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A dynamic stochastic programming model for international portfolio management ☆

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Abstract

We develop a multi-stage stochastic programming model for international portfolio management in a dynamic setting. We model uncertainty in asset prices and exchange rates in terms of scenario trees that reflect the empirical distributions implied by market data. The model takes a holistic view of the problem. It considers portfolio rebalancing decisions over multiple periods in accordance with the contingencies of the scenario tree. The solution jointly determines capital allocations to international markets, the selection of assets within each market, and appropriate currency hedging levels. We investigate the performance of alternative hedging strategies through extensive numerical tests with real market data. We show that appropriate selection of currency forward contracts materially reduces risk in international portfolios. We further find that multi-stage models consistently outperform single-stage models. Our results demonstrate that the stochastic programming framework provides a flexible and effective decision support tool for international portfolio management.

Keywords: Stochastic programming; Risk management; International portfolios; Scenarios

1. Introduction

Active management of international portfolios is of particular interest to multinational firms, financial intermediaries, institutional investors (e.g., banks, insurance firms, mutual funds, pension funds) and high net-worth individuals. Investments in foreign securities are becoming accessible to a wide range of investors. This is a consequence of liberalization in capital flows, and advancements in information technology that provide instantaneously information from remote financial markets and facilitate the execution of transactions. International investments provide certain benefits: (a) the prospect for higher profit in the event of favorable performance of foreign markets, (b) wider scope for diversification, (c) reduced exposure to

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systematic risk due to generally low correlations of international securities.

The potential benefits of international diversification were recognized early; see, for example, Grubel [11], Levy and Sarnat [18], and Solnik [22]. Due to comparably low correlations of international securities, improvements in reward-to-risk performance can be achieved by holding internationally diversified portfolios. Despite a general increase in the correlation of international markets in recent years as a result of financial globalization and market integration, international investments continue to provide a wider scope for portfolio diversification than is available in a domestic market. Optimal management of international portfolios, especially in a dynamic setting, continues to be a problem of great interest and practical significance.

Eun and Resnick [8] pointed out that international diversification decisions should take into account the exposure to currency risk. Namely, the risk stemming from fluctuations of foreign exchange rates, as uncertain future exchange rates directly affect the translation of returns from foreign investments into the investor's base-currency returns. Given the volatility of exchange rates in some periods, currency risk can have a major impact on international portfolios.

Means to effectively hedge currency risk are actively studied. In a key early contribution, Black [3] suggested a universal hedge ratio for all investors, implying that all foreign investments should be hedged to the same proportion. Subsequent studies examined more flexible currency hedging strategies. A number of empirical studies evaluated the merits of various approaches for hedging exchange rate risk, usually on the basis of forward currency rates. We do not review this extensive literature here. Despite a general agreement that currency hedging can mitigate portfolio risk, there is not a consensus as to a universally optimal currency hedging rule. The literature presents somewhat different views as to the optimal course of action for hedging currency risk in international portfolios, depending on the focus of each study with regard to various factors, such as: the investment opportunity set, the risk aversion preference and time horizon of the decision maker, the reference currency of the investor, the investment strategy (passive vs active), the distribution and stationarity of asset returns and exchange rates in the timeframe of the study (i.e. the historical data used in calibrating the distributions), the realized market conditions (trends, volatilities, correlations between markets) at the time of simulations, the specific hedging strategies that were compared.

The overall conclusion is that the relative merits of alternative hedging strategies remain mostly an empirical issue, they depend on the factors mentioned above and are usually problem-dependent. These observations point to the need for flexible decision approaches that can be adjusted in each case to the problem at hand. This study develops a general and flexible model to support international portfolio management decisions.

In practice, and more often than not in the literature as well, the international portfolio management problem is addressed in an overlay manner. First, the capital allocation to various markets is decided at the strategic level. The management of the capital allocations is then assigned to different trading desks with expertise in the respective markets. Each trading desk has independent jurisdiction to tactically select specific securities within its assigned market, and operationally manage its own portfolio. Performance is typically compared against some benchmark. Currency hedging is often viewed as a subordinate decision, and is taken last to cover specific exposures of foreign asset positions that were decided previously. A global view of the problem is not often taken, and possible cross-hedging effects among portfolio positions are frequently ignored. Changes in the portfolio structure are not always coordinated with corresponding adjustments to the currency hedging positions. Important and interrelated decisions are considered separately and sequentially.

Jorion [16] criticized this overlay approach and showed that it is suboptimal to a holistic view that considers all the interrelated decisions in a unified manner. Here, we develop a comprehensive modelling framework that jointly addresses the capital allocation, the portfolio selection, and the currency hedging decisions.

Filatov and Rappoport [9] were the first to suggest a selective hedging approach in which the hedge ratios may vary across currencies. They showed that selective hedging can yield different optimal hedge ratios for each currency. Owing to this flexibility, the selective hedging approach dominates other strategies that tie in specific ways the hedge ratios across currencies (e.g., unitary and partial hedging, which selective hedging encompasses as special cases). In their empirical tests, they also found that the optimal selective hedging policy can be different for investors with different reference currencies. Beltratti et al. [2] incorporated selective hedging decisions within a single-stage portfolio optimization model and empirically confirmed the superiority of selective hedging to unitary hedging. Their model used a mean-absolute-deviation objective, and bootstrapping of historical data for scenario generation. Topaloglou et al. [25] extended this model for international asset allocation by using a more appropriate scenario generation procedure, and by adopting a risk measure that is suitable for the asymmetric distributions exhibited by international asset returns and exchange rates.

Glen and Jorion [10] considered dynamic hedging strategies, and observed that the optimal hedging policy may be temporally unstable. They concluded that benefits can be obtained by following a state contingent policy that varies the level and the structure of hedging depending on the investment opportunity set and on evolving information regarding the distributions of asset returns and exchange rates.

Building on the findings of these studies, we develop a general and flexible modelling framework for the active management of international portfolios in a dynamic setting. The primary features of our model are: a multi-period decision framework that considers information and decision dynamics, the ability to capture arbitrary stochastic evolutions of the random factors by means of a discrete representation (scenario tree), state-contingent decisions in conformity with the projected outcomes of the random variables, dynamic adjustments of portfolio positions and currency hedging levels in response to changing information on prevailing economic conditions (distributions of the random variables as reflected in the scenario tree), operationalization of currency hedging decisions by means of explicit forward exchange contracts for each currency that fully encompass a selective hedging approach, a holistic view of the problem that accounts for decision interactions and considers the total risk exposure of the international portfolio, a risk measure that is suitable for skewed and leptokurtic distributions as have been observed for international indices and exchange rates, consideration of transaction costs. Alternative objectives to reflect the decision maker's risk bearing preferences, as well as other operational issues (e.g., managerial and regulatory conditions) can be easily incorporated with appropriate adjustments to the model.

The paper is organized as follows. In the next section we briefly describe the problem and introduce the modelling choices for our solution approach. In Section 3 we describe the representation of uncertainty in the multi-stage portfolio optimization model by means of a scenario tree. We present the key statistics of the random variables based on historical data and discuss the scenario generation procedure. In Section 4 we formulate the stochastic programming model for international portfolio management and examine its features. In Section 5 we describe the computational tests and we present the empirical results. In Section 6 we discuss the findings of this study and directions for further research.

2. Problem description and modeling approach

We consider the problem of a decision maker who is concerned with the active management of a set of financial assets (stock and bond indices in this study) denominated in multiple currencies, so as to generate profit while at the same time controlling the downside risk exposure. The problem has a dynamic structure that involves portfolio rebalancing decisions at periodic intervals in response to new information on market conditions (i.e., changing perceptions regarding the distributions of random asset prices and exchange rates). Rebalancing decisions are manifested in a sequence of successive revisions of holdings through sales and purchases of assets and currency exchange transactions in the spot market. Currency forward contracts can be employed to hedge the currency risk of foreign investments; we consider forward contracts with a term of a single period at each decision stage.

The decision maker starts with an initial portfolio and has full knowledge of the current asset prices and exchange rates. Thus, individual asset holdings, as well as the entire portfolio, can be accurately valued. The decision maker must assess the potential movements of the asset prices and the exchange rates that affect the future value and risk exposure of his portfolio. His perceptions for such market movements are expressed in terms of a joint probability distribution of the random variables; he must project the contingent evolution of the random variables over the entire planning horizon. The portfolio rebalancing and currency hedging decisions should conform to these projections.

Clearly, the initial portfolio restructuring decision has a direct effect in subsequent periods. Portfolio rebalancing decisions at later periods depend on the portfolio composition at that time, the prevailing market conditions at the time, and the perception for subsequent potential movements of the random variables. Conversely, the flexibility for subsequent portfolio rebalancing influences earlier decisions. Thus, a multi-stage model is needed to capture the information and decision dynamics and their interaction.

Practical issues, such as transaction costs, managerial and regulatory requirements should be incorporated in the decision model. Transaction costs do play a role in portfolio management and their effect must be considered. Their omission can lead to high portfolio turnover with a consequent reduction in gains and a potential increase in risk exposure. The impact of transaction costs and the potential benefits of diversification are more properly captured in a multi-period decision model.

The paradigm of multi-stage stochastic programs with recourse is particularly suitable for this problem. Uncertainty in input parameters of stochastic programs is represented by discrete scenarios that depict the joint co-variation of the random variables. In multi-stage problems the progressive evolution of the random variables is expressed in terms of a scenario tree. Scenarios can be generated with various approaches, and they are not restricted to any particular distributional assumption. Asymmetric and fat-tailed distributions that are often observed in practice can be captured by discrete approximations. Indeed, this flexibility in the representation of uncertainty in input parameters is a major advantage of stochastic programs.

Stochastic programs can accommodate different objective functions to capture the decision maker's risk bearing preferences (e.g., utility functions, penalties on shortfalls and other risk measures, etc.) Moreover, they can incorporate managerial and regulatory requirements, especially when such requirements are expressed in terms of linear constraints. Because of their flexibility, stochastic programs have attracted the attention of researchers and practitioners alike and are being increasingly applied to diverse practical problems. Various applications of stochastic programs are documented in the volume edited by Wallace and Ziemba [30].

Financial modeling is a particularly fertile application domain for stochastic programming. Numerous important contributions have been made during the last 20 years. Financial applications of stochastic programming models have twice been among the finalists for the Edelman Prize for best achievement in Management Science in recent years (see, [5,19]). There have been numerous notable contributions of stochastic programming applications to such diverse problems as asset and liability management, portfolio management, insurance, pension funds, credit risk management, etc. Some recent collections contain several representative contributions (e.g., [31,30,28,32]).

We adopt the multistage stochastic programming framework for the international portfolio management problem because of its flexible features that we discussed above. Multi-stage models help decision makers gain useful insights and adopt more effective decisions. They shape decisions based on longer-term potential benefits and avoid myopic reactions to short-term market movements that may prove risky. They determine appropriate dynamic contingency (recourse) decisions under changing economic conditions that are represented by scenario trees. Our model encompasses all the important aspects of the problem and jointly determines the allocation of capital to international markets, the selection of specific asset holdings within each market, and currency hedging decisions with forward contracts. Thus, all interrelated decisions, that are traditionally considered separately, are cast in a unified and flexible framework.

As we indicate in Section 3.2, the returns of several international indices and the movements of exchange rates exhibit asymmetric distributions with fat tails. Consideration of these features is important in our study that aims to devise methods for controlling downside risk. We capture skewness and excess kurtosis in the distributions of the random variables by applying a moment-matching scenario generation procedure. This procedure generates scenarios so that key statistics (specifically, the first four marginal moments and the correlations) of the random variables match specified target values. We estimated the target values for these statistics on the basis of historical data. We verified through exhaustive tests that the scenario sets also satisfied the required no-arbitrage conditions.

We employed the conditional value-at-risk (CVaR) metric in the objective function in order to minimize the expected shortfall beyond the value-at-risk (VaR) of portfolio losses at the end of the horizon. The choice of the objective function was made in light of the observed asymmetries and excess kurtosis of key random variables. CVaR is a coherent risk measure, and is suitable for asymmetric distributions. This objective function enabled the effective control of downside risk exposure. Alternative means can be employed to control downside risk; for example, piecewise-linear convex penalties on shortfall levels [5].

This study extends the work in Topaloglou et al. [25] in several important directions. First, it develops a multi-stage stochastic programming model that allows the solution of dynamic international portfolio management models. Second, it operationalizes the currency hedging decisions by incorporating explicit variables that transparently determine the appropriate level of forward contracts for each currency. Third, it adopts a flexible and effective scenario generation procedure that produces scenario sets that closely approximate the empirical joint distributions of asset returns and exchange rates.

3. Representation of uncertainty

The representation of uncertainty in input parameters of the portfolio optimization model is a critical step in the modelling process. The key random inputs in the international portfolio management problem are the asset prices (or, equivalently, their returns) and the currency exchange rates at the trading dates within the planning horizon. Plausible evolutions of the random parameters during the planning horizon are specified in terms of discrete joint outcomes that are mapped to the nodes of a scenario tree, such as the one shown in Fig. 1.

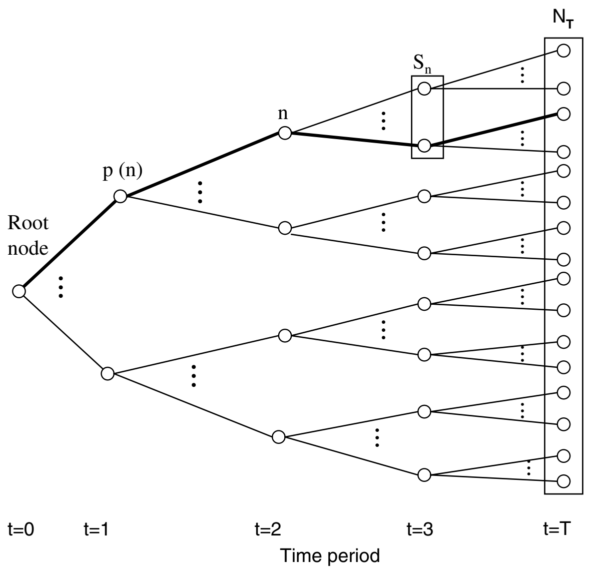

Fig. 1. General form of a scenario tree.

A major advantage of stochastic programs is that they are not restricted to any particular distributional assumption for the random variables. They can accommodate arbitrary discrete distributions that are expressed by means of a scenario tree. Alternative scenario generation procedures for stochastic programs are reviewed in Dupačová et al. [6]. The key decisions in a scenario generation method are the choice of a statistical model and the calibration of its parameters, usually on the basis of market data or subjective expert opinions. The scenario set must conform to financial principles; specifically, it must be free from arbitrage opportunities.

3.1. The scenario tree

The planning horizon is divided to periods $t = 0, 1, \ldots, T$ corresponding to the times at which portfolio rebalancing decisions can be made. The tree has a depth equal to the number of periods (decision stages); we use monthly trading periods. The root node ($n = 0$) corresponds to the initial state at the present time ($t = 0$). All input data associated with the root node are known with certainty. The tree branches out from the root to depict progressive outcomes in the values of the random variables at subsequent periods. The branches emanating from the root reflect the possible outcomes during the first period ($t = 1$). Each postulated outcome is associated with an immediate successor node. Similarly, the branches emanating from any subsequent node represent the discrete, conditional distribution of the random variables during the next time period. Each node reflects a possible state at the corresponding time period, and captures a joint realization of the random variables at that time.

Each scenario distinguishes a sequence of joint realizations of the random variables during the planning horizon. Thus, it has a one-to-one correspondence with a terminal node (leaf) of the tree. The constituent realizations of the random variables at each period are identified by the nodes on the unique path from the root to the leaf node associated with the scenario (such as the highlighted path in Fig. 1). The scenario tree need not be symmetric, and it is rarely binomial as shown in Fig. 1 simply for illustration purposes.

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We use the following notation:

$\boldsymbol{N}$ the set of nodes of the scenario tree;
$n \in \boldsymbol{N}$ a typical node of the scenario tree ($n = 0$ denotes the root node at $t = 0$);
$\boldsymbol{N}_t \subset \boldsymbol{N}$ the set of distinct nodes of the tree at time period $t = 0, 1, \ldots, T$;
$\boldsymbol{N}_T \subset \boldsymbol{N}$ the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios;
$p(n) \in \boldsymbol{N}$ the unique predecessor node of node $n \in \boldsymbol{N} \backslash \{0\}$;
$\boldsymbol{S}_n \subset \boldsymbol{N}$ the set of immediate successor nodes of node $n \in \boldsymbol{N} \backslash \boldsymbol{N}_T$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n;
p_t^n the conditional probability for the outcome associated with the transition from the predecessor node $p(n)$ to node $n \in \boldsymbol{N}$;
p_n the probability of the state associated with node $n \in \boldsymbol{N}$.

The probability, p_n, of a certain node (state) $n \in \boldsymbol{N}$ is determined by multiplying the conditional probabilities of the outcomes on the path from the root to the specific node; i.e., by compounding the conditional probabilities of the constituent outcomes that lead to the specific state. The probabilities of all distinct nodes at any decision stage sum to one (i.e., $\sum_{n \in N_t} p_n = 1, t = 0, 1, \ldots, T$). Also, the probability of a node is equal to the sum of the probabilities of its immediate successor nodes (i.e., $p_n = \sum_{m \in \boldsymbol{S}_n} p_m, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T$).

The scenario tree represents the evolution of the multi-variate random variables over the planning horizon. We define distinct (recourse) variables, and associated constraints, to model the portfolio rebalancing decisions at each intermediate node of the scenario tree (i.e., $\forall n \in \boldsymbol{N} \backslash \boldsymbol{N}_T$). At the leaf nodes, $\boldsymbol{N}_T$, we only compute the corresponding terminal value of the portfolio under the respective scenarios. The size of the resulting multi-stage stochastic program grows substantially with the number of tree nodes. In multi-stage problems attention must be paid to limit the branching factor from each node—and, consequently, the total number of scenarios—in order to keep the size of the optimization program within computationally tractable limits. Scenario reduction techniques have been proposed for this purpose (see, [7,12]).

Stochastic programs must conform to the logical requirement for non-anticipative decisions. That is, scenarios that share common information history (outcomes) up to a particular time period—i.e., have common subpath of the scenario tree up to that period—must yield identical decisions up to that period. The non-anticipativity condition is explicitly enforced in our model as decision variables are defined for each node—instead of each path.

3.2. Scenario generation

We consider portfolios of stock and bond indices denominated in different currencies. Statistical analysis of market data reveals that the random variables (index returns and currency exchange rates) are correlated. Moreover, their historical values do not conform to normal distributions; they exhibit asymmetries and heavy tails. These observations are consistent with the findings of other studies that considered the returns of international financial assets; see, for example, Prakash et al. [20] and references therein. These features should be reflected in the postulated scenario sets that should capture the statistical characteristics of the random variables' empirical distribution. Effectively capturing the observed skewness and excess kurtosis in the distribution of the random variables, as well as their correlations, becomes important. This becomes all the more necessary as we are concerned with controlling the downside risk in the tail of the portfolio's return distribution.

In the empirical tests we consider investments in four markets: United States (US), Great Britain (UK), Germany (GR) and Japan (JP), comprised of the following instruments in each market: a stock index, denoted as `Stk`, and bond indices with three different maturity bands: short—(1–3 years), intermediate—(3–7 years) and long-term (7–10 years), denoted `Bnd1`, `Bnd3` and `Bnd7`, respectively. Thus, a total of 16 assets are considered in each portfolio. The problem is viewed from the perspective of a US investor. Repositioning of investments between markets entails spot currency exchange transactions. Moreover, forward currency exchange contracts—with a term equal to a decision period, i.e., one month—are incorporated in the portfolio optimization model to (partly) hedge the currency risk of foreign investments. Hence, data for the spot and forward exchange rates of the foreign currencies to USD are also needed.

Market values of the stock indices were obtained from the Morgan Stanley Capital International, Inc. database (www.mscidata.com). The values for the

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bond indices and the currency exchange rates were collected from DataStream. All time series have a monthly time-step and cover the period from April 1988 through December 2001. From the data of index prices we computed their corresponding monthly returns (in domestic terms). Similarly, from the observed series of spot exchange rates we computed their corresponding monthly appreciation rates (proportional changes).

The statistics in Table 1 show that both the domestic returns of the indices and the proportional changes of exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean (especially the stock indices). Jarque–Bera tests on these data indicate that the normality hypothesis cannot be accepted for most of them.[1] The normal distribution cannot properly capture the observed joint behavior of the financial time series. Extreme values are encountered more often than predicted by the normal distribution.

The correlations of the random variables over the period 01/1990–12/2001 are shown in Table 2. Observe that correlations of asset returns are much lower across markets than within markets. Hence, international diversification can reduce the total risk of a portfolio.

Given the statistical characteristics of the random variables, we apply the moment-matching procedure of Høyland and Wallace [15] and Høyland et al. [14] to generate sets of scenarios so that key statistics of the random variables match specified target values. Specifically, we match the following statistics: the first four marginal moments (mean, variance, skewness, and kurtosis), as well as the correlations of the monthly asset returns and currency exchange appreciation rates. We estimate the target values to be matched on the basis of historical data.

The user specifies a-priori the desired number of scenarios, thus controlling the size of the resulting portfolio optimization program. The model presented in the next section is dynamic and involves an arbitrary number of decision stages. We generate the scenarios incrementally, one stage at a time.

Table 1
Statistical characteristics of historical monthly data for domestic returns of assets and proportional changes of spot currency exchange rates over the period 01/1990–12/2001

Asset class	Mean (%)	Std. Dev. (%)	Skewness	Kurtosis	Jarque–Bera statistic
Statistical characteristics of monthly domestic returns of assets					
US.Stk	1.211	4.097	−0.413	3.649	6.11
UK.Stk	1.025	4.254	−0.018	3.371	0.61
GRStk	1.167	5.807	−0.561	4.301	13.92
JP.Stk	−0.170	6.881	0.017	3.836	2.45
US.Bnd1	0.567	0.504	−0.006	2.765	0.74
US.Bnd3	0.654	1.118	−0.112	2.626	1.03
US.Bnd7	0.710	1.670	−0.090	2.912	0.30
UK.Bnd1	0.680	0.714	0.995	7.461	150.82
UK.Bnd3	0.741	1.349	0.510	4.631	29.45
UK.Bnd7	0.793	1.880	0.134	3.390	1.53
GR.Bnd1	0.493	0.480	0.288	4.451	31.24
GR.Bnd3	0.552	0.930	−0.255	3.162	4.58
GR.Bnd7	0.583	1.372	−0.663	3.887	22.87
JP.Bnd1	0.283	0.481	0.663	4.708	12.98
JP.Bnd3	0.416	1.121	−0.099	4.401	26.37
JP.Bnd7	0.503	1.678	−0.519	5.434	47.62
Exchange rate					
Statistical characteristics of monthly proportional spot exchange rate changes					
US to UK	−0.116	2.894	−0.755	5.672	102.39
US to GR	−0.124	3.091	−0.215	3.399	5.69
US to JP	0.030	3.615	0.942	6.213	105.27

From observed market data we estimate the target statistics of the random variables for a monthly time period—equal to a decision period of the model. We apply the moment-matching procedure to generate the joint realizations (a discrete multivariate distribution) of the random variables for the first period ($t = 1$)—i.e., for the branches emanating from the root node. Similarly, for each subsequent node we generate a number of joint outcomes of the random variables for the next stage that match the same statistical properties. We generate equiprobable outcomes from each node; this is not an absolute requirement as the moment-matching method can also generate outcomes with differing probabilities, at the expense of higher computational effort.

This scenario generation procedure does not account for possible intertemporal dependencies of the random variables (e.g., mean reversion, volatility clustering, etc.). This should be the subject of further research. Consideration of such effects is not critical for problems with fairly short planning horizons (<1 year) and monthly decision periods,

[1] The Jarque–Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.99 and 9.21, respectively. The normality hypothesis is rejected when the Jarque–Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

Table 2
Correlations of monthly asset returns and proportional changes of spot exchange rates computed from historical market data over the period 01/1990–12/2001

	US.Stk	UK.Stk	GR.Stk	JP.Stk	US.Bnd1	US.Bnd3	US.Bnd7	UK.Bnd1	UK.Bnd3	UK.Bnd7	GR.Bnd1	GR.Bnd3	GR.Bnd7	JP.Bnd1	JP.Bnd3	JP.Bnd7	US to UK	US to GR	US to JP
US.Stk	1																		
UK.Stk	0.69	1																	
GR.Stk	0.58	0.60	1																
JP.Stk	0.40	0.39	0.37	1															
US.Bnd1	0.18	0.07	−0.08	−0.02	1														
US.Bnd3	0.21	0.09	−0.05	0.01	0.96	1													
US.Bnd7	0.24	0.11	−0.02	0.01	0.89	0.97	1												
UK.Bnd1	0.13	0.40	0.03	0.00	0.38	0.37	0.35	1											
UK.Bnd3	0.21	0.48	0.11	0.03	0.41	0.44	0.44	0.94	1										
UK.Bnd7	0.26	0.50	0.16	0.05	0.42	0.47	0.49	0.84	0.97	1									
GR.Bnd1	0.05	0.20	0.03	−0.03	0.36	0.39	0.38	0.59	0.59	0.56	1								
GR.Bnd3	0.13	0.25	0.12	0.01	0.42	0.48	0.50	0.54	0.63	0.64	0.92	1							
GR.Bnd7	0.17	0.27	0.20	0.02	0.45	0.53	0.56	0.48	0.61	0.67	0.77	0.94	1						
JP.Bnd1	0.10	0.12	0.02	0.05	0.21	0.21	0.20	0.33	0.31	0.29	0.40	0.36	0.30	1					
JP.Bnd3	0.11	0.09	0.01	0.05	0.22	0.25	0.26	0.24	0.27	0.27	0.37	0.38	0.33	0.91	1				
JP.Bnd7	0.12	0.10	0.02	0.05	0.25	0.29	0.30	0.22	0.26	0.27	0.34	0.38	0.34	0.78	0.94	1			
US to UK	−0.02	−0.20	−0.10	0.10	0.18	0.17	0.13	−0.17	−0.16	−0.10	−0.09	−0.08	−0.05	0.05	0.08	0.07	1		
US to GR	−0.09	−0.22	−0.22	−0.07	0.25	0.25	0.19	−0.09	−0.10	−0.09	0.03	−0.02	0.00	0.10	0.12	0.13	0.71	1	
US to JP	0.08	0.05	−0.15	0.04	0.11	0.13	0.08	0.12	0.11	0.10	0.14	0.08	0.04	0.07	0.06	0.07	0.36	0.46	1

as we consider in this study. It becomes more important for problems with short decision stages (e.g., daily) or very long planning horizons.

At the root node we know with certainty the market asset prices, the spot exchange rates, and the forward rates for the next period (i.e., one month). Using the projected asset returns and currency appreciation rates for the first period, we compute the joint asset prices and spot currency exchange rates for each node at the first stage ($t = 1$). Similarly, knowing the asset prices and spot exchange rates at a subsequent node of the scenario tree, as well as the projected asset returns and currency appreciation rates for the branches emanating from that node, we determine the asset prices and spot exchange rates at the end of the stage for each immediate successor node. For each node we also specify forward exchange rates for one-month forward currency transactions. These forward rates are set equal to the expected value of the respective spot exchange rates at the end of the period—i.e., by taking the expectation of the spot exchange rates over the immediate successor nodes. This is done so as to ensure that the no-arbitrage conditions are met for forward currency exchanges.

The multi-stage stochastic program can be extended to include longer-term forward currency exchanges (i.e., spanning multiple decision stages). Additional variables would be needed to represent these forward currency contracts, coupling the cash balance conditions in non-successive decision periods. The longer-term forward exchange rates would have to be carefully modelled consistently with the projected spot exchange rates and the shorter-term forward exchange rates. We leave such an extension for a subsequent study.

In a critique of the moment-matching method, Klaasen [17] argued that it is possible to match the moments of random asset returns with a coarse discrete distribution that violates the no-arbitrage principle; he provided a small example with a triangular distribution for a univariate random variable. We tested exhaustively all the scenario sets used in our numerical experiments and empirically verified that the no-arbitrage conditions were always satisfied.

The stochastic programming model in the next section is not restricted to the moment-matching scenario generation procedure. A user can employ an alternative approach—such as the method of Hochreiter and Pflug [13]—that he finds preferable to generate the scenarios of asset prices and exchange rates. Of course, the scenarios must effectively reflect the intended distributions of the random variables and must satisfy fundamental financial principles (e.g., must be arbitrage free). The moment-matching procedure that we employed in the numerical validation tests meets these requirements.

4. International portfolio management model

The model determines a sequence of investment decisions at discrete points in time (monthly intervals). The portfolio manager starts with an initial portfolio and with a set of postulated outcomes regarding future states of the economy. This information is incorporated into a portfolio restructuring decision. At the beginning of the next period the manager has at hand a seasoned portfolio. The composition of the portfolio depends on the transactions at the previous decision point; its value depends on the outcome of asset returns and exchange rates in the interim period. For every projected realization of the random variables we end up at a different state at the end of the period—associated with a descendant node. Another portfolio restructuring decision is made at that node based on the portfolio at hand, and taking into account the subsequent possible outcomes of the random variables.

The problem is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. Without loss of generality, no *direct* exchanges between foreign currencies are executed—either in the spot or in the forward market—in order to simplify the formulation of the model and to reduce its data needs, as well as the number of decision variables. All currency exchanges are executed with respect to the base currency. To reposition his investments from one market (currency) to another, the investor must first convert to base currency the proceeds of foreign asset sales in the market in which he reduces his presence and then purchase the foreign currency in which he wishes to increase his investments. The spot exchange rates of foreign currencies to USD applicable at the decision state (node) are used for the currency exchange transactions. At the end of the holding period we compute the state-dependent value of asset holdings using their projected prices at the respective tree node. The USD-equivalent value is determined by applying the estimate of the appropriate spot exchange rate to USD at the same node.

The portfolio is exposed to market risk in the domestic and foreign markets, as well as to currency risk for the foreign investments. To (partly) hedge currency risk at each decision node, the investor may enter into forward currency exchange contracts. We allow forward currency exchanges with a term (maturity) of one period, i.e., one month. The key decision is the amount of the contract. The actual exchange takes place at the end of the period using the forward rate that is specified at the time the decision is made (i.e., at the beginning of the period). The forward currency exchange contracts constitute hedging decisions to mitigate the respective exposure of foreign investments to currency risk. The optimal selection of currency forward contracts is integrally incorporated in the portfolio management model.

The notion of "full hedging" that is often referred to in the literature is not attainable exactly with forward exchanges. This is because the value of foreign asset holdings at the end of any period is not known with certainty at the beginning of the period, i.e., at the time that a forward contract is decided. This value depends on the realized asset returns during the interim period. Yet, the amount of the forward contract must be decided before the realized asset returns are observed. Hence, forward currency exchanges cannot hedge fully the currency risk exposure of foreign investments.

The stochastic programming model takes as input the representation of uncertainty as captured by the scenario tree. The model minimizes the conditional value-at-risk (CVaR) of portfolio losses at the end of the planning horizon. That is, it minimizes the expected value in the tail (beyond a specific percentile, α) of the portfolio losses at the end of the planning horizon. CVaR is the conditional expectation of excess shortfall beyond the value-at-risk of losses (VaR) at the respective percentile level. Unlike VaR, CVaR is a coherent risk measure in the sense of Artzner et al. [1], and is receiving increasing attention in financial applications. The incorporation of the CVaR function in the stochastic programming model is done along the lines of Rockafellar and Uryasev [21] (see, also, Topaloglou et al. [25]).

We define the following additional notation:

Sets:

$\boldsymbol{C}_0$ set of markets (synonymously, countries, currencies);

$\ell \in \boldsymbol{C}_0$ index of investor's base (reference) currency (in our case USD);

$\boldsymbol{C}$ set of foreign markets; $C = \boldsymbol{C}_0 \setminus \{\ell\}$;

$\boldsymbol{I}_c$ set of available investments (asset classes) in market $c \in \boldsymbol{C}_0$.

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User-specified parameters:

α critical percentile for VaR and CVaR;

μ target (minimum) expected portfolio return over the planning horizon.

Deterministic input data:

h_c^0 initially available cash in currency $c \in \boldsymbol{C}_0$ (deficit if negative);

b_{ic} initial position (in number of units of face value) in asset $i \in \boldsymbol{I}_c$ of market $c \in \boldsymbol{C}_0$;

P_{ic}^0 current market price of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$ (in units of domestic currency c);

e_c^0 current spot exchange rate of currency $c \in \boldsymbol{C}_0$;

φ_c^0 currently quoted one-month forward exchange rate for foreign currency $c \in \boldsymbol{C}$;

γ_{ic} proportional transaction cost for sales or purchases of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$;

V_0 total value of initial portfolio (i.e., initial wealth, in units of reference currency): $V_0 = \sum_{c \in \boldsymbol{C}_0}(h_c^0 + \sum_{i \in \boldsymbol{I}_c} b_{ic}P_{ic}^0)e_c^0$.

Scenario-dependent data:

p_n probability of node $n \in \boldsymbol{N}$—we generate symmetric trees with equiprobable scenarios, thus $p_n = \frac{1}{|\boldsymbol{N}_t|}, \ \forall n \in \boldsymbol{N}_t, t = 0, 1, \ldots, T$;

h_c^n exogenous cash inflow (liability if −ve) of currency $c \in \boldsymbol{C}_0$ at node $n \in \boldsymbol{N}$;

P_{ic}^n price of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$ at node $n \in \boldsymbol{N}$ (in units of domestic currency c);

e_c^n spot exchange rate for foreign currency $c \in \boldsymbol{C}$ at node $n \in \boldsymbol{N}$;

φ_c^n one-month forward exchange rate for foreign currency $c \in \boldsymbol{C}$ at node $n \in \boldsymbol{N}$;

Φ_c^n upper bound on a forward contract in currency $c \in \boldsymbol{C}_0$ at node $n \in \boldsymbol{N}$ (in units of the base currency).

Decision variables:

Portfolio rebalancing decisions are made at nodes of the scenario tree except for the leaves; thus separate variables are defined for each node $n \in \boldsymbol{N} \backslash \boldsymbol{N}_T$ to reflect the decisions made at the respective node:

x_{ic}^n units of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$ purchased;

v_{ic}^n units of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$ sold;

w_{ic}^n units of asset $i \in \boldsymbol{I}_c, c \in \boldsymbol{C}_0$ held in the portfolio after revision;

g_c^n expenditure of base currency, ℓ, for purchase of currency $c \in \boldsymbol{C}$ in the spot market;

q_c^n revenue in base currency, ℓ, from sale of foreign currency $c \in \boldsymbol{C}$ in the spot market;

f_c^n amount of base currency, ℓ, collected from sale of currency $c \in \boldsymbol{C}$ in the forward market (i.e., amount of forward contract, in units of the base currency). A negative value indicates a purchase of the foreign currency in the forward market. These decisions are made at node $n \in \boldsymbol{N} \backslash \boldsymbol{N}_T$, but the actual transaction is executed at the end of the respective period, i.e., at the successor nodes $\boldsymbol{S}_n$.

Variables at the leaf nodes $n \in \boldsymbol{N}_T$:

V_n final value of the portfolio held at the end of the planning horizon (in units of the base currency, ℓ);

R_n return of the portfolio over the planning horizon;

L_n portfolio loss over the planning horizon.

Auxiliary variables:

y_n portfolio shortfall in excess of VaR at leaf node $n \in \boldsymbol{N}_T$;

z variable in definition of CVaR—equals to VaR at the optimal solution.

All exchange rates are expressed as the equivalent amount of the base currency, ℓ, for one unit of the foreign currency. Obviously, $e_\ell^n = \varphi_\ell^n = 1, \ \forall n \in \boldsymbol{N}$. Also $g_\ell^n = q_\ell^n = f_\ell^n = 0, \forall n \in \boldsymbol{N}$; these variables are omitted in the actual implementation.

We formulate the multi-stage stochastic programming model for the dynamic international portfolio management problem as follows:

$$\min \quad \zeta = z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N}_T} p_n y_n, \tag{1a}$$

$$\text{s.t.} \quad w_{ic}^0 = b_{ic} + x_{ic}^0 - v_{ic}^0, \quad \forall c \in \boldsymbol{C}_0, \ \forall i \in \boldsymbol{I}_c, \tag{1b}$$

$$w_{ic}^n = w_{ic}^{p(n)} + x_{ic}^n - v_{ic}^n, \quad \forall c \in \boldsymbol{C}_0, \ \forall i \in \boldsymbol{I}_c, \quad \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N}_T \cup 0\}, \tag{1c}$$

$$h_\ell^0 + \sum_{i \in \boldsymbol{I}_\ell} v_{i\ell}^0 P_{i\ell}^0 (1 - \gamma_{ic}) + \sum_{c \in \boldsymbol{C}} q_c^0 = \sum_{i \in \boldsymbol{I}_\ell} x_{i\ell}^0 P_{i\ell}^0 (1 + \gamma_{ic}) + \sum_{c \in \boldsymbol{C}} g_c^0, \tag{1d}$$

$$h_c^0 + \sum_{i \in \boldsymbol{I}_c} v_{ic}^0 P_{ic}^0 (1 - \gamma_{ic}) + \frac{g_c^0}{e_c^0} = \sum_{i \in \boldsymbol{I}_c} x_{ic}^0 P_{ic}^0 (1 + \gamma_{ic}) + \frac{q_c^0}{e_c^0}, \quad \forall c \in \boldsymbol{C}, \tag{1e}$$

$$h_\ell^n + \sum_{i \in \boldsymbol{I}_\ell} v_{i\ell}^n P_{i\ell}^n (1 - \gamma_{ic}) + \sum_{c \in \boldsymbol{C}} (q_c^n + f_c^{p(n)}) = \sum_{i \in \boldsymbol{I}_\ell} x_{i\ell}^n P_{i\ell}^n (1 + \gamma_{ic}) + \sum_{c \in \boldsymbol{C}} g_c^n, \quad \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N}_T \cup 0\}, \tag{1f}$$

$$h_c^n + \sum_{i \in \boldsymbol{I}_c} v_{ic}^n P_{ic}^n (1 - \gamma_{ic}) + \frac{g_c^n}{e_c^n} = \sum_{i \in \boldsymbol{I}_c} x_{ic}^n P_{ic}^n (1 + \gamma_{ic}) + \frac{q_c^n}{e_c^n} + \frac{f_c^{p(n)}}{\varphi_c^{p(n)}}, \quad \forall c \in \boldsymbol{C}, \ \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N}_T \cup 0\}, \tag{1g}$$

$$R_n = \frac{V_n}{V_0} - 1, \quad \forall n \in \boldsymbol{N}_T, \tag{1h}$$

$$L_n = -R_n, \quad \forall n \in \boldsymbol{N}_T, \tag{1i}$$

$$y_n \geqslant L_n - z, \quad \forall n \in \boldsymbol{N}_T, \tag{1j}$$

$$y_n \geqslant 0, \quad \forall n \in \boldsymbol{N}_T, \tag{1k}$$

$$x_{ic}^n \geqslant 0, \quad w_{ic}^n \geqslant 0, \ \forall c \in \boldsymbol{C}_0 \ , \forall i \in \boldsymbol{I}_c, \quad \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{1l}$$

$$0 \leqslant v_{ic}^0 \leqslant b_{ic}, \quad \forall c \in \boldsymbol{C}_0 \ , \forall i \in \boldsymbol{I}_c, \tag{1m}$$

$$0 \leqslant v_{ic}^n \leqslant w_{ic}^{p(n)}, \quad \forall c \in \boldsymbol{C}_0, \ \forall i \in \boldsymbol{I}_c, \quad \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N}_T \cup 0\}, \tag{1n}$$

$$g_c^n \geqslant 0, q_c^n \geqslant 0, \quad \forall c \in \boldsymbol{C}, \ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{1o}$$

$$f_c^n \leqslant \Phi_c^n, \quad \forall c \in \boldsymbol{C}, \ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{1p}$$

$$\sum_{n \in \boldsymbol{N}_T} p_n R_n \geqslant \mu, \tag{1q}$$

where

$$V_n = h_\ell^n + \sum_{i \in \boldsymbol{I}_\ell} w_{i\ell}^{p(n)} P_{i\ell}^n + \sum_{c \in \boldsymbol{C}} \left\{ f_c^{p(n)} + e_c^n \left(h_c^n + \sum_{i \in \boldsymbol{I}_c} w_{ic}^{p(n)} P_{ic}^n - \frac{f_c^{p(n)}}{\varphi_c^{p(n)}} \right) \right\}, \quad \forall n \in \boldsymbol{N}_T. \tag{2}$$

The model is a multi-stage stochastic linear program with recourse. Decisions at any node explicitly depend on the corresponding postulated outcome for the random variables and directly impact on decisions at descendant nodes. The model minimizes the conditional value-at-risk of the portfolio losses at the end of the horizon, while it parametrically sets a minimum target, μ, for the portfolio's expected return over the planning horizon. Expectations are computed over the set of terminal states (leaf nodes). The objective value, ζ, measures the CVaR of portfolio losses at the end of the planning horizon, while the corresponding VaR of portfolio losses (at percentile α) is captured by the variable z; see [21].

Eqs. (1b) and (1c) are the balance conditions for each asset, in each market, at the first and subsequent decision stages, respectively. Eqs. (1d) and (1e) impose the cash balance conditions at the first stage; the former for the base currency ℓ and the latter for the foreign currencies $c \in \boldsymbol{C}$. In each case, availability of funds stems from initially available reserves, revenues from asset sales, and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of funds cover the expenditure for the purchase of assets and outgoing currency exchanges in the spot market. No holdings in cash are allowed after portfolio restructuring. Hence, we do not need to explicitly model the interest rates in each market. Similarly, Eqs. (1f) and (1g) impose the state-dependent cash balance conditions in every currency at subsequent decision stages. These equations additionally account for forward currency exchange contracts that are decided at the predecessor node.

Eqs. (1h) and (1i) define the portfolio return and the portfolio loss at leaf node $n \in \boldsymbol{N}_T$, respectively. The constraints (1j) and (1k) define the portfolio's excess shortfall, $y_n = \max[0, L_n - z]$, over the planning horizon for each scenario. The constraints in (1l), (1m) and (1n) disallow short positions in the assets and ensure that sales cannot exceed the quantities in the portfolio at hand. Constraints (1o) ensure that currency transactions in the spot market are nonnegative. Constraint (1q) sets a minimum target μ on the portfolio's expected return over the planning horizon. By parametrically changing this target level we generate solutions that trade off expected return against total risk exposure (as captured in the objective function).

The final value of the portfolio at leaf node $n \in \boldsymbol{N}_T$ is computed in (2). This equation takes into consideration exogenously available cash and revenues from the liquidation of asset holdings in each market. The contribution of foreign investments to the total value of the portfolio accounts for the settlement of any outstanding forward contracts. The residual amount is valued in terms of the base currency by using the projected spot exchange rates.

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The constraints in (1p) impose limits on the currency forward contracts at every decision state. We consider alternative hedging policies by appropriately setting these bounds:

$$\text{(i) } \Phi_c^n = \sum_{i \in \boldsymbol{I}_c} e_c^n (w_{ic}^n P_{ic}^n), \quad \forall c \in \boldsymbol{C},\ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{3}$$

$$\text{(ii) } \Phi_c^n = \sum_{m \in \boldsymbol{S_n}} p_m e_c^m \Big(\sum_{i \in \boldsymbol{I}_c} w_{ic}^n P_{ic}^m\Big), \quad \forall c \in \boldsymbol{C},\ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{4}$$

$$\text{(iii) } \Phi_c^n = \infty, \quad \forall c \in \boldsymbol{C},\ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T, \tag{5}$$

$$\text{(iv) } \Phi_c^n = 0, \quad \forall c \in \boldsymbol{C},\ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T. \tag{6}$$

The first case permits a forward contract to cover up to the current value of total asset holdings in the respective currency, with the foreign holdings valued at the time that the forward contract is decided. This alternative ignores the fact that the value of the foreign asset holdings will change due to the uncertain returns and exchange rates during the period. In the second case, the level of a forward contract is bounded by the expected value (in units of the base currency) of the respective foreign asset holdings in the portfolio at the end of the decision period—thus, the expectation is taken over the outcomes at the successor nodes—so as to reflect the expected exposure of the foreign asset positions. In the third case, no restriction is explicitly imposed on the level of forward contracts, which are then treated simply as alternative investment opportunities. In this case, forward positions are allowed regardless of the value of asset holdings in the respective currency. The last case sets the currency forward contracts identically equal to zero (in fact, we eliminate from the model the variables f_c^n in this case). This case corresponds to totally unhedged international investments.

The proportional transaction costs γ_{ic} for the assets create proportional bid-ask spreads $2\gamma_{ic}P_{ic}^n, \forall c \in \boldsymbol{C}_0, \forall i \in \boldsymbol{I}_c, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N}_T$. The effective purchase price for an asset in a cash balance equation for node $n \in \boldsymbol{N} \setminus \boldsymbol{N}_T$ is $P_{ic}^n(1+\gamma_{ic})$, while the effective sale price for the same asset is $P_{ic}^n(1-\gamma_{ic})$. As defined in the formulation, the model permits a different transaction cost for each asset. In the computational tests we used a constant proportional transaction cost $\gamma_{ic} = 0.05\%$ for all assets (i.e., $\forall c \in \boldsymbol{C}_0,\ \forall i \in \boldsymbol{I}_c$). For currency exchanges (both spot and forward) a transaction cost 0.01% was incorporated in the respective exchange rates.

Starting with an initial portfolio, the multi-stage portfolio optimization model determines optimal decisions for the contingencies of the scenario tree. The decisions at each node of the tree specify not only the allocation of funds across markets but also the specific asset holdings in each market. Moreover, currency forward contracts are determined so as to (partly) hedge the currency risk exposure of the foreign investments during the holding period (i.e., until the next portfolio rebalancing decision). Thus, a number of decisions that are usually treated separately are cast here in a unified framework. Simple variants of the model can reflect alternative policies for hedging currency risks with the use of currency forward contracts. The model provides a testbed so as to analyze empirically its effectiveness in managing international portfolios of stock and bond indices, and to compare the performance of risk hedging alternatives.

5. Empirical results

We implemented the multistage stochastic programming model in the General Algebraic Modeling System (GAMS) [4]. We solved single-stage and two-stage instances of the model. The aims of the numerical experiments are: (i) to investigate the efficacy of the stochastic programming model as a practical decision support tool for managing international investment portfolios, (ii) to test alternative risk hedging strategies so as to assess their relative performance in controlling currency risk, and (iii) to contrast the performance of the two-stage stochastic programming model with that of its single-stage counterpart.

We examine the performance of various decision strategies in static as well as in dynamic tests to identify the most promising tactics. In static tests we compare the risk-return profiles (efficient frontiers) generated with appropriate variants of the model at a certain point in time. The optimization models were run with alternative bounds on currency forward contracts as specified in Eqs. (3)–(6). The static tests considered portfolio selection problems. The initial portfolio involved only a cash endowment in the base currency—which the models apportioned optimally to the available assets. The models were repeatedly run for various levels of minimum expected return, μ. The solutions trace the corresponding efficient frontiers of expected portfolio return vs. the CVaR risk metric of portfolio losses (at the $\alpha = 95\%$ percentile) over the planning horizon. The efficient frontiers are determined in-sample; that is, with respect to the postulated scenarios.

The static tests yield useful insights regarding the potential of the various decision strategies with respect to the postulated distributions (scenarios) at a certain point in time. However, this potential does not necessarily materialize in practice. We additionally ran dynamic tests to assess the performance of the models in backtesting simulations. The models were run, on a rolling horizon basis, at each successive month in the period 04/1998–11/2001 (i.e., for a total of 43 months). Starting with an initial cash endowment in the base currency in April 1998, each model was executed to decide the initial portfolio composition. The statistics of the random variables, computed from their observed market values during the previous 10 years, were matched in generating the scenarios. Each model was solved and the first-stage decisions were recorded. The clock then advanced one month. The realized return of the optimal portfolio was determined on the basis of the revealed market prices of the assets and the exchange rates. Any outstanding currency forward contracts were settled and the resulting cash positions in each currency were updated accordingly. A new set of scenarios was then generated by matching the statistics of the random variables to their estimates from their market values during the previous ten years. With the new scenarios as input, and using the portfolio composition and cash positions resulting from the previous decisions as a starting point, the model was solved again. The process was repeated for each successive month and the ex post realized returns were recorded. Thus, the backtesting simulations demonstrate the actual returns that would have been realized had the decisions of the models been implemented during the simulation period 04/1998–11/2001.

5.1. Assessment of hedging strategies

Fig. 2 presents the efficient frontiers for alternative allowable levels of currency forward positions — controlled by the respective bounds in (3)–(6). This figure shows the tradeoffs between the expected return and the `CVaR` measure of losses during a two-month planning horizon for optimal portfolios of the two-stage stochastic program. The two-stage model used 15,000 scenarios composed of 150 joint realizations of the random variables in the first stage, each followed by a set of 100 further outcomes in the second stage. These tests considered portfolio selection problems in August 2001. The

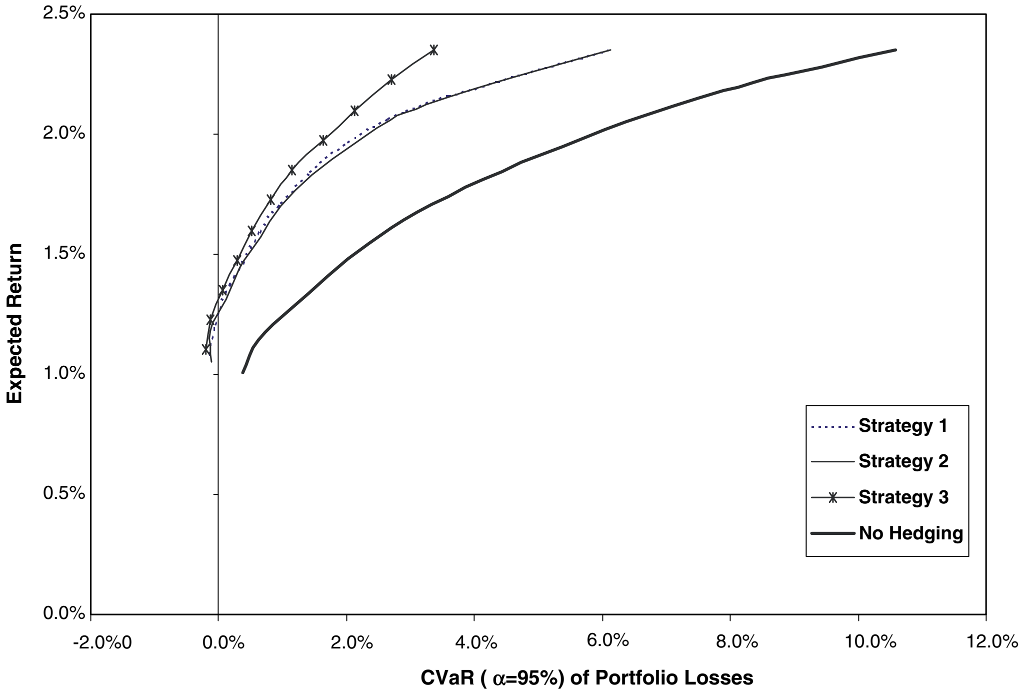

Fig. 2. Risk-return efficient frontiers with two-stage models for alternative hedging strategies (August 2001).

scenarios were generated so that the statistical properties of the random variables in each stage matched their empirical values for the 10 preceding years. The results obtained at this time are typical of the model's observed behavior at other periods.

The efficient frontier of the optimal unhedged portfolios—i.e., without currency forward contracts—is clearly dominated by the efficient frontiers of the optimal, selectively hedged portfolios. Incorporating decisions for currency forward positions in the portfolio management model improves the risk-return profile of the resulting optimal portfolios as indicated by the shift of the efficient frontier to the left. That is, for any value of target expected return, the optimal hedged portfolios exhibit a lower level of risk. This potential benefit of risk reduction is increasing for more aggressive targets of expected portfolio return.

The three strategies that permit currency forward contracts reflect alternative views of the selective hedging approach, as they allow the hedge ratios to be different—and indeed they come out to be different—across currencies. The third strategy exhibits the dominating risk-return profile, as it allows unrestricted use of currency forward positions. The other two strategies restrict the use of currency forward contracts so as to (partly) cover the exposure in foreign currency investments. The first strategy (Eq. (3)) limits the levels of currency forward contracts to the respective current values of foreign investments in the revised portfolio, while the second strategy (Eq. (4)) bounds the levels of such contracts to the expected value of the respective foreign asset holdings at the end of the decision period. These two strategies exhibited almost indistinguishable risk-return profiles.

We applied the same hedging tactics in backtesting experiments to investigate whether their potential in the static tests actually materializes in practice. Fig. 3 contrasts the ex post realized returns of the different hedging policies over backtesting simulations covering the period 04/1998–11/2001. The results were generated by successively applying the respective two-stage models in each month of the simulation period and implementing each time the first-stage optimal decisions of the model. Each instance of the model again used 15,000 scenarios (150 outcomes for the first stage, each associated with 100 further outcomes for the second period) that were generated as described above.

The first graph in Fig. 3 presents the results for the minimum risk case—i.e., when the models simply minimize the CVaR risk measure at the end of the planning horizon without imposing a target on expected portfolio return. The results in the second graph were generated when a target $\mu = 2\%$ for expected return over the two-month horizon was imposed at each instance of the two-stage portfolio optimization model (i.e., for an aggressive investor).

In the minimum risk case, all three selective hedging strategies resulted in essentially the same performance. Only the most liberal (third) strategy fell a little behind in the fall of 1998, but traced closely the performance of the other two strategies in all other periods. The performance of optimal unhedged portfolios was not very different either in this case. Optimal hedged portfolios exhibited only a slight advantage in comparison to the optimal unhedged portfolios, in the minimum risk case, as they demonstrated slightly more stable return paths. The optimal portfolios were positioned almost exclusively in short-term government bond indices throughout these simulations. The models selected diversified portfolios of short-term international bond indices that weathered the storm of the September 11, 2001 crisis unscathed, and actually generated profits during that period. That crisis affected primarily the stock markets for a short period and had no material impact on the bond markets, especially the international bond markets.

The differences in the performance of the alternative hedging strategies are more pronounced when we use a more aggressive target for expected return, as shown in the second graph of Fig. 3. In this case all three selective hedging strategies demonstrate material benefits from the reduction of currency risk through the use of currency forward positions for hedging purposes. Their realized return paths over the simulation period are discernibly more stable than the corresponding path of the optimal unhedged portfolios. The international hedged portfolios were affected much less than the undhedged portfolios during market downturns (e.g., 08/1998, 01–02/2000, 04/2001, 08–10/2001).

Again, the first two strategies demonstrated very similar ex post performance; with the second strategy being a very slight favorite. The third strategy—with unrestricted positions in currency forwards—lagged a bit behind, particularly in periods of down markets. In these simulations the models selected portfolios that varied more substantially

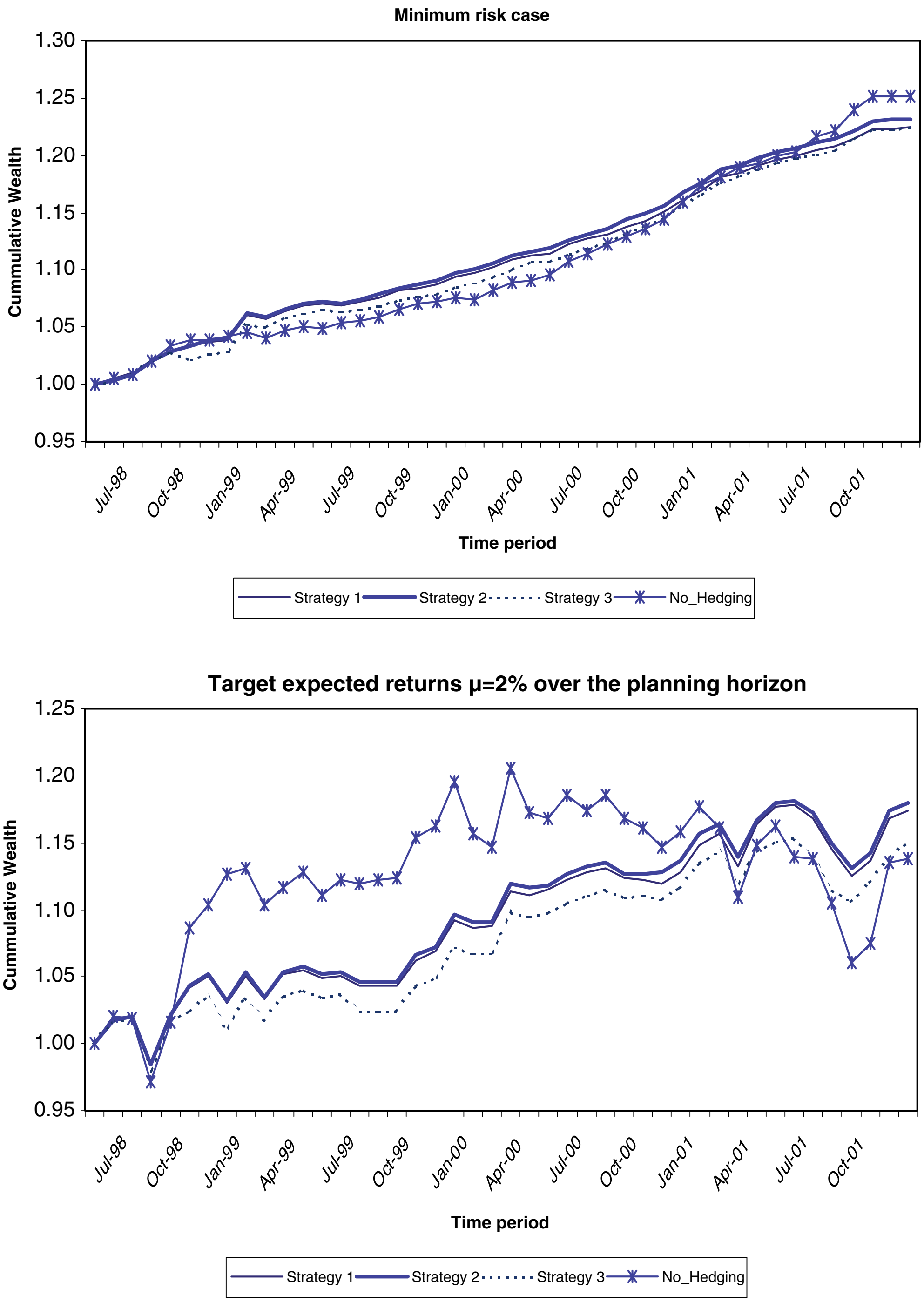

Fig. 3. Ex-post realized performance of alternative hedging strategies in two-stage models. The first graph corresponds to the minimum risk case, while for the second graph the target expected return at the horizon is $\mu = 2\%$.

over time, in comparison to the runs for the minimum risk case, in their attempt to meet the high expected return target. The optimal portfolios also involved sizable positions in the US stock index for most of the simulation period, and thus did not avoid the effects of the crisis in September 2001—even if more mildly than the unhedged portfolios. The strategies with controlled currency hedging consistently attained more stable (less volatile) return paths compared to the returns of unhedged portfolios and also had lower loses in times of severe market downturns.

The results of the static and the dynamic tests show that benefits can be gained, in terms of risk reduction, by internalizing decisions for currency forward positions within the portfolio management models. Regardless of minor details on how permissible currency forward contracts may be controlled in the portfolio optimization models, we observe that the controlled use of forward contracts has a positive impact on reducing risk. The stochastic programming models prove to be useful and practical decision support tools for international portfolio management. They provide a flexible framework for incorporating alternative risk hedging strategies in a dynamic decision setting.

We adopted the second strategy for the tests that are presented below as it exhibited more effective performance than the other hedging alternatives. Recall that in this strategy the allowable positions in one-month currency forwards are bounded by the expected value of asset holdings in the respective currency during the same term.

5.2. Comparison of single- and two-stage models

We now turn to a comparative assessment of single- and two-stage variants of the stochastic programming models. We first examine the performance of the models in static tests on August 2001. The 15,000 scenarios generated for the previous static tests were used as input in these tests as well. We set up and solved two-stage and single-stage instances of the portfolio selection model. The single-stage model had the same horizon (two months) and used the same scenarios as the two-stage model. Thus, the two models used exactly the same information content in terms of the outcomes of the random variables (scenarios), and optimized the same risk measure (CVaR of portfolio losses at the end of the two-month horizon), starting with the same initial portfolio—a cash endowment in the base currency only. The only fundamental difference was that, unlike the single-stage model, the two-stage model allowed portfolio rebalancing decisions during the interim month.

The resulting risk-return efficient frontiers of the two models are shown in Fig. 4. The two-stage model exhibits a dominating risk-return profile over the common two-month horizon of the two models;

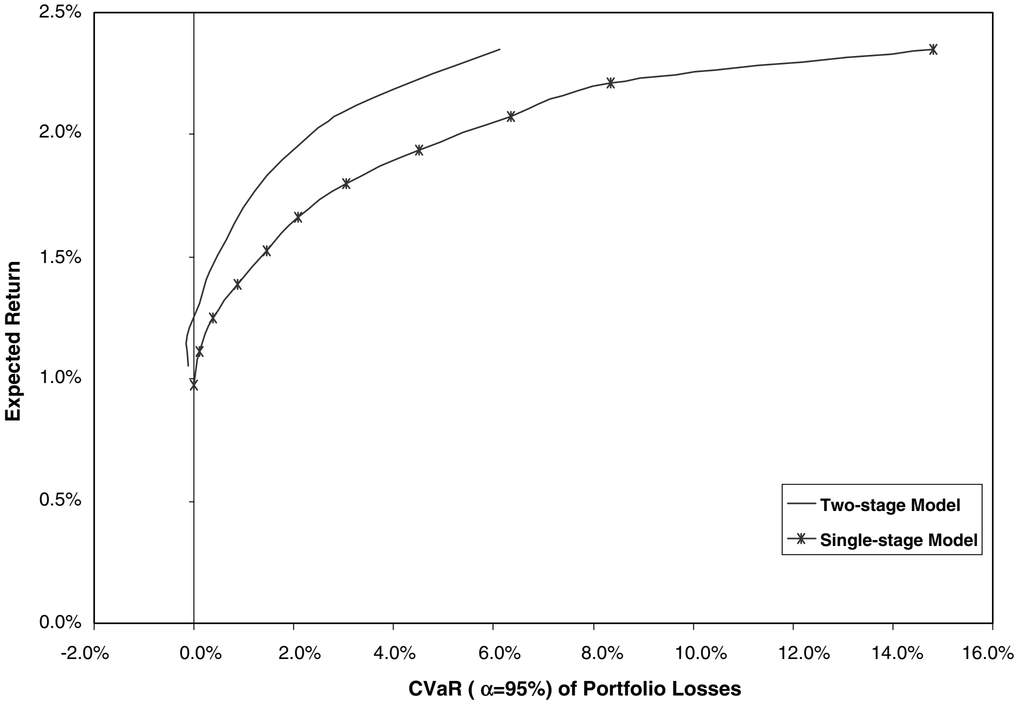

Fig. 4. Efficient frontiers for single- and two-stage models (August 2001).

for any expected return target, the two-stage model yields optimal solutions with lower risk than the solutions of the respective single-stage model. These results confirm the intuition that when the two models have the same information content and the same horizon, then the two-stage model should produce superior results owing to its additional flexibility to incorporate rebalancing decisions at an intermediate stage.

The intermediate decisions permit the reconfiguration of the portfolio during the planning horizon for each descendant node of the root in the scenario tree, i.e., in response to each specific outcome of the first stage. The same principle applies to extensions of the model with additional decision stages, as long as the postulated outcomes (scenarios) remain the same over the planning horizon.

At the minimum-risk end of the frontier the benefit of the two-stage model over the single-stage model seems marginal. This is because at the minimum risk case both models exhibit a "flight to safety", i.e., they select very similar conservative portfolios composed of short-term international bond indices. However, the potential benefits for risk reduction by adopting the two-stage model in comparison to its single-stage counterpart increase for increasingly aggressive (higher) targets of expected return that dictate the selection of riskier portfolios. In these cases, the flexibility of an interim readjustment of the portfolio during the planning horizon in response to changing economic conditions carries a higher incremental value.

The results indicate that for a given representation of uncertainty, and a specific horizon, it is preferable to allow portfolio rebalancing in as many stages as captured in the scenario tree rather than to aggregate decision stages. Of course, this modelling choice has significant implications on the size and computational complexity of the resulting stochastic programs.

Finally, we contrast the performance of the single- and two-stage models in dynamic, backtesting experiments with real market data. Again, the models were set up and executed repeatedly at successive time periods according to the procedure we have explained earlier. From each instance of the models we kept and implemented only the optimal decisions for the first stage. The single-stage model has a horizon of one month, while the two-stage model has a horizon of two months, partitioned into two monthly decision stages. The scenarios were generated on a rolling horizon basis; at each month the scenarios were generated on the basis of the historical data during the prior 10 years.

In order to assess the effect of adding information (i.e., additional outcomes in the scenario set) we experimented with the following variants of the models:

- a two-stage model that used 15,000 scenarios (150×100) generated as described before,
- a single-stage model that used the 150 first-stage outcomes of the two-stage model as its scenario set,
- a single-stage model with a finer representation of uncertainty for its monthly horizon, comprised of 15,000 scenarios.

The first two models share the same information regarding potential outcomes in the month ahead, but the two-stage model incorporates additional information for potential outcomes in the following month. The third model uses a much finer representation for the distribution of the random variables in the month ahead.

The ex post realized returns of these models during backtesting simulations are shown in Fig. 5. The first graph corresponds to the experiments with minimum risk models, i.e., models that minimize the risk measure without any constraint on target expected return. The second graph represents the use of more aggressive return targets in the models (a target expected return $\mu = 1\%$ over the monthly horizon of the single-stage models, and a target $\mu = 2\%$ over the two-month horizon of the two-stage model). In all cases, the two-stage model achieved superior performance, while the single-stage model with the limited set of 150 scenarios produced the worst performance. Clearly, the addition of information in the representation of uncertainty—either with more scenarios for the single-stage model, or with the extension of the horizon to consider further outcomes in a second decision period in a two-stage model—resulted in performance improvements. Hence, there is an added value to finer specifications of distributions (scenarios) in these stochastic programming models.

We now compare the performance of single- and two-stage models that both use 15,000 scenarios. In the minimum risk case, their performance is quite similar. They achieve quite stable growth paths, with slight losses in only few instances during the simulations. As we explained earlier, this is a consequence of the selection of very similar portfolios

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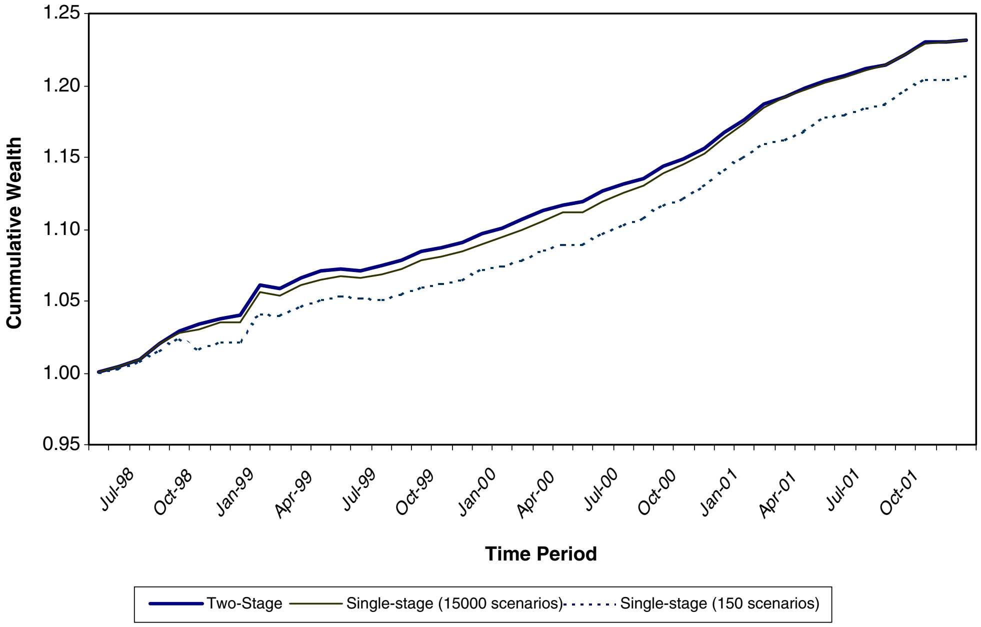

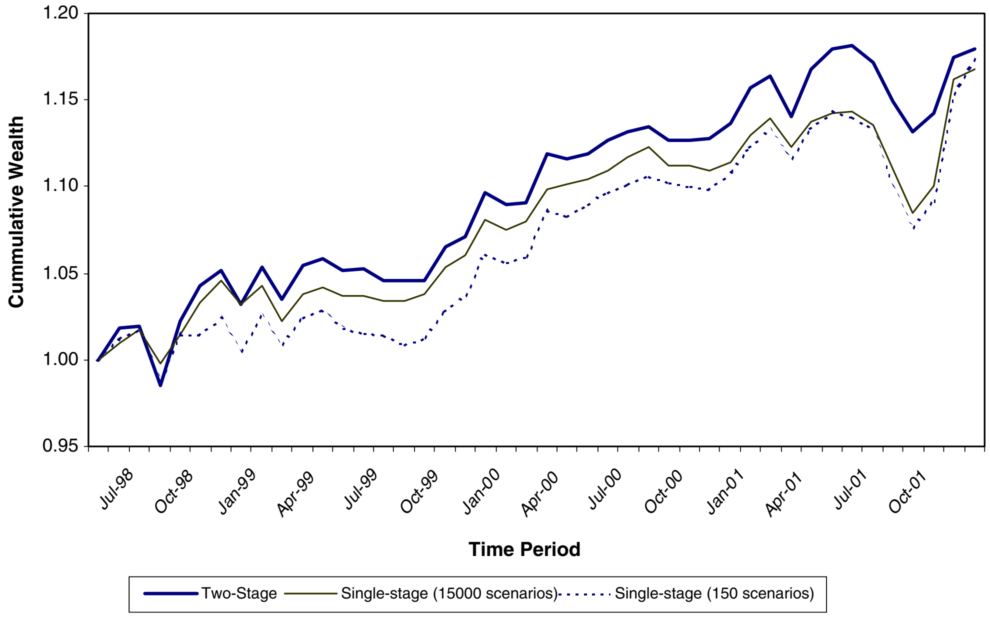

Fig. 5. Ex-post realized performance of single- and two-stage models. The first graph corresponds to the minimum risk case, while for the second graph the target expected returns during the planning horizon are $\mu = 1\%$ for the single-stage, and $\mu = 2\%$ for the two-stage models.

(composed of the most secure assets) by both models. But, as expected, the differences in realized performance are more evident when higher target returns are imposed, forcing the selection of riskier portfolios. In this case, the two-stage model yields a clearly superior performance.

Figs. 6 and 7 compare the compositions of the optimal portfolios for the single- and two-stage models throughout the simulation period. Observe that in the minimum risk case (Fig. 6) both models select very similar portfolios. These consist primarily of positions in the short-term US bond index

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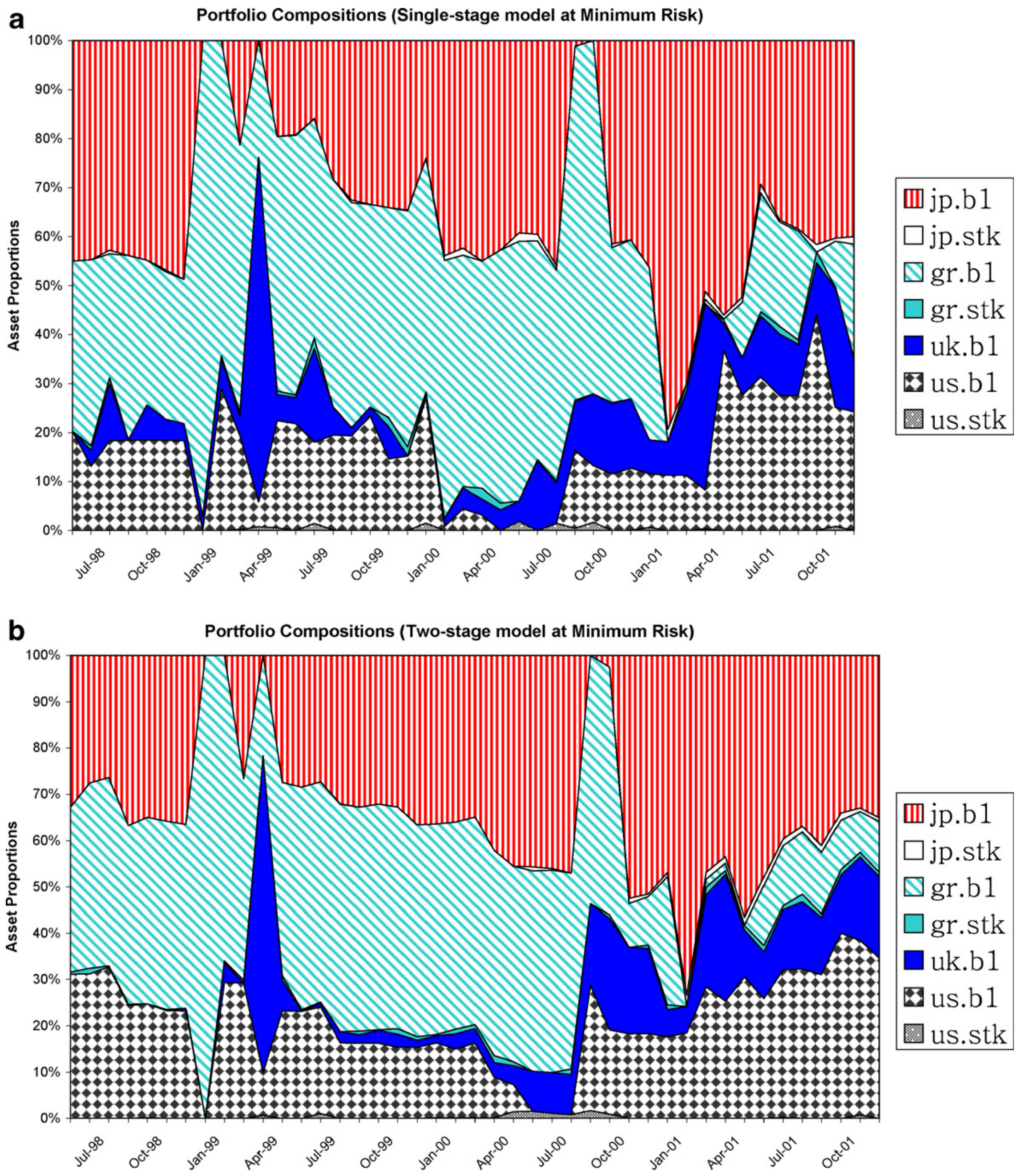

Fig. 6. Compositions of selectively-hedged international portfolios (minimum risk case) during backtesting simulations. The first graph shows the portfolios selected by the single-stage model, while the second graph shows the portfolios selected by the two-stage model.

and hedged positions in the short-term government bond indices of the other three countries; these instruments exhibited the most stable performance over the backtesting period. For the more aggressive targets of expected return, Fig. 7 shows that the models resort to more diversified portfolios that include also holdings of stock indices (particularly in the US stock index). The optimal portfolios of the two-stage model are more diversified and more stable over time compared to those of the single-stage model, thus indicating less active portfolio turnovers. This can be attributed to the look-ahead

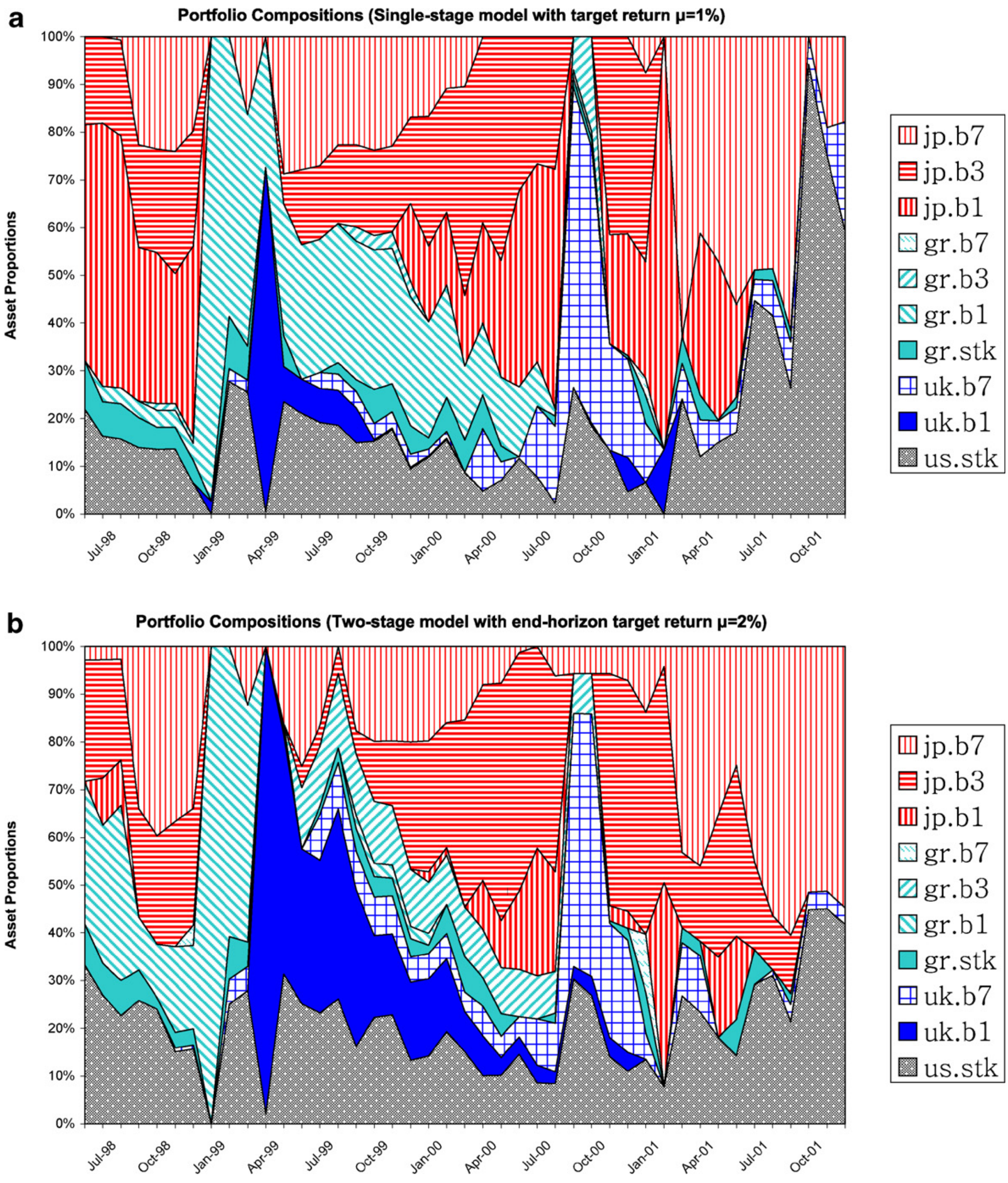

Fig. 7. Compositions of selectively-hedged international portfolios (aggressive portfolios) during backtesting simulations. The first graph shows the portfolios selected by the single-stage model, while the second graph shows the portfolios selected by the two-stage model.

feature of this model that has a longer horizon and considers longer-term effects in comparison to the myopic model when deciding the portfolio composition.

Finally, we compute some measures to compare the overall performance of the models. Specifically, we consider the following measures of the ex-post realized monthly returns over the simulation period: geometric mean, standard deviation, Sharpe ratio, and the upside potential and downside risk (UP) ratio proposed by Sortino and van der Meeer [23]. This ratio contrasts the upside potential against a

Table 3
Statistical characteristics of realized monthly returns

	Two-stage model	Single-stage model (15,000 scenarios)	Single-stage model (150 scenarios)
Performance measures of monthly realized returns (aggressive models)			
Geometric mean	0.460%	0.432%	0.430%
Standard deviation	0.0124	0.0139	0.0143
Sharp ratio	−0.0120	−0.0307	−0.0314
UP_ratio	0.9692	0.9501	0.8317
Performance measures of monthly realized returns (minimum risk)			
Geometric mean	0.581%	0.579%	0.484%
Standard deviation	0.0025	0.0027	0.0042
Sharp ratio	0.4132	0.3796	0.0201
UP_ratio	12.451	11.135	10.377

specific target (benchmark) with the shortfall risk against the same target. We use the risk-free rate of one-month T-bills as the benchmark. The UP_ratio is computed as follows. Let r_t be the realized return of a portfolio in month $t = 1, \ldots, k$ of the simulation, where $k = 43$ is the number of months in the simulation period 04/1998–11/2001. Let ρ_t be the return of the benchmark (riskless asset) at the same period. Then the UP_ratio is

$$\text{UP\_ratio} = \frac{\frac{1}{k}\sum_{t=1}^{k}\max[0, r_t - \rho_t]}{\sqrt{\frac{1}{k}\sum_{t=1}^{k}(\max[0, \rho_t - r_t])^2}} \tag{7}$$

The numerator is the average excess return compared to the benchmark, reflecting the upside potential. The denominator is a measure of downside risk, as proposed in Sortino et al. [24], and can be thought of as the shortfall risk compared to the benchmark.

The performance measures from the simulation results are reported in Table 3. The aggressive models exhibited the worst measures. Each model has a lower geometric mean and higher standard deviation in the aggressive case than its minimum risk instance. The Sharpe ratios also have small negative values for the aggressive models, and the UP_ratios are small, indicating a low upside potential relative to downside risk. The expected return targets (≈12.6% annually) that have been used for the models in the aggressive case proved overly ambitious and unattainable during the particular simulation period. We used them simply to distinguish the relative behavior of the models at different levels of risk tolerance. The aggressive models attained average annual returns of 5.7% (two-stage model) and 5.3% (singe-stage model) during the simulation period. The models fared much better in the minimum risk case, yielding average annual returns of 7.2% (two-stage) and 6.0% (single-stage), while also exhibiting significantly lower volatility of returns during this period as can be seen from the low standard deviation values (see also Fig. 5). The reported returns of the models are net of transaction costs.

These results show that the two-stage model dominated its single-stage counterparts; it consistently achieved the best performance in terms of both higher growth and greater stability of returns. In all measures, the single-stage model with the fewer scenarios had the worst performance, while the performance of the single-stage model with the finer scenario set was fairly close to that of the two-stage model. This observation implies that there is incremental benefit from increasing information content in stochastic programs, with a higher branching factor in the first stage but also with the addition of a subsequent stage. The question of the sensitivity of the results to the relative branching factors at different stages of a stochastic program, considering also the implications of this decision on the size and computational complexity of the resulting stochastic programs, remains a problem-dependent empirical issue.

Table 4
Size and solution times of models

Model	Number of constraints	Number of variables	Number of nonzeros	Approx. solution time (seconds)
Single-stage (150 scenarios)	325	360	3860	0.1
Single-stage (15,000 scenarios)	30,026	30,060	375,111	55
Two-stage (15,000 scenarios)	36,782	39,969	444,499	88

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Table 4 presents the size and computational effort required to solve the stochastic linear programs. The reported solution times reflect an average for typical problem instances. The models were solved with IBM's Optimization Subroutine Library (OSL) on an IBM RS/6000 44P worksstation (Model 170 with a 400 MHz Power 3 Risk processor, 1Gb of RAM, running AIX 4.3). Observe the almost proportional increase in solution time with the increasing number of scenarios for the single-stage model, and the larger size and required solution time for the two-stage model in comparison to a single-stage model with the same number of scenarios. These solution times are by no means prohibitive for realistic instances of the models with today's available computing technologies. The problems can be solved much more efficiently by employing specialized algorithms that exploit the structure of stochastic programs, and especially by resorting to parallel computing systems (e.g., see [29]). Issues of computational efficiency are not of primary concern in this study.

6. Conclusions and further research

We developed a stochastic optimization approach for managing international portfolios of financial assets in a dynamic setting and demonstrated its practical viability through extensive computational experiments using real market data. We formulated a multi-stage stochastic programming model to address active, international portfolio management problems. The model provides a useful and flexible decision support framework as it encompasses many practical features. It considers decisions over multiple periods to capture decision dynamics, it accommodates a multi-stage representation of the stochastic evolution of the random variables by means of a scenario tree, and it accounts for transaction costs.

Our implementation of the model controls the portfolio's total risk exposure by minimizing the conditional value-at-risk (CVaR) of losses at the end of the planning horizon, while imposing a target on expected return over the planning horizon. The CVaR objective minimizes excess shortfall beyond VaR. CVaR is a coherent risk measure and is appropriate for asymmetric distributions. So, its choice is consistent with the asymmetric and leptokurtic return distributions exhibited by market data of international stock and bond indices, as well as by currency exchange rates. However, the model can be easily modified to accommodate alternative objective functions to reflect the decision maker's risk-bearing preferences, or to incorporate additional practical constraints (e.g., managerial or regulatory requirements).

We employed a flexible and realistic scenario generation method based on principles of moment-matching. This method generates scenarios so that key statistics of the random variables match their observed values so as to closely approximate their empirical distributions. The scenarios of international stock and bond index returns and exchange rates reflect the correlations, as well as the skewness and excess kurtosis that these financial variables exhibit in historical data. Through exhaustive tests, we verified that the no-arbitrage conditions were satisfied for all scenario sets that we used in numerical experiments. We have not observed a case in which the scenario generation method failed to meet the no-arbitrage conditions. Other appropriate scenario generation methods can also be used in conjunction with the stochastic portfolio optimization model.

A contribution of this study is the extension of the stochastic programming model for international portfolio management to a multi-stage, dynamic setting. Another contribution concerns the internalization of decisions for currency forward positions in the portfolio management model, as means for mitigating currency risk. The inclusion of explicit decisions for currency forward contracts enables the determination of optimal selective hedging decisions to control currency risk exposure of foreign investments.

The model determines jointly the allocation of capital to international markets, the transactions to achieve an optimal portfolio composition (i.e., the specific asset holdings in each market), as well as the levels of appropriate currency forward contracts to minimize the portfolio's total risk. Thus, the asset allocation, the portfolio selection, and the currency risk hedging decisions, that are traditionally considered separately, are here cast in a unified decision framework.

The stochastic programming model provides a flexible framework to assess alternative risk hedging tactics. We used variants of the model to investigate the performance of alternative decision strategies in extensive empirical tests, both in static and dynamic experiments. We demonstrated that controlled use of currency forward contracts materially contributes to the reduction of risk of international portfolios. Selective hedging strategies proved effective in controlling risk and generating stable return paths in backtesting simulations with real market data. Our results also demonstrated the incremental benefits that can be gained from the adoption of dynamic portfolio optimization models instead of myopic, single-period models. Two-stage variants of the model exhibited superior performance in comparison to corresponding single-stage models, both in static as well as in dynamic tests. The two-stage models produced more diversified and stable portfolios, with lower volatility of returns and higher resilience during market downturns, and with lower turnover compared to the decisions of myopic models. Moreover, our results showed that finer specifications of distributions (scenarios) of uncertain input parameters resulted in improved solutions of the stochastic programs.

The multi-stage stochastic programming model provides the foundation for the implementation and empirical investigation of additional decision strategies. Building on the fundamental structure of the stochastic programming model, we have developed extensions that incorporate different types of options in the international portfolio management model. We use the stochastic programming framework to empirically examine the effectiveness of option-based strategies to control exposure to different risk factors. In [26] we focus on the use of straight options, and quantos, on international stock indices as means of mitigating market risk, while in [27] we incorporate currency options in the stochastic programming model and contrast their risk hedging performance with that of currency forward contracts.

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