in the first stage of the life-cycle. Consequently, stock market participation starts later in life, between ages 45 and 55, but it picks up quite fast.

It is important to remember that the fixed cost in our model is proportional to the level

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<sup>34</sup>This is consistent with the results of Cocco (2000). He introduces housing in a life-cycle model of asset allocation and finds that this improves the model's ability to match the participation rate. However, conditional on participation, the model now predicts even larger equity holdings.

of income, reflecting an opportunity cost of time devoted to the gathering and processing of information. However, even though we expect that the absolute value of the opportunity cost increases with the income level, we also expect that the relative cost will decrease with the education level. By setting a lower value of  $F$  for college graduates and a higher value for the no high-school group, we would obtain a more intuitive prediction: higher participation rates for more educated households at every given age. Overall, the results are fairly similar to the ones obtained in the benchmark case and therefore, for the remainder of the paper, we will only report results for high-school graduates.

## 6 Extensions

Our baseline model can match stock-market participation better than previous life-cycle asset allocation models but, conditional on participation, it still predicts that households will invest virtually all of their wealth in stocks. Furthermore, it still predicts that all investors will eventually hold stocks, at some stage of their life. In this section we try to improve upon these predictions by considering different extensions of the baseline model.<sup>35</sup>

### 6.1 Fat Tails in the Distribution of Labor Income Shocks

Empirical evidence suggests that labor income innovations have non-normal distributions. In particular, earnings are characterized by negative skewness (see, for instance, Chamberlain and Hirano (1999)). Carroll (1992) estimates that with a probability ( $p$ ) equal to 0.5 percent, the transitory labor income shock equals zero (at an annual frequency). The stochastic process for the innovations to the transitory log labor income shocks is then given by:

$$\ln(U_{it}) = \begin{cases} \ln(U_{it}^*) \sim N(-0.5\sigma_{u^*}^2, \sigma_{u^*}^2) & \text{with probability } 1 - p \\ \eta & \text{with probability } p \end{cases} \quad (23)$$

where  $\eta$  is a very small number, and in particular we can consider  $\eta = 0$ . Carroll motivates this adverse shock as representing layoff risk, but we can also think of it as a reduced form

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<sup>35</sup>Given the results in the previous section we will always restrict the analysis to the case of CRRA utility. Therefore, since we have  $\psi = 1/\rho$ , we will refer to the value of  $\rho$ .

representation a more complicated process, such as the one identified in Chamberlain and Hirano (1999).

Figures 5.1 and 5.2 plot the life-cycle asset allocation obtained when considering the specification given by (23), for  $\rho = 2$  and  $\rho = 5$ , respectively. For  $\rho = 2$ , the co-existence of cash and stocks in the portfolio begins to be generated, but the magnitude of this effect is very small. The participation rate also remains close to what is obtained in the baseline model. Given that the agent is not very risk averse, the effect of dramatically increasing background labor income risk is negligible. On the other hand, when risk aversion is stronger ( $\rho = 5$ ), the possibility of a disastrous labor income shock crowds out stock-holding by a significant amount. However, the stock market participation rate rises very quickly with age, implying that the lower participation rates observed in the data cannot be explained by this alternative specification.

These results suggest that a more detailed calibration of labor income uncertainty is not likely to improve the performance of the model. If we increase the background risk that the agent faces, then we can obtain co-existence of cash and stocks in the portfolio, but we cannot explain non-participation. Risk aversion is driving both results. In the face of a disastrous labor income shock, albeit transitory, the agent builds up a buffer stock of riskless asset holdings, generating the co-existence result. However, she also saves more and this creates a stronger incentive to enter the stock market. Therefore stock market participation takes place earlier than in the benchmark case. This trade-off was already visible when we compared the results obtained with different coefficients of risk aversion. These results are consistent with the ones obtained in sections 5.1 and 5.2, and suggest that any other extension along the same lines, namely introducing uncertainty about the parameters of the labor income process, or introducing consumption risk (taste shocks, for instance), is likely to face the same trade-off.<sup>36</sup>

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<sup>36</sup>One natural extension along these lines would be to consider explicitly the role of housing consumption or family-size shocks. Cocco (2000) introduces housing in life-cycle model of asset allocation and finds that this improves the model's ability to match the participation rate. However, conditional on participation, the model now predicts even larger equity holdings. Hu (2001) introduces a renting option in the housing model, and she requires households to pay a cost if they increase the mortgage size. These two features generate a positive demand for cash and reduce the demand for equity. However, in her model, there is no

## 6.2 Correlation Between Labor Income Shocks and Stock Returns

The previous results suggest that if we want to improve the empirical performance of the model, we need to reduce the desirability of owning stocks without, however, simultaneously increasing the demand for precautionary (or retirement) savings, so as to keep the participation rate over the life-cycle as in the baseline model. Positive correlation between stock returns and permanent earnings shocks is a good candidate since it does not affect the consumption policy rule (and therefore does not affect total precautionary saving) but does have substantial effects on the portfolio allocation between stocks and cash. Moreover, a number of recent theoretical papers argue for the importance of positive correlation between stock returns and earnings shocks on portfolio choice.<sup>37</sup> Motivated by this literature we extend the model to allow for positive correlation between permanent labor income shocks and stock returns.<sup>38</sup> More formally, we let the permanent earnings innovation follow:

$$\ln N_{it} = \left(\phi \frac{\varepsilon_t}{\sigma_\varepsilon} + (1 - \phi^2)^{1/2} \ln N_{it}^*\right) \sigma_n \quad (24)$$

where  $\ln N_{it}^*$  follows a standard normal, and  $\phi$  is the correlation coefficient between  $\ln N_{it}$  and  $\frac{\varepsilon_t}{\sigma_\varepsilon}$ .

It should be noted that the existing empirical evidence is mixed. Davis and Willen stock market entry cost. If such a cost did exist, then naturally, the participation results would be weaker than the ones obtained in Cocco (2000). Again, the same trade-off between matching participation rates and asset allocation conditional on participation is obtained.

<sup>37</sup>Viceira (2001), Michaelides (2001), Heaton and Lucas (2000), Campbell, Cocco, Gomes and Maenhout (2000), Cocco, Gomes and Maenhout (1999), Haliassos and Michaelides (2002) conclude that when stock returns are positively correlated with labor income shocks, the implied negative hedging demands can be quite large, thus significantly crowding-out stock holdings, without a first-order impact on the consumption decision. Large positive correlation between wages and stock returns is also generated endogenously in general equilibrium models with a Cobb-Douglas production technology (Storesletten, Telmer, Yaron (2001)) and substantially affects the optimal asset allocation over the life-cycle.

<sup>38</sup>We consider correlation with the permanent labor income shocks as opposed to transitory shocks for two reasons. First, theory suggests that this correlation should lead to substantial hedging demands (we have indeed considered correlation between stock returns and transitory labor income shocks, and found that it has virtually no impact on the optimal asset allocation). Second, this is consistent with the existing empirical evidence (Davis and Willen (2001)).

(2001) estimate significant positive correlations, while Campbell et. al. (2001) and Heaton and Lucas (2000), obtain mostly small and insignificant values, except for some subgroups of the population (e.g. self-employed households or households with private businesses).

The results are reported in Figures 6.1 and 6.2.<sup>39</sup> For  $\rho = 5$ , the negative hedging demand is substantial and stockholdings are significantly reduced (figure 6.2). Given that total saving is not affected, the stock market participation decision remains roughly unchanged and all households participate in the stock market by age 35. However, with a lower risk aversion coefficient, ( $\rho = 2$ ), the correlation does not generate a substantial hedging demand since the agent is not very risk averse (consistent with the results of Viceira (2001)). Therefore, the stock market participation decision also remains unchanged from the baseline case, and predicts an upward sloping participation profile over the life-cycle.

We conclude that, even though positive correlation can generate co-existence of cash and stocks over the life-cycle for higher levels of risk aversion, the stock market participation decision remains roughly unchanged from the benchmark model for these risk aversion parameter choices, implying (counterfactually) that all households participate in the stock market by around age 35.<sup>40</sup>

### 6.3 Introducing a third financial asset: long-term bonds

In this section we introduce a third financial asset in model, which we will call (long-term) bonds. The return process is calibrated to match the historical mean and standard deviation

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<sup>39</sup>We set the correlation coefficient ( $\phi$ ) equal to 0.5, a magnitude that we view as an upper bound, given the empirical evidence.

<sup>40</sup>Another extension along the same lines would be to allow for uncertainty about the structural parameters of the model. Barberis (2000), Lewellen and Shanken (2000), Pástor (2000) and Kandell and Stambaugh (1996) among others, have shown how parameter uncertainty can have a strong impact on the optimal portfolio allocation. The results (not reported here, but available upon request) are virtually the same as for the positive correlation case. For low values of risk aversion ( $\gamma = 2$ ) there is no significant impact. For moderately high levels of risk aversion ( $\rho = 5$ ) the co-existence of cash and stocks can be obtained. Nevertheless, this prediction comes again at the cost of a stock-market participation profile that peaks very early (by age 40 everyone participates in the stock market).

of long-term government bonds in the US:<sup>41</sup>

$$\tilde{R}_{t+1}^b - R_f = \mu^b + \varepsilon_{t+1}^b \quad (25)$$

where  $\varepsilon_t^b \sim N(0, \sigma_{\varepsilon^b}^2)$ ,  $\mu^b = 2\%$ , and  $\sigma_{\varepsilon^b} = 8\%$ .<sup>42</sup>

The investor must pay the fixed cost if she wishes to invest in either bonds or stocks. The motivation is the same as before: the fixed cost represents the direct financial cost and the opportunity of setting up a broker's account and "understanding" how the risky assets work.<sup>43</sup>

The simulation results are shown in figures 7.1 and 7.2, respectively for  $\rho = 2$  and  $\rho = 5$ , and can be summarized as follows. First, immediately after paying the fixed cost, households invest almost all of their wealth in stocks. The desire to pay the cost is motivated by the willingness to hold stocks, not long-term bonds. Second, the participation rate (defined as the percentage of households that have paid the fixed cost) is virtually the same as in the 2-asset model. This follows as a natural consequence of the first result. Third, for the households that have paid the fixed cost, long-term bonds are a substitute for cash, rather than a substitute for stocks. The share of wealth invested in stocks is not affected by the presence of this new asset, while cash is replaced by bonds with its share dropping almost to zero.

These results rationalize the asset allocation puzzle identified by Canner, Mankiw and Weil (1997): popular financial advisors recommend that more risk-averse investors should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. This is inconsistent with the predictions of static and frictionless Capital Asset Pricing Model which implies that all investors should hold the same combination of risky assets. Their risk aversion

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<sup>41</sup>For this calibration we used the returns on 20-year notes as reported by CRSP. The values are not very sensitive to the choice of the maturity date.

<sup>42</sup>With this specification we abstract from term structure considerations since we are not interested in market timing issues (see, among others, Campbell, Chan and Viceira (2002), Brennan and Xia (2002), Campbell and Viceira (2001), or Brennan, Schwartz and Lagnado (1997)).

<sup>43</sup>One could argue that the fixed cost is smaller for government bonds than for stocks and this could potentially help to improve the predictions of the model. Since the cost is already hard to calibrate, we decided not to pursue this possibility further at this stage.

should only determine the size of their investment to the risky assets as a whole. Our results show that the presence of undiversifiable human capital rationalizes this puzzle.

Canner et. al. (1997) also restate this puzzle as implying that the ratio of bonds to stocks should increase when the ratio of stocks to total financial wealth falls. In the context of our model we do not need heterogeneity in risk aversion to generate this prediction. The ratio of stocks to total financial wealth falls at mid-life, as the ratio of human capital to financial wealth falls and the investor is less willing to take risks. The decrease in stock holdings is compensated by higher bond holdings, thus leading to an increase in the ratio of bonds to stocks, as recommended by financial consultants.

## 7 Conclusion

This paper studies optimal consumption and portfolio choice over the life-cycle in the presence of non-diversifiable idiosyncratic labor income risk, borrowing and short sale constraints and fixed, one-time, transaction costs associated with stock market participation. We find that, for plausibly calibrated parameters, stock market participation takes place later in life typically peaking at around ages 45 – 65. Given that surveys usually sample the population between ages 25 and 75, we view this result as moving the model in the right direction for explaining the median stock-holding puzzle. Moreover, given that entry occurs late in life, small initial savings is first invested in the riskless asset market and not in the stock market as the frictionless model would predict.

On the negative side, the model still predicts that all households will have paid the stock market cost by the time of retirement. Furthermore, conditional on stock market participation, the share of wealth invested in equities is counterfactually high, generating in most cases a complete portfolio specialization in stocks. Attempts to generate co-existence of cash and stocks in the portfolio by increasing the degree of background risk or risk aversion are moderately successful along that dimension, but fail in generating non-participation, even very early in working life. Allowing for a (counterfactually) large positive correlation between stock returns and labor income shocks generates very large riskless asset holdings only for large values of risk aversion, which are associated with very high participation rates.

For low values of risk aversion the impact of the added uncertainty is of second order.

Introducing long-term government bonds in the model, does not alter the basic prediction of the model that, conditional on stock market participation, households invest almost all of their wealth in stocks. The desire to pay the fixed cost is driven by willingness to hold stocks, not long-term bonds; as a result, the participation rate (defined as the percentage of households which have paid the fixed cost) is virtually the same as in the 2-asset model. Nevertheless, stock-holders in the 3-asset model, substitute cash with long-term bonds and increase the share of bond holdings as a fraction of total financial wealth as they age, helping to rationalize the asset allocation puzzle identified by Canner, Mankiw and Weil (1997).

Overall, the results in this paper suggest that explaining participation rates and asset allocation simultaneously is still challenging and, in particular, that explanations based solely on the role of background risk are unlikely to succeed.



## A Numerical Solution

We first simplify the solution by exploiting the scale-independence of the maximization problem and rewriting all variables as ratios to the permanent component of labor income ( $P_{it}$ ). The equations of motion and the value function can then be rewritten as normalized variables and we use lower case letters to denote them (for instance,  $x_{it} \equiv \frac{X_{it}}{P_{it}}$ ). This allows us to reduce the number of state variables to three; one continuous state variable (cash on hand) and two discrete state variables (age and participation status). The relevant policy functions can then be computed as functions of the three state variables: age ( $t$ ), normalized cash-on-hand ( $x_{it}$ ), and participation status (whether the fixed cost has already been paid or not). In the last period the policy functions are trivial (the agent consumes all available wealth) and the value function corresponds to the indirect utility function. We can use this value function to compute the policy rules for the previous period and given these, obtain the corresponding value function. This procedure is then iterated backwards.

We discretize the state-space (cash-on-hand) and use Gaussian quadrature to approximate the distributions of the innovations to the labor income process and risky asset returns (Tauchen and Hussey (1991)). In every period  $t$  prior to  $T$ , and for each admissible combination of the state variables, we compute the value associated with each level of consumption, the decision to pay the fixed cost of entering the stock market, and the share of liquid wealth invested in stocks. This value is equal to current utility plus the expected discounted continuation value. To compute this continuation value for points which do not lie on the grid for the state space we use a univariate cubic spline interpolation procedure along the cash on hand dimension. The participation decision is obtained by comparing the value functions for stock market participants (adjusting for the entry cost) and for non-participants. We optimize over the different choices using grid search.

## B Computing the Transition Distributions

To find the distribution of cash on hand, we first compute the relevant optimal policy rules; bond and stock policy functions for stock market participants and non-participants and the  $\{0, 1\}$  participation rule as a function of cash on hand. Let  $b^I(x)$  and  $s^I(x)$  denote the cash and stock policy rules for individuals participating in the stock market respectively and let  $b^O(x)$  be the savings decision for the individual out of the stock market. We assume that households start their working life with zero liquid assets. During working life, for the households that have not paid the fixed cost, the evolution of normalized cash on hand is given by<sup>44</sup>

$$\begin{aligned} x_{t+1} &= [b^O(x_t)R_f] \left\{ \frac{P_t}{P_{t+1}} \frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))} \right\} + U_{t+1} \\ &= w \left( x_t \middle| \frac{P_t}{P_{t+1}}, \frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))} \right) + U_{t+1} \end{aligned}$$

where  $w(x)$  is defined by the last equality and is conditional on  $\{\frac{P_t}{P_{t+1}}\}$  and the deterministically evolving  $\frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))}$ . Denote the transition matrix of moving from  $x_j$  to  $x_k$ ,<sup>45</sup> conditional on being in the bond market as  $T_{kj}^O$ . Let  $\Delta$  denote the distance between the equally spaced discrete points of cash on hand. The random permanent shock  $\frac{P_t}{P_{t+1}}$  is discretized using Gaussian quadrature with  $m^*$  points:  $\frac{P_t}{P_{t+1}} = \{N_m\}_{m=1}^{m^*}$ .  $T_{kj}^O = \Pr(x_{t+1}=k|x_t=j)$  is found using<sup>46</sup>

$$\sum_{m=1}^{m^*} \Pr \left( x_{t+1}|x_t, \frac{P_t}{P_{t+1}} = N_m \right) * \Pr \left( \frac{P_t}{P_{t+1}} = N_m \right) \quad (26)$$

Numerically, this probability is calculated using

$$T_{kjm}^O = \Pr \left( x_k + \frac{\Delta}{2} \geq x_{t+1} \geq x_k - \frac{\Delta}{2} \middle| x_t = x_j, \frac{P_t}{P_{t+1}} = N_m \right)$$

Making use the approximation that for small values of  $\sigma_u^2$ ,  $U \sim N(\exp(\mu_u + .5 * \sigma_u^2), (\exp(2 * \mu_u + (\sigma_u^2)) * (\exp(\sigma_u^2) - 1)))$ , and denoting the mean of  $U$  by  $\bar{U}$  and its standard deviation

<sup>44</sup>To avoid cumbersome notation, the subscript  $i$  that denotes a particular individual is omitted in what follows.

<sup>45</sup>The normalized grid is discretized between  $(x \text{ min}, x \text{ max})$  where  $x \text{ min}$  denotes the minimum point on the equally spaced grid and  $x \text{ max}$  the maximum point.

<sup>46</sup>The dependence on the deterministically evolving  $\frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))}$  is implied and is omitted from what follows for expositional clarity.

by  $\sigma$ , the transition probability conditional on  $N_m$  equals

$$T_{kjm}^O = \Phi\left(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_m) - \bar{U}}{\sigma}\right) - \Phi\left(\frac{x_k - \frac{\Delta}{2} - w(x_t|N_m) - \bar{U}}{\sigma}\right)$$

where  $\Phi$  is the cumulative distribution function for the standard normal. The unconditional probability from  $x_j$  to  $x_k$  is then given by

$$T_{kj}^O = \sum_{m=1}^{m=m^*} T_{kjm}^O \Pr(N_m) \quad (27)$$

Given the transition matrix  $\mathbf{T}^O$  (letting the number of cash on hand grid points equal to  $J$ , this is a  $J$  by  $J$  matrix;  $T_{kj}^O$  represents the  $\{k\text{th}, j\text{th}\}$  element), the next period probabilities of each of the cash on hand states can be found using

$$\pi_{kt}^O = \sum_j T_{kj}^O * \pi_{jt-1}^O \quad (28)$$

We next use the vector  $\mathbf{\Pi}_t^O$  (this is a  $J$  by 1 vector representing the mass of the population out of the stock market at each grid point;  $\pi_{kt}^O$  represents the  $\{k\text{th}\}$  element at time  $t$ ) and the participation policy rule to determine the percentage of households that optimally choose to incur the fixed cost and participate in the stock market. This is found by computing the sum of the probabilities in  $\mathbf{\Pi}_t^O$  for which  $x > x^*$ ,  $x^*$  being the trigger point that causes participation ( $x^*$  is determined endogenously through the participation decision rule). These probabilities are then deleted from  $\mathbf{\Pi}_t^O$  and are added to  $\mathbf{\Pi}_t^I$ , appropriately renormalizing both  $\{\mathbf{\Pi}_t^O, \mathbf{\Pi}_t^I\}$  to sum to one. The participation rate ( $\theta_t$ ) can be computed at this stage as

$$\theta_t = \theta_{t-1} + (1 - \theta_{t-1}) * \sum_{x_j > x^*} \pi_{t,j}^O$$

The same methodology (but with more algebra and computations) can then be used to derive the transition distribution for cash on hand conditional on being in the stock market  $\mathbf{T}_t^I$ . For stock market participants, the normalized cash on hand evolution equation is

$$\begin{aligned} x_{t+1} &= [b(x_t)R_f + s(x_t)\tilde{R}_{t+1}] \left\{ \frac{P_t}{P_{t+1}} \frac{\exp(f(t, Z_t))}{\exp(f(t+1, Z_{t+1}))} \right\} + U_{t+1} \\ &= w\left(x_t | \tilde{R}_{t+1}, \frac{P_t}{P_{t+1}}\right) + U_{t+1} \end{aligned} \quad (29)$$

where  $w(x)$  is now conditional on  $\{\tilde{R}_{t+1}, \frac{P_t}{P_{t+1}}\}$ <sup>47</sup>. The random processes  $\tilde{R}$  and  $\frac{P_t}{P_{t+1}}$  are discretized using Gaussian quadrature with  $m^*$  points:  $R = \{R_l\}_{l=1}^{l=m^*}$  and  $\frac{P_t}{P_{t+1}} = \{N_m\}_{m=1}^{m=m^*}$ .  $T_{kj}^I = \Pr(x_{t+1}=k|x_t=j)$  is found using

$$\sum_{l=1}^{l=m^*} \sum_{m=1}^{m=m^*} \Pr\left(x_{t+1}|x_t, \tilde{R}_{t+1} = R_l, \frac{P_t}{P_{t+1}} = N_m\right) * \Pr(\tilde{R}_{t+1} = R_l) * \Pr\left(\frac{P_t}{P_{t+1}} = N_m\right) \quad (30)$$

where the independence of  $\frac{P_t}{P_{t+1}}$  from  $\tilde{R}_{t+1}$  was used.<sup>48</sup> Numerically, this probability is calculated using

$$T_{kjl}^I = \Pr\left(x_k + \frac{\Delta}{2} \geq x_{t+1} \geq x_k - \frac{\Delta}{2} | x_t = x_j, \frac{P_{it}}{P_{it+1}} = N_m, R_{t+1} = R_l\right)$$

The transition probability conditional on  $N_m$  and  $R_l$  equals

$$T_{kjl}^I = \Phi\left(\frac{x_k + \frac{\Delta}{2} - w(x_t|N_m, R_l) - \bar{U}}{\sigma}\right) - \Phi\left(\frac{x_k - \frac{\Delta}{2} - w(x_t|N_m, R_l) - \bar{U}}{\sigma}\right)$$

The unconditional probability from  $x_j$  to  $x_k$  is then given by

$$T_{kj}^I = \sum_{l=1}^{l=10} \sum_{m=1}^{m=10} T_{kjl}^I \Pr(N_m) \Pr(R_l) \quad (31)$$

Given the matrix  $\mathbf{T}^I$ , the probabilities of each of the states are updated by

$$\pi_{kt+1}^I = \sum_j T_{kj}^I * \pi_{jt}^I \quad (32)$$

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<sup>47</sup>The dependence on the non-random earnings component is omitted to simplify notation.

<sup>48</sup>The methodology can be applied for an arbitrary correlation structure between the stock market and permanent shock innovation.

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Figure 1.1 - Equity Holdings as a Fraction of Total Financial Wealth  
(Ameriks and Zeldes, 2001 - OLS Regressions)

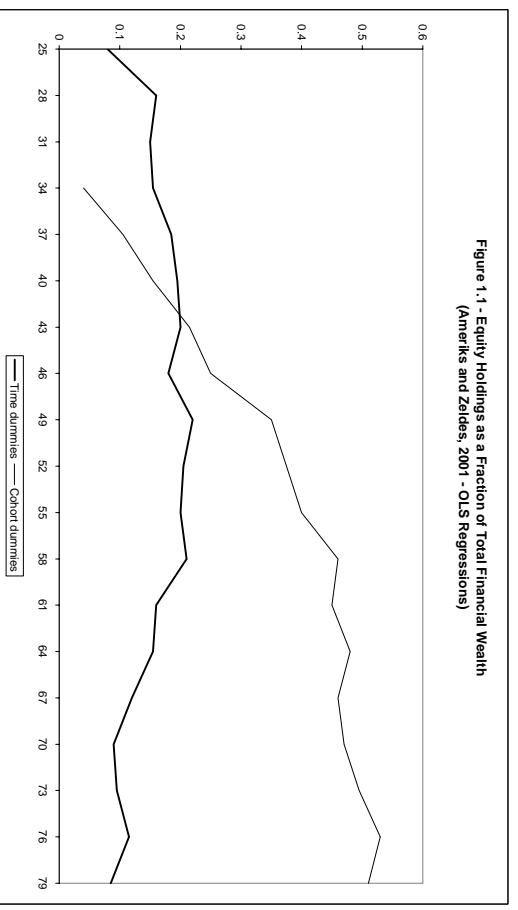


Figure 1.2 - Stock Market Participation  
(Ameriks and Zeldes, 2001 - Probit Regressions)

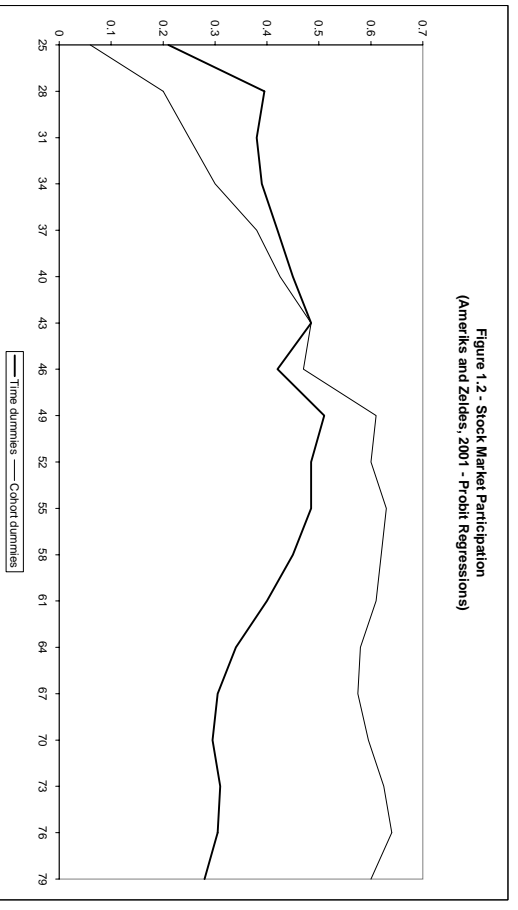


Figure 1.3 - Equity Holdings as a Fraction of Total Financial Wealth, Conditional on Participation  
(Guiso, Hallassos and Jappelli, 2001)

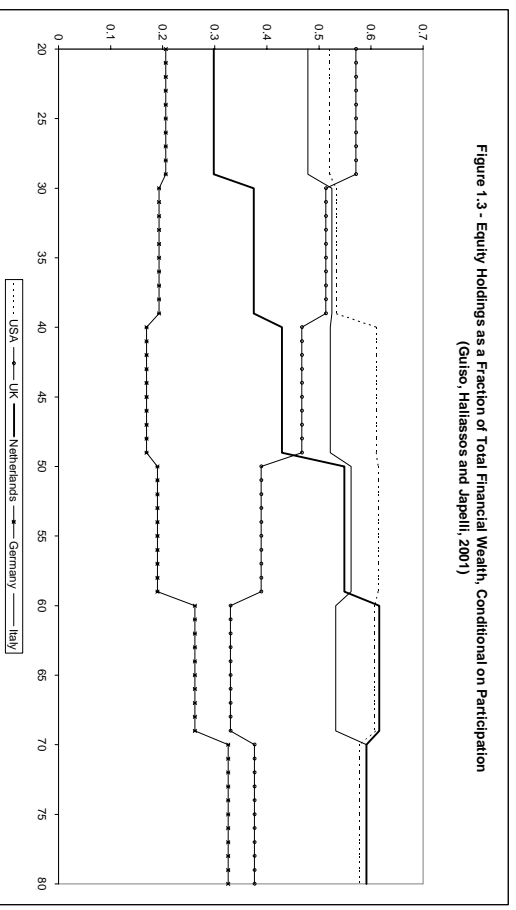


Figure 1.4 - Stock Market Participation  
(Guiso, Hallassos and Jappelli, 2001)

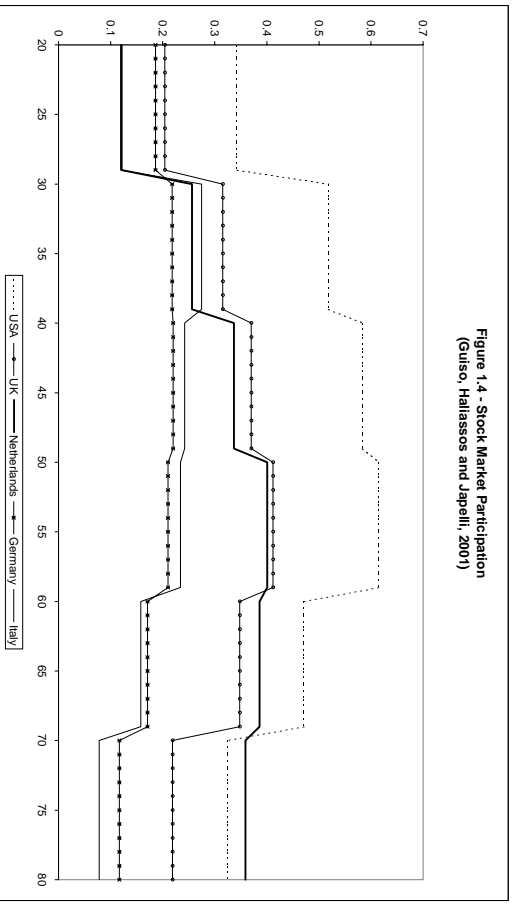


Fig 2.1: Distributions for Normalized Cash on Hand Model without fixed cost

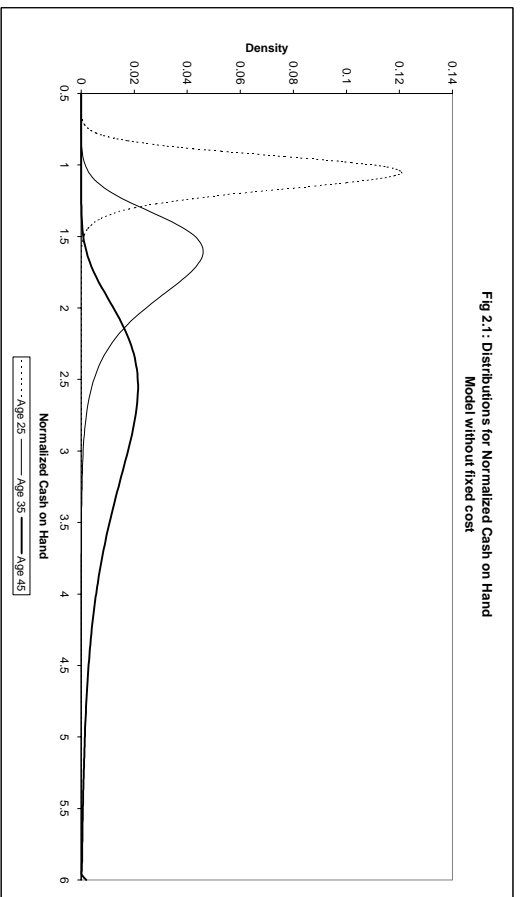


Fig 2.2: Distributions for Normalized Cash on Hand (Retirement stage) Model without the fixed cost

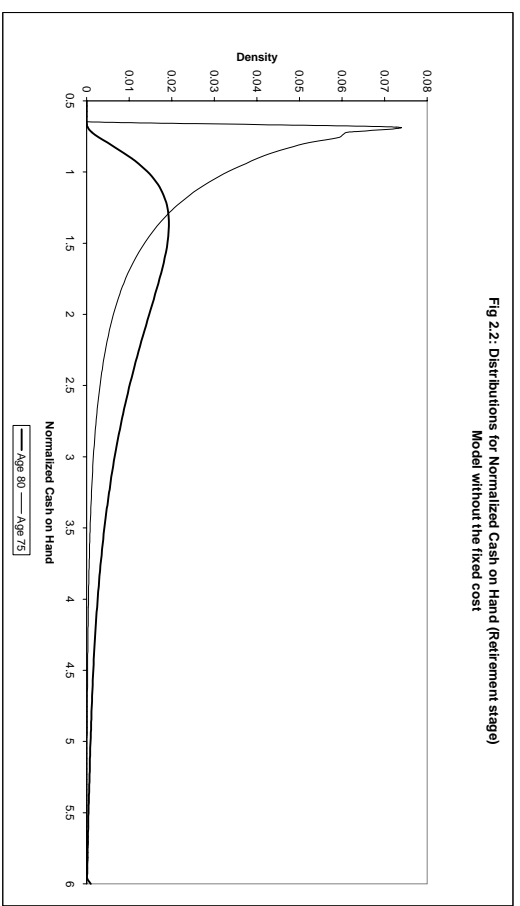


Fig 2.3: Consumption and Wealth Model without the fixed cost

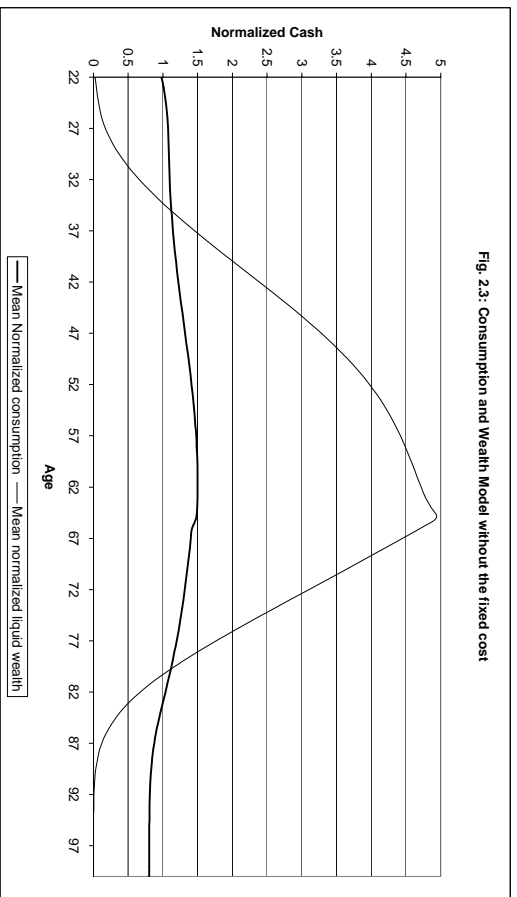


Fig 2.4: Portfolio Allocation Model Without the Fixed Cost

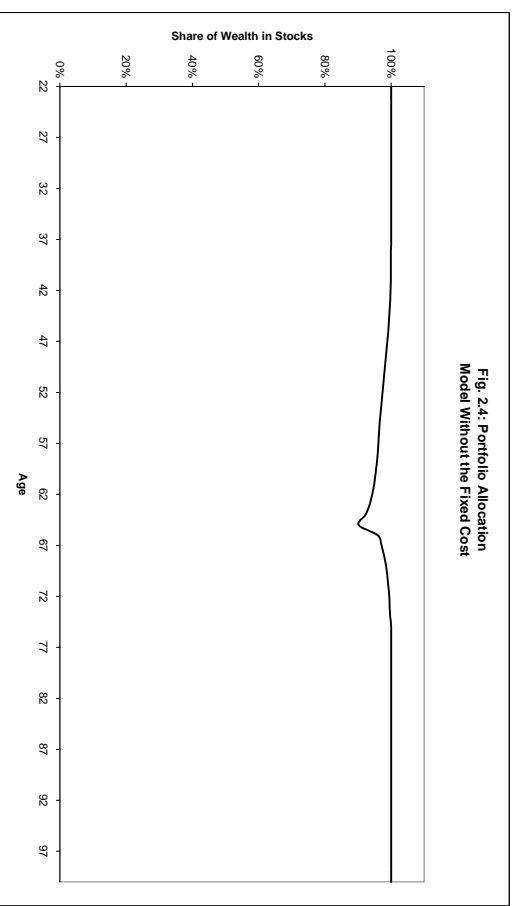


Fig. 3.1: Distributions for Normalized Cash on Hand  
Model with Fixed Cost

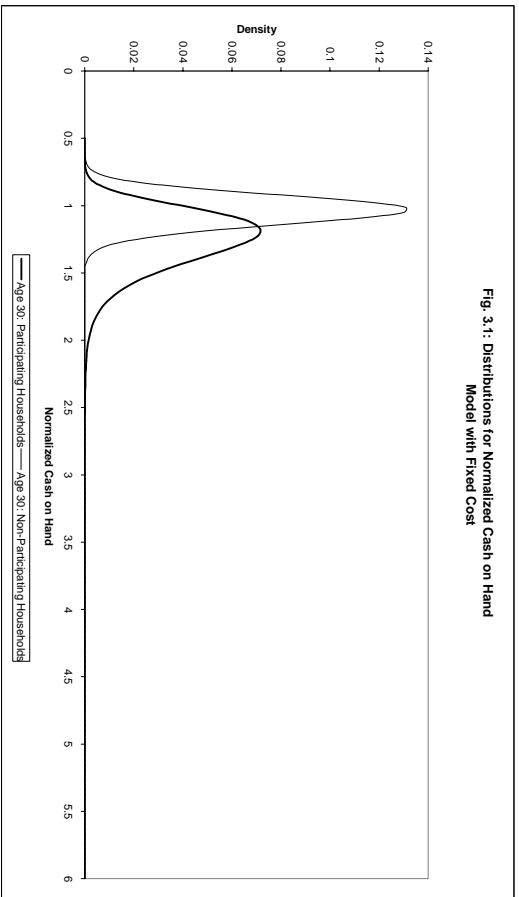


Fig. 3.2: Distributions for Normalized Cash on Hand  
Model with Fixed Cost

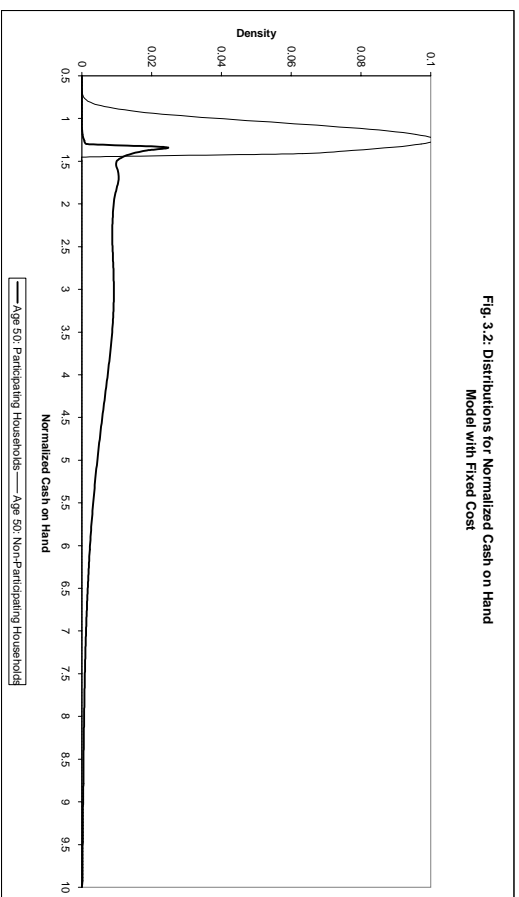


Fig. 3.3: Consumption and Wealth  
Model with the Fixed Cost

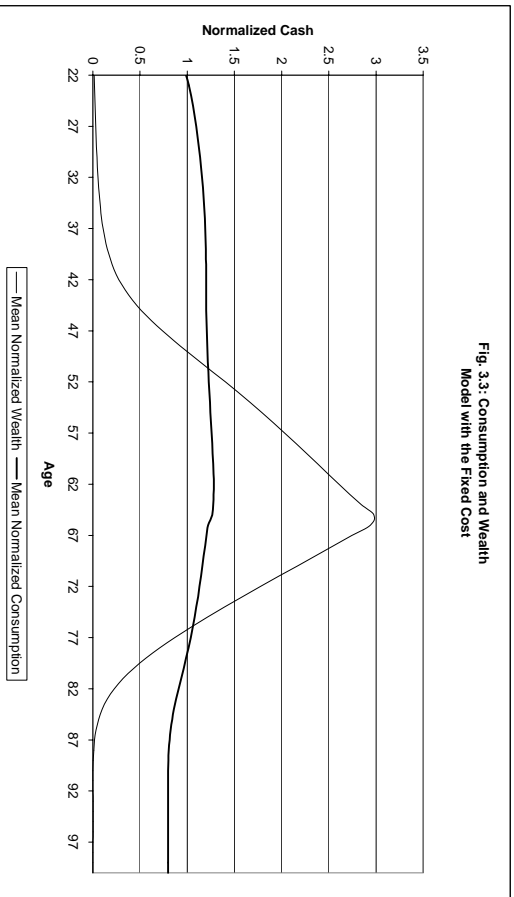


Fig. 3.4: Portfolio Allocation over the Life-Cycle with the fixed cost

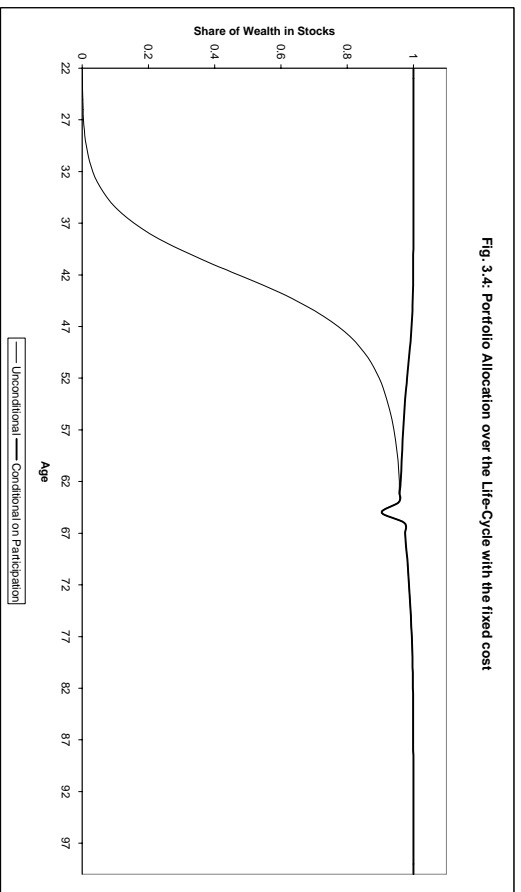


Fig. 3.5: Stock Market Participation Rate With the fixed cost

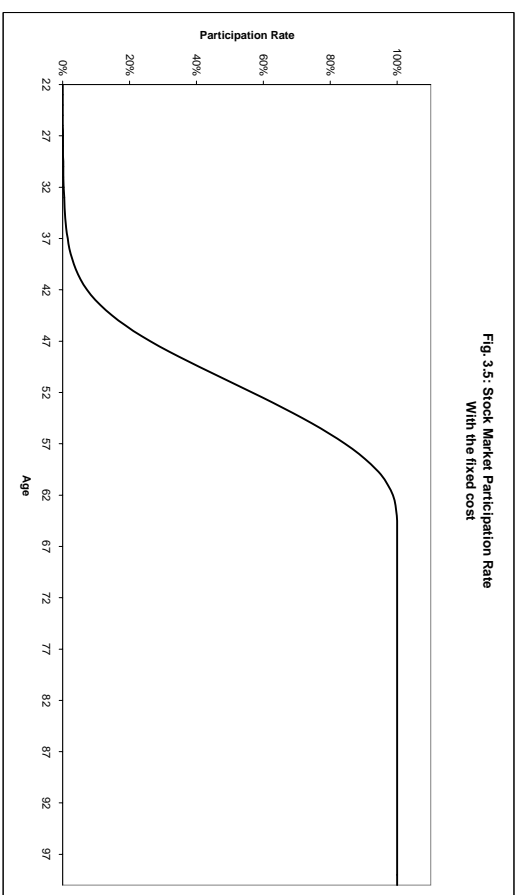


Fig 3.6: Wealth Comparison Models with and without fixed costs of participation

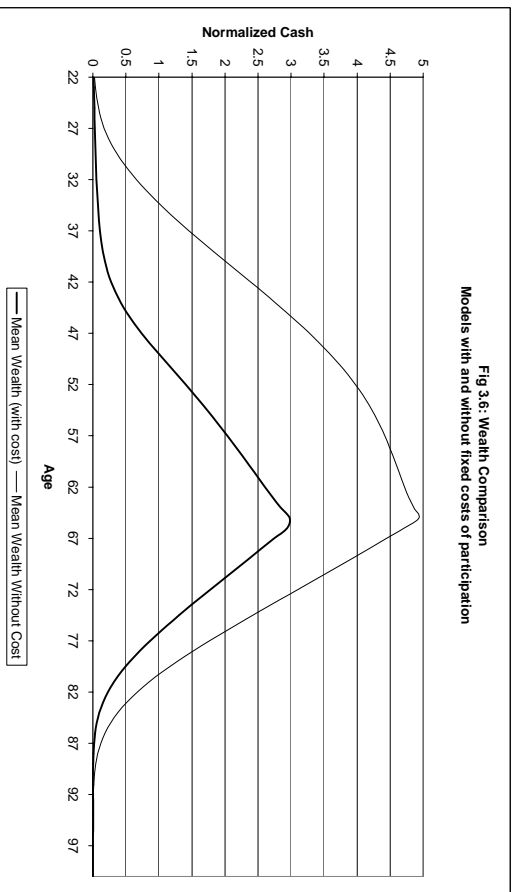


Fig. 4.1: Stock Market Participation Rates with fixed cost ( $F=0.1$ )  
 $\rho=2$  vs  $\rho=5$

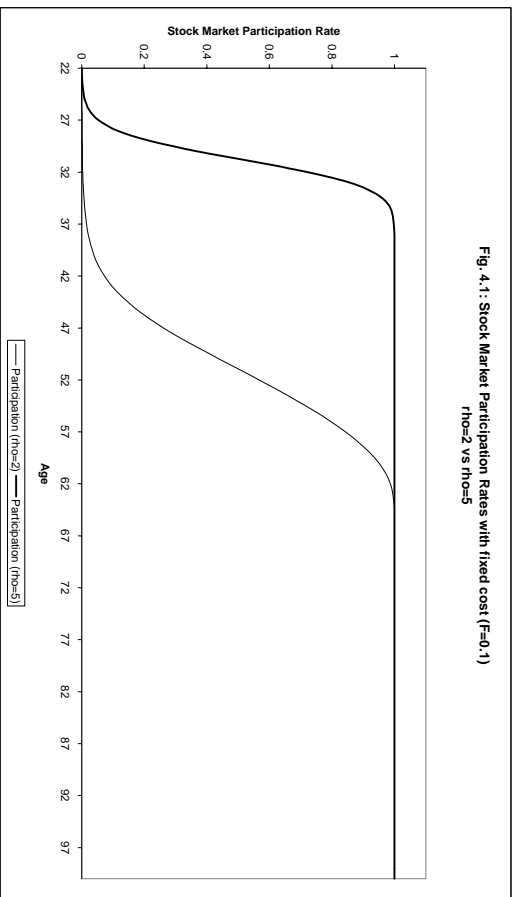


Fig. 4.2: Unconditional Portfolio Allocations ( $\rho=2$  vs  $\rho=5$ )

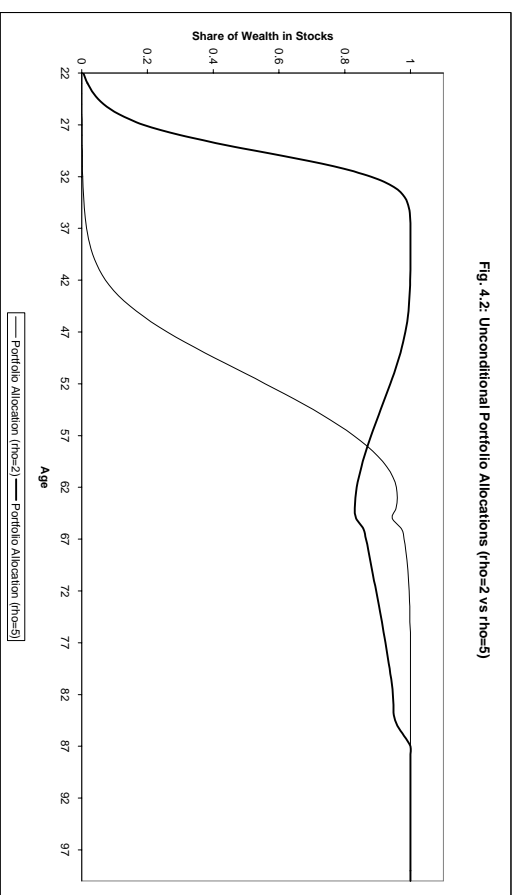


Fig. 4.3: Portfolio Allocation Conditional on Participation  
 $(\rho=2$  vs  $\rho=5)$

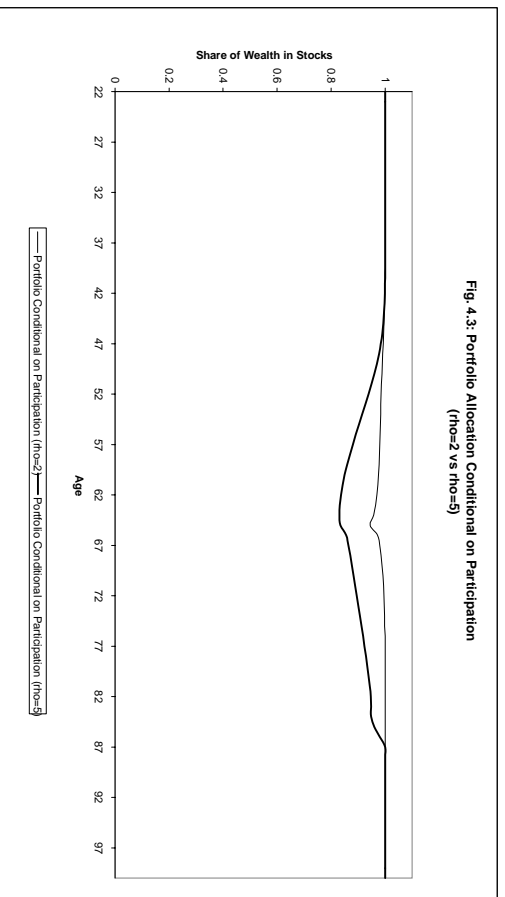




Figure 4.4 - Portfolio Allocation and Stock Market Participation  
Beta = 0.7 and Rho = 10

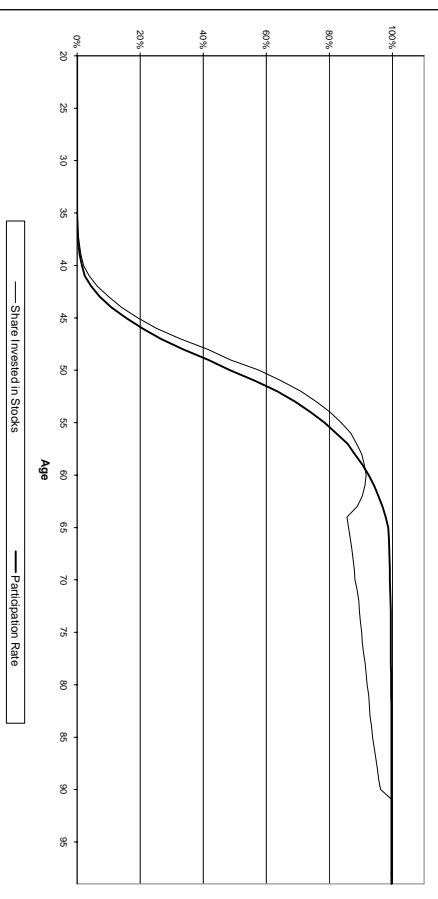


Figure 4.5 - Portfolio Allocation and Stock Market Participation  
College Graduates (CG) and Households with no High-School Degree (NHSD)

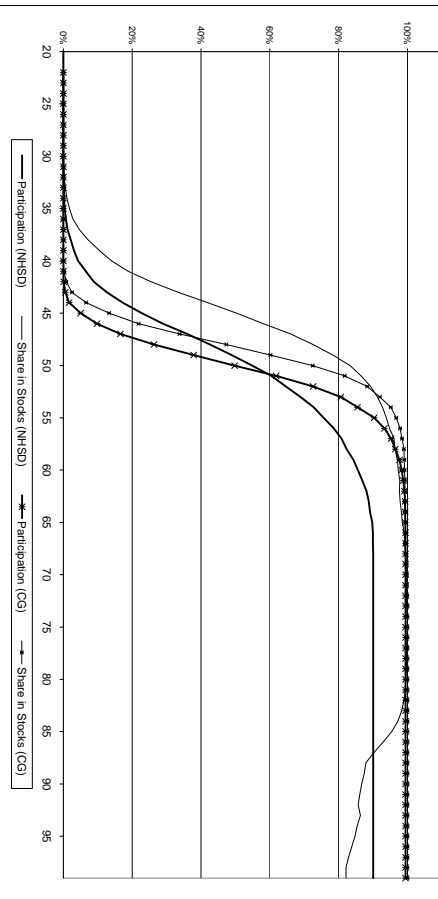


Figure 5.1 - Portfolio Allocation and Stock Market Participation (RRA = 2)  
(with probability of large negative income shock)

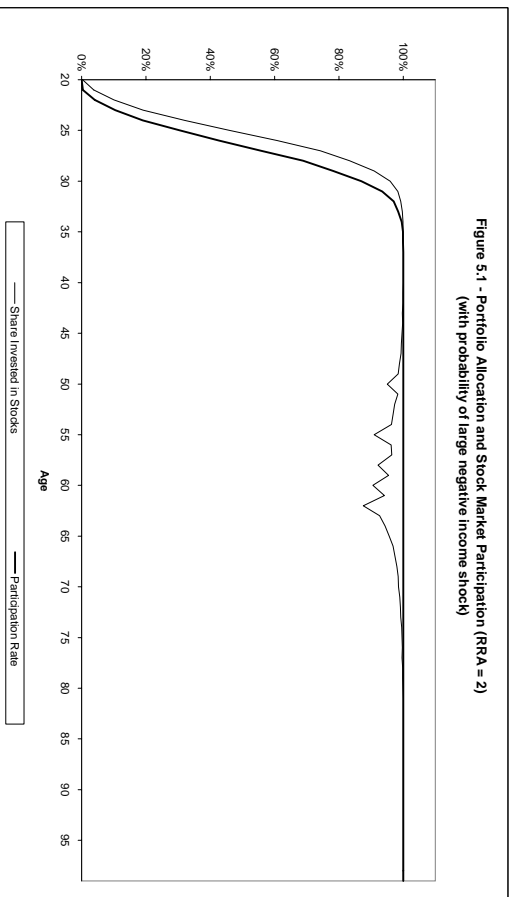


Figure 6.1 - Portfolio Allocation and Stock Market Participation (RRA = 2)  
(with correlation between stock return and permanent labor income shocks equal to 0.5)

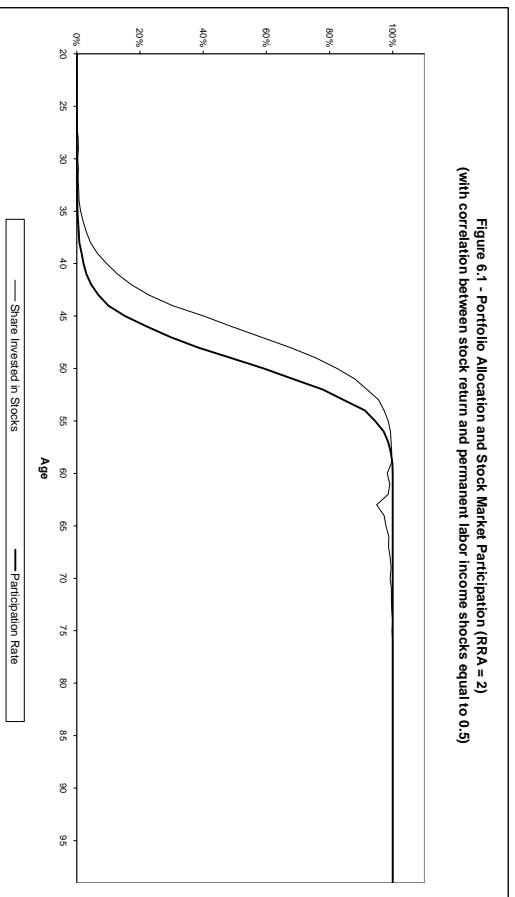


Figure 5.2 - Portfolio Allocation and Stock Market Participation (RRA = 5)  
(with probability of large negative income shock)

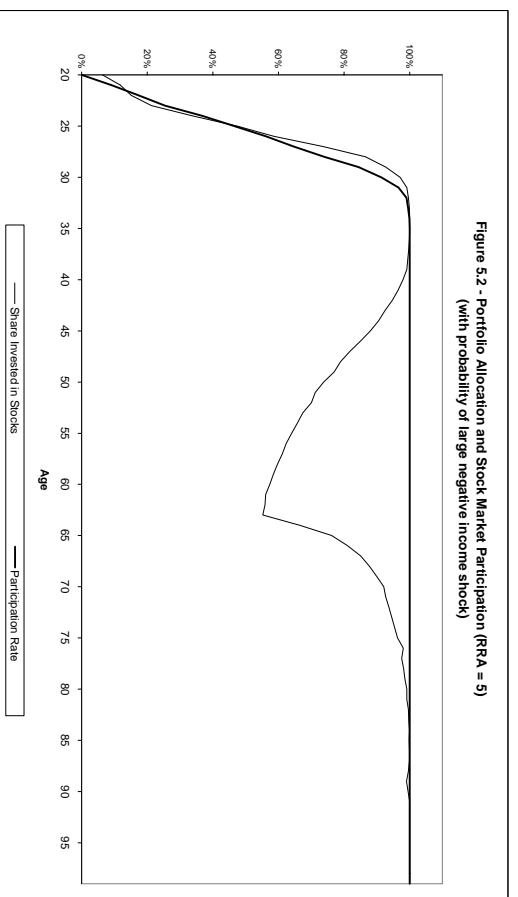


Figure 6.2 - Portfolio Allocation and Stock Market Participation (RRA = 5)  
(with correlation between stock return and permanent labor income shocks equal to 0.5)

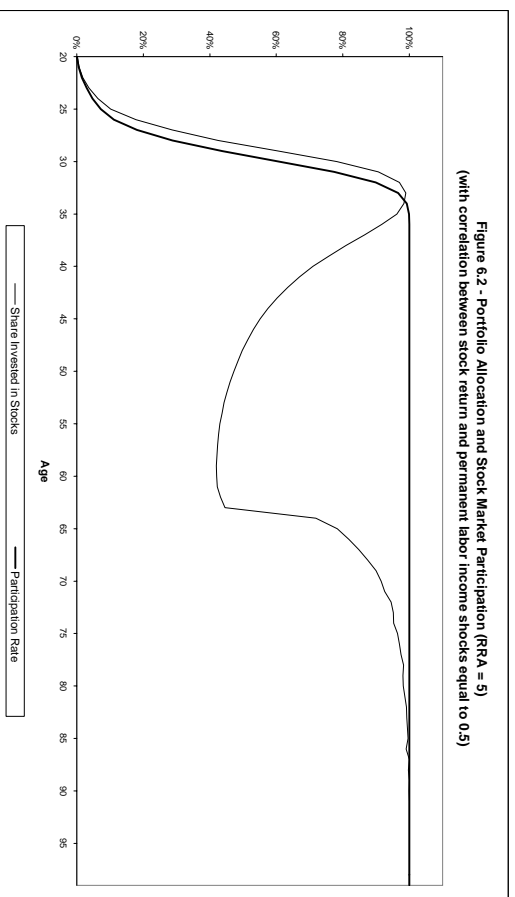


Figure 7.1 - Portfolio Allocation and Stock Market Participation (RRR = 2)  
(model with three assets)

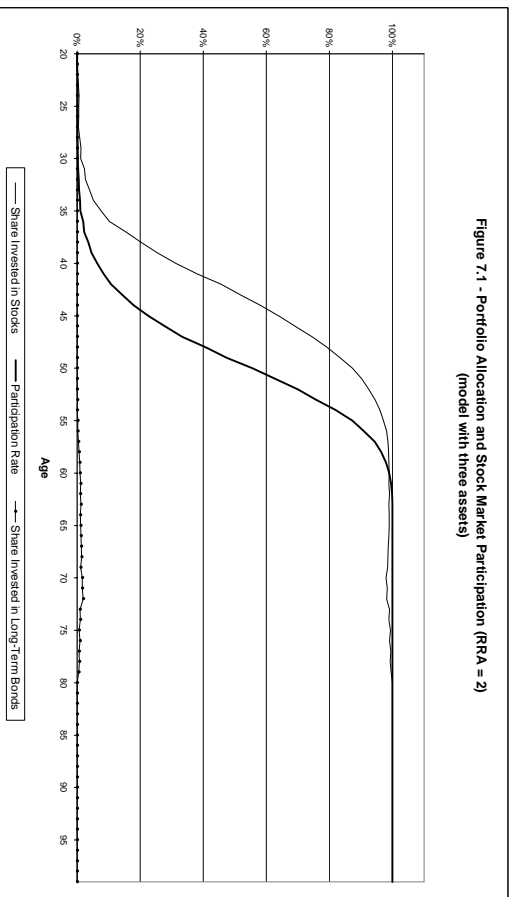


Figure 7.2 - Portfolio Allocation and Stock Market Participation (RRR = 5)  
(model with three assets)

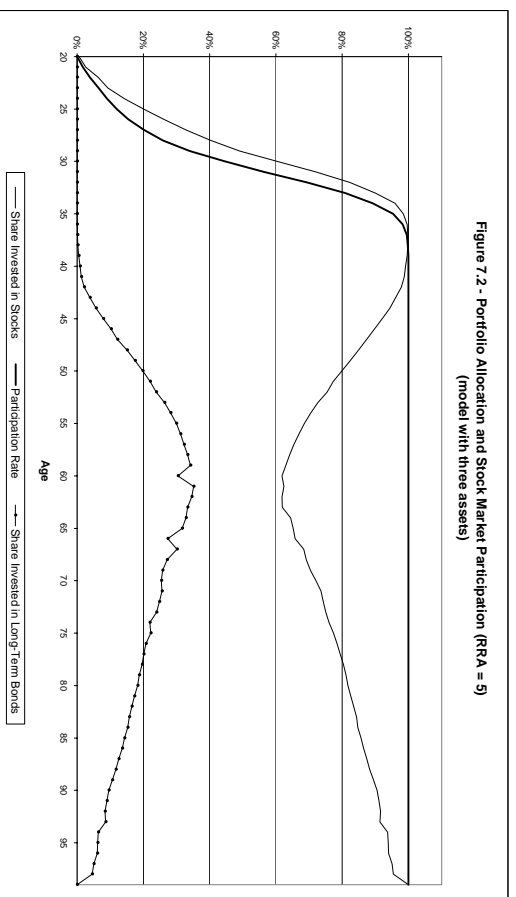


Table 1.1 - Mean Normalized Consumption for different age groups. Results are shown for different values of the elasticity of intertemporal substitution (in columns), and different values of the coefficient of relative risk aversion (in rows).

**Ages 20 to 35**

	0.75	0.5	0.25	0.1
4/3	1.401	1.402	1.399	1.400
2	1.365	1.399	1.397	1.392
4	1.287	1.735	1.725	1.339
10	1.237	1.249	1.251	1.227

**Ages 36 to 50**

	0.75	0.5	0.25	0.1
4/3	1.775	1.782	1.785	1.786
2	1.675	1.697	1.733	1.746
4	1.835	2.069	1.999	1.726
10	1.912	1.984	1.899	1.766

**Ages 51 to 65**

	0.75	0.5	0.25	0.1
4/3	1.566	1.619	1.627	1.629
2	1.991	1.699	1.662	1.665
4	2.167	1.921	1.879	1.944
10	2.175	2.159	2.143	1.962

**Ages 66 to 80**

	0.75	0.5	0.25	0.1
4/3	1.163	1.036	1.029	1.026
2	1.873	1.539	1.285	1.184
4	1.959	1.213	1.255	1.839
10	1.849	1.668	1.781	1.880

Table 1.2 - Mean Normalized Wealth for different age groups.

Results are shown for different values of the elasticity of intertemporal substitution (in columns), and different values of the coefficient of relative risk aversion (in rows).

**Ages 20 to 35**

	0.75	0.5	0.25	0.1
4/3	0.03	0.04	0.05	0.06
2	0.14	0.05	0.06	0.08
4	0.66	0.29	0.24	0.27
10	1.56	1.45	1.41	1.44

**Ages 36 to 50**

	0.75	0.5	0.25	0.1
4/3	0.14	0.14	0.17	0.19
2	2.94	0.67	0.52	0.61
4	5.44	4.13	3.61	3.38
10	7.49	6.91	7.31	7.83

**Ages 51 to 65**

	0.75	0.5	0.25	0.1
4/3	0.64	0.14	0.11	0.10
2	7.60	3.83	2.07	1.45
4	9.39	8.72	8.26	7.88
10	10.23	7.59	10.00	11.42

**Ages 66 to 80**

	0.75	0.5	0.25	0.1
4/3	0.66	0.06	0.04	0.04
2	6.16	3.13	1.23	0.65
4	7.02	6.97	7.02	7.09
10	5.76	4.75	6.97	10.08

Table 1.3 - Mean Share of Wealth Invested in Stocks for different age groups. Results are shown for different values of the elasticity of intertemporal substitution (in columns), and different values of the coefficient of relative risk aversion (in rows).

**Ages 20 to 35**

	0.75	0.5	0.25	0.1
4/3	0%	0%	0%	0%
2	51%	4%	0%	0%
4	78%	55%	31%	38%
10	77%	76%	77%	76%

**Ages 36 to 50**

	0.75	0.5	0.25	0.1
4/3	14%	0%	0%	0%
2	99%	60%	29%	24%
4	100%	100%	99%	99%
10	80%	92%	87%	54%

**Ages 51 to 65**

	0.75	0.5	0.25	0.1
4/3	58%	0%	0%	0%
2	97%	99%	95%	87%
4	92%	92%	90%	85%
10	56%	70%	52%	33%

**Ages 66 to 80**

	0.75	0.5	0.25	0.1
4/3	98%	0%	0%	0%
2	99%	100%	100%	100%
4	97%	95%	94%	91%
10	73%	78%	67%	57%

Table 1.4 – Stock-Market Participation Rate for different age groups.

Results are shown for different values of the elasticity of intertemporal substitution (in columns), and different values of the coefficient of relative risk aversion (in rows).

**Ages 20 to 35**

	0.75	0.5	0.25	0.1
4/3	0%	0%	0%	0%
2	17%	1%	0%	0%
4	60%	33%	26%	40%
10	75%	75%	78%	80%

**Ages 36 to 50**

	0.75	0.5	0.25	0.1
4/3	2%	0%	0%	0%
2	86%	34%	22%	23%
4	100%	99%	100%	100%
10	100%	100%	100%	100%

**Ages 51 to 65**

	0.75	0.5	0.25	0.1
4/3	27%	0%	0%	0%
2	100%	98%	91%	84%
4	100%	100%	100%	100%
10	100%	100%	100%	100%

**Ages 66 to 80**

	0.75	0.5	0.25	0.1
4/3	85%	0%	0%	0%
2	100%	100%	100%	98%
4	100%	100%	100%	100%
10	100%	100%	100%	100%

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