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Teachers' attempts to address both mathematical challenge and differentiation in whole class discussion

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Research focus - Research question

- Mathematics' teachers attempts to balance work on mathematically demanding tasks and differentiation in lesson planning and enactment.
- Whole class discussion terrain provides fertile opportunities to study the intersection of mathematical challenge and differentiation (Sullivan, Mousley & Zevenbergen, 2004).
- The research question: "How do teachers attempt to balance mathematical challenge and differentiation in whole class settings?"

Theoretical framework: challenging tasks

- Challenging tasks require students to:
  - process multiple pieces of information and make connections between them
  - explain their strategies and justify their thinking
  - engage with important mathematical ideas
  - extend their knowledge and thinking in new ways (Stein et al., 2000).
- Working with challenging tasks is demanding for teachers:
  - Lesson planning involves design of tasks that support a learning trajectory and extend students' thinking (Sullivan et al., 2006).
  - Enactment of challenging tasks during whole class sessions requires:
    - Actions: revoicing, repeating, reasoning and adding on (Chapin et al., 2009)
    - Key practices: anticipating, monitoring, selecting, sequencing, and making connections between student responses (Stein et al., 2008).
    - Ways to handle the demands of challenging tasks : lowering, maintaining, or increasing the challenge. Stein et al. (2000)

Theoretical framework: Differentiation

- Differentiation is a process of aligning learning targets, tasks, activities, resources to individual learners' needs, styles and paces (Beltramo, 2017).
- Approaches to differentiation:
  - A triplet to study differentiation in teaching is focusing on content, process, and product (Tomlinson, 2014)
  - Use of enabling and extending prompts (Sullivan et al., 2006).
  - Building of a sense of communal experience (Sullivan et al., 2006).

Theoretical framework: whole class discussion

- Both emergent and collaborative
  - the outcome cannot be predicted in advance
  - it is the outcome of the collective activity between the teacher and the students (Dooley, 2009)
- Tensions involved in teachers' attempts to build on students' ideas
  - maintain the student-centered process of mathematical discourse
  - direct the content of the mathematical outcomes (Sherin, 2002)

The context of the study

- EDUCATE: ***Enhancing Differentiated Instruction and Cognitive Activation in Mathematics Lessons by Supporting Teacher Learning*** (4 partners from 4 different countries)
  - Professional development (PD) Erasmus+ project aiming to support teachers to engage all their students in challenging tasks
  - Development of teacher education and PD activities and materials
- Introductory phase of the project
  - Partners' aim: explore teachers' needs and challenges in relation to working with challenging mathematical tasks and differentiation without being engaged in any teacher education/PD activity

Participants

- Six experienced, qualified secondary school mathematics teachers
- Teaching experience ranged from 10-25 years in both lower (3) and upper (3) secondary schools with different qualifications
- No prior PD experience related to the aim of the project
- Goal: to design challenging tasks and enact them as part of their everyday teaching aiming to engage all their students
- Focus on difficulties encountered

Data collection

- Video-recording of two lessons for each teacher (12 video-recordings in total)
- Pre- and post-lesson reflections/interviews (12 in total)
- Teachers' designs for their lessons (e.g., worksheets, digital resources)
- Students' work

Data analysis

- Grounded theory approach (Charmaz, 2006)
- Looking for balance of mathematical challenge and differentiation in teachers' designs and enactment in the setting up of the task and the discussion of students' solutions
- Teaching actions indicating mathematical challenge and differentiation were identified (line by line) and they were linked to students' participation in the mathematical discourse
- Episodes where the mathematical challenge was made accessible to students were identified and a categorization of teaching practices was carried out looking for emerging patterns

Results: How mathematical challenge and differentiation were balanced in lesson planning

- Most tasks (21 out of 25) can be characterized as challenging offering opportunities to students to:
  - model an everyday situation through arithmetic, algebraic and geometrical relations (Markos - 7th grade, Adonis – 7th grade, Gianna - 10th grade, Eugenia - 8th grade)
  - link algebraic and geometrical representations (Kosmas, 10thgrade)
  - conjecture and prove a geometrical property (Kosmas - 10th grade, Takis - 10th grade).

Results: How mathematical challenge and differentiation were balanced in lesson planning

(a) *Designing tasks with multiple solutions and different entry points*

- The use of both algebraic and geometrical ways to solve equations with absolute values
  - to meet students' different capabilities ("Maria could easily handle the geometrical way while they had difficulties in the algebraic")
  - To develop deeper understanding of the meaning of absolute value
- Encouraging exploration in the context of open and/or modelling problems. E.g. estimating the height of the classroom by using trigonometry

Results: How mathematical challenge and differentiation were balanced in lesson planning

(b) *Using different kinds of resources manipulatives, digital applets, diagrams, typical and non-typical measuring instruments)* to facilitate the making of connections between different representations.

- Example: The use of digital tools (*Algebra Arrows* applet) to facilitate students' focus on the structure of arithmetic and algebraic expressions.

- Use of a hand-made protractor (a measuring instrument originated in the ancient Greek mathematics) to explore the critical role of this tool in conceptualizing the notion of tangent ratio.

Results: How mathematical challenge and differentiation were balanced in lesson planning

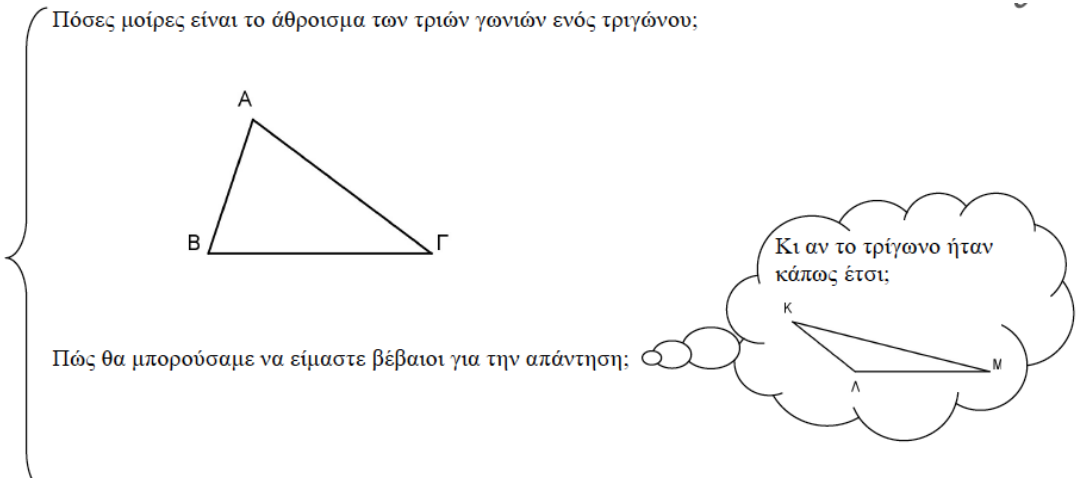
(c) *Creating an inclusive and mathematically challenging learning environment* by encouraging students to share their work in groups and in whole class discussions and avoiding evaluative comments.

- Example: attributing roles and responsibilities to the students in each mixed ability group according to their mathematical backgrounds and interests.

Results - Lesson enactment: Episode 1

Episode 1: *Stimulating the key mathematical idea by exploring the validity of students' responses*

- 10th grade classroom
- Task "How many degrees is the sum of the three angles of a triangle? How can we be sure about the answer?



Results - Lesson enactment Episode 1

- T: Are we sure that the sum of the angles of a triangle is 180 degrees? (*stimulating the challenge*).
- [most students say that they know it from previous grades] Alexis: [having usually limited participation in the lesson] To measure the angles
- T: If you measure the angles do you think that you will find 180 degrees?[to all the students] Draw triangles and measure their angles using a protractor (*valuing students' ideas by addressing them to the whole class; encouraging empirical solutions)*.
- [Alexis finds 179 degrees and other students 178, 179, 180, 181. The teacher writes these responses on the board (*recording and discussing all students' answers*)]
- T: How can we be sure? It seems that we cannot be sure by measuring" (*refuting the empirical solutions)*.

Analysis and interpretation of Episode 1

- The teacher
  - points out the key mathematical idea by building on Alexis' suggestion for an empirical measurement.
  - accepts Alexis' suggestion and invites all students to perform measurements in different triangles.
  - records all students' answers on the board as a way to question the validity of the approach.

"The discussion was what I wanted as the responses varied. It also went well since most students were involved, also by working in groups felt less exposed to evaluation" (Kosmas' post-lesson reflection).

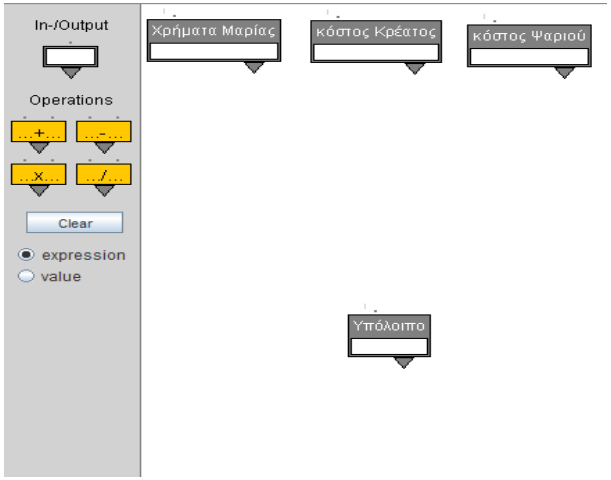
Interpretation of Episode 1: Emerging pattern

- Stimulates the mathematical challenge of the task
- Encourages students to explore an inappropriate idea coming from a student
- Summarizes students' responses
- Provokes the refutation and reinforces the challenge

Results - Lesson enactment: Episode 2

Using digital resources to address mathematical challenge and students' difficulties

- 7th grade classroom .
- Task: Maria has 500€ in her bank account. She bought meat that cost 10€. She also bought fish that cost 20€. She used her debit card and received a message from her bank on her mobile, informing her that her account balance is 470€.
- 1) Using the application *Algebra Arrows* construct two different arithmetic expressions to describe the account balance (the result being the amount left in the account). 2) What is the relation between the two expressions and why? 3) If Maria's money is unknown what are the two expressions and their relation. Justify.



Results - Lesson enactment: Episode 2

- The episode took place during the whole class discussion after they students had worked with the task in groups without the applet)
- [A group of students (4 girls) (group 1) had written correctly two expressions using variable on their worksheet – i.e. x-(10+20) and x-10-20 – but they could not consider them as equal]
- [The teacher constructs the two expressions with the applet and asks students to provide the answer before this is projected (*using multiple and interconnected digital representations to justify an answer*)].
- Next, he comes back to the group 1's
- T: Girls, I remember that earlier you were concerned if it is the same 'x' that appears here [in the expression x-(10+20)] and the 'x' that appears there [in the expression x-10-20)]. This is a question for the whole class. I ask: Does this 'x' express the same thing? (*posing an individual student difficulty to the whole class; stimulating all students to reflect on the provided representations*).
- Nick: They are the same as they are connected with the same field – called Maria's Money: x - in the applet
- The teacher indicates that both expressions are built by using the same field symbolized as 'x' (*revoicing the correct answer)*: "
- T: Look the arrows starting from the cell containing Maria's initial amount of money. We speak about Maria's money in both expressions *(linking the digital representations to the realistic context)*.
- In this episode, the teacher brings an individual student's difficulty in the whole class through the use of digital representations, stimulates all students to reflect on the provided representations, revoices a student's correct answer, strengthens the challenge by providing links to the realistic context.

Results - Lesson enactment: Episode 2

- Pattern:

- the teacher brings an individual student's difficulty in the whole class through the use of digital representations
- stimulates all students to reflect on the provided representations
- revoices a student's correct answer
- strengthens the challenge by providing links to the realistic context.

Conclusions: Balance of mathematical challenge and differentation

- Designing mathematically demanding tasks and using different teaching resources to engage students in exploring, connecting and reflecting.
- Stimulating the challenge involved building on students' ideas as well as scaffolding by simplifying and extending it
- Valuing students' contributions and using them as the basis for communal reflection

  recording all students' answers

  inviting students to connect different solutions

  questioning proposed ideas

  favoring the development of an inclusive learning environment (e.g., encouraging silent students to participate)

Conclusions: Balance of mathematical challenge and differentation

Emerging patterns indicate the complexity of the process of balancing that is beyond existing characterizations (lowering/maintaining/increasing the challenge)

An example of a pattern:

a dynamic interplay of actions moving back and forth between providing challenging questions and prompts.

A ‘zig-zag’ pattern related to the complexity of teaching practice when the teacher aims to keep the challenge and at the same time to maximize learning opportunities for all students.

Thank you