A STOCHASTIC PROGRAMMING FRAMEWORK FOR INTERNATIONAL PORTFOLIO MANAGEMENT

NIKOLAS TOPALOGLOU

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ADVISORS:
HERCULES VLADIMIROU AND STAVROS A. ZENIOS

DEPARTMENT OF PUBLIC AND BUSINESS ADMINISTRATION
SCHOOL OF ECONOMICS AND MANAGEMENT
UNIVERSITY OF CYPRUS

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ABSTRACT

In this thesis we develop a stochastic programming framework for the risk management of international portfolios. Risk management has been recognized to play an increasingly important role in financial problems such as the international asset allocation, where widespread deregulation has entailed a substantial increase in asset price and currency volatility. In this framework, risk encompasses market as well as currency risk. At the same time, the emergence of international diversification has enhanced the range and scope of derivative securities available in addition to standard assets, for hedging and speculation purposes. Financial contracts such as stock and currency options have been tailored to the risk positions of international investors.

This thesis concerns the development, implementation and validation of scenario-based stochastic programming models for actively managing international investment portfolios of stock and bond indices in multiple currencies. Stochastic programs posses several attractive features that make them applicable in international portfolio management problems. Forward contracts and European options (including stock options, quantos and currency options) are incorporated in the portfolio optimization models as risk hedging instruments. The models address jointly the volatility of asset returns, the volatility of exchange rates, and their correlations. Integrated simulation and optimization procedures are employed to address these issues in a common framework. Appropriate scenario generation methods are used to generate jointly scenarios for asset returns and exchange rates, so as to capture their interdependencies. The scenarios represent discrete, joint distributions of the random variables and constitute the primary inputs to the stochastic optimization models. The models determine jointly the portfolio positions (not only across the different markets, but also to specific assets within each market) and the levels of hedging with "fairly" priced derivative securities. The optimization models incorporate institutional considerations (no short sales, transaction costs) and determine a solution that is optimal, according to specific objectives, with respect to the postulated scenario set. Thus, several interrelated decisions that were previously examined separately, are cast in a common framework with consequent benefits to the investors.

A primary issue is the adaptation of suitable methods for pricing and incorporating options in scenario-based stochastic programming models. The resulting option prices are consistent with the postulated scenario set. The option pricing methods are not dependent on any explicit assumption regarding the discrete distribution that represents the stochastic prices of the underlying security. In principle, they can be applied to any arbitrary discrete distribution (scenario tree) for the prices of the underlying. We apply empirical tests to validate the proposed option pricing methods.

The thesis develops suitable modelling frameworks for addressing problems of managing optimally international portfolios and controlling the associated risks. The major aim is to investigate the relative effectiveness of alternative hedging instruments and strategies for controlling the primary risks that affect the performance of international portfolios. The thesis gradually leads to the integrated framework of dynamic international portfolio management. We start with a single-stage model for international asset allocation where forward rates are used to (partly) hedge the currency risk, while the market risk is not explicitly addressed except via international diversification. Then we extend this model into a multistage setting. Next, we adopt methods for pricing and incorporating options in scenario-based stochastic programming models. The next step uses options to hedge the risks in isolation. Finally, a general integrated portfolio risk management framework is developed. It is shown through extensive tests that increasingly integrated views towards total risk management are more effective compared to consideration of constituent risk components in isolation. Multistage stochastic programming models incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively. These models shape the overall portfolio risk profile accounting for both the market and the currency risks, thus contributing directly to the objective of total risk management. Empirical results indicate the superiority of this integrated risk management scheme. The simultaneous use of stock and currency options yields the best ex post performance of international portfolios. Moreover, multistage models clearly outperform their myopic (single-stage) counterparts, regardless of the trading strategy and the hedging instruments that are used. Hence, we establish that multi-stage stochastic programming models are effective risk decision support tools. The integrated modelling framework of this thesis provides a complete basis to comprehensively address all relevant decisions in international portfolio management in a unified manner.

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Contents

Chapter 1

Introduction

The theory of optimal portfolio selection was developed by Harry Markowitz in the 1950's. His work [139, 140] formalized the diversification principles in portfolio selection and earned him the 1990 Nobel prize in economics. Since then, mathematical programming techniques have become essential tools in financial management, and thus are being increasingly applied in practice. The need to adopt sophisticated analytical tools in financial management is being compounded by the increasing diversity of complex financial instruments and the realization that multiple uncertain factors have a concerted effect on the risk-performance characteristics of securities. Consequently, financial managers realizing the multi-dimensional nature of risk, are increasingly taking an integrated view towards managing their total risk exposure.

The essence of financial management is the study of allocation and deployment of economic resources, both spatially and across time, in an uncertain environment. To capture the influence and dynamic interaction of uncertain risk factors effectively, requires sophisticated analytical tools. Mathematical models of modern finance include elegant applications of optimization. Over the past decades, financial optimization had a direct and important influence on financial practices.

Using mathematical programming techniques, we can reduce financial risks that affect the performance of portfolios, by diversifying away the nonsystematic risk of these portfolios. The diversification principle simply states that the investments should be distributed across various assets so that the exposure to the risk of any particular asset is limited. By placing one's eggs in many baskets, overall portfolio risk actually may be less than the risk of any component security considered in isolation, as the returns of the assets are not perfectly correlated.

1.1 Thesis Theme and Motivation

Given a relatively low correlation among national equity markets, investors can improve their reward-to-risk ratio by holding assets in several countries. Optimal international diversification is one of the major topics in global asset management. The benefits from such diversification (were originally shown by Grubel [83] and Levy and Sarnat [133]) seem to be clear: the more assets with low correlation among each other are added to the portfolio, the lower the overall portfolio risk. In other words, international diversification helps to improve the risk-adjusted performance of a domestic portfolio. The main goal of diversification is to reduce the total risk of a portfolio. Although the risk of a foreign stock market may be larger than that of the domestic market, the addition of risky foreign assets to a purely domestic portfolio may still reduce its total risk if the foreign assets have negative (very low) correlation with the domestic market. Thus, institutional investors often turn to foreign markets to obtain a wider scope for diversification than is possible in a domestic market. With the internationalization of portfolios, however, also comes an additional risk factor, that is, foreign exchange risk.

Risk in this context stems from the uncertainty surrounding the value of international portfolios due to unforeseen and unforecastable future events. The primary risk factors are in this case two: The market and the currency risk. Market risk is the risk associated with changes in the market prices of the assets over time. Hence, the price of a stock may drop over time due to adverse movements of the stock market as reflected in market index changes, or the value of a fixed-income security may drop over time due to increases in the level of interest rates. Currency risk is the risk that the value of a foreign security may change with time due to changes in the exchange rate. It is the impact on a portfolio's total return due to the volatility of the exchange rates of foreign currencies to the investor's reference currency, i.e., the detrimental effect to an investor due to potential depreciation of foreign currencies in which investments are held. Potential benefits from international diversification may be reaped but with exposure to this additional risk (see, Eun and Resnick [68]). Foreign exchange rate fluctuations induce changes in overall portfolio returns because uncertain future exchange rates directly affect the translation of returns from foreign-currency-denominated investments into base-currency returns. Diversification of portfolio holdings across several countries can help mitigate market risk, as well as currency risk, as long as correlations remain fairly low, or negative.

Many studies have noted that the correlations of stock returns in major international markets appear to have increased in recent years, potentially reducing the diversification benefits of international investments. Plausibly, this is the result of globalization — the process of integration of the global economy. How should investors react to global integration? Some analysts argue that rising correlations weaken the case for international diversification. If all markets move together, why not keep your money safely at home? This argument, while apparently reasonable, creates a paradox of globalization. It implies that investors should ignore the trend of global integration and invest domestically, neglecting the potential advantages from a presence in diverse markets. Despite the observed increases in the correlations of major international equity markets, these are still at lower levels than the correlations of domestic securities, especially in similar industrial sectors. In this work we show that the case for international diversification remains strong.

Correlations between stock markets have indeed increased since the early 1990's, the volatility of the US stock market has also increased while the volatility of the Japanese market has declined over the same period. Changes in volatility of markets imply greater benefits from international diversification, offsetting the effect of the rising correlations.

It is important to realize that the only reason why investors are willing to take risk is their perception of a positive expected return (in excess of the risk free rate). In an international setting, for instance, many investors may not have strong convictions about future asset price and currency movements. If this is the case, these movements induce additional risk in the portfolio that may not be appropriately remunerated by a positive risk premium. It is clear that there is a need for proper diversification. However, diversification alone may not be sufficient to serve the investor's risk management objectives. Risk reduction (or the risk management function) may be augmented with the judicious use of suitable hedging instruments and associated trading strategies. This is the motivation behind the modelling studies and empirical investigations contained in this thesis. The overall aim is to develop effective and practically useful decision support tools for total risk management in portfolios of international financial assets.

Hedging is a way of reducing some of the risks involved in holding international portfolios. It refers to the use of appropriate instruments and strategies that aim to minimize the potential effects of the various risk factors, thereby minimizing the uncertainty in the future value of investments and providing some stability of estimated earnings and cash flows. There are two primary risks in international financial portfolios against which one can hedge (i.e., market and currency risk) and several alternative means for hedging these risks, either separately or jointly. Hedging is, in a sense, equivalent to insurance. A hedge is just a way of insuring investments against risk. The basic idea behind constructing a hedge is to identify the exposures of a portfolio to various risks, and to construct positions with the opposite exposure. The central problem in financial risk management is to employ appropriate decision strategies so that the portfolio achieves a desirable combination between expected return and risk. Thus, hedging decisions are an integral part of the overall investment structuring decision so as to achieve a balance between expected return and risk.

When an investor or a financial institution, hedges his exposure to risk, the primary objective is to minimize risk, not to maximize profit from speculation regarding potential movements in the underlying random variables (asset returns and exchange rates). A hedged position will, therefore, forgo the benefit of a favorable movement, but at the same time will protect the hedger from potential losses due to an unfavorable movement.

Essential for hedging market risk are the options on stocks, that are traded in many exchanges. Options are contracts which give the investor the right to buy (call option) or to sell (put option) a specific amount of an underlying asset for a specific price (exercise price) only at the end (European option) of the specified expiration date. By construction, options have highly asymmetric payoffs that can effectively cover against adverse price movements of the underlying security. The investor decides a hedge strategy consisting of holding an underlying asset and simultaneously buying an option (or a combination of options) on that underlying asset. This combination leads to an asymmetric gain/loss position at the expiration date: on the one hand there is a hedge against potential losses and on the other hand it is possible to participate in potential gains of the underlying asset. Thus, one important use of options is to hedge risks or to sell risks off to another party. Derivative securities are playing an increasingly important role as hedging instruments.

Currency risk is an important aspect of international investment portfolios, as globalization and integration of financial markets worldwide gained momentum over the past two decades. Since the end of the Bretton Woods system in February 1973, when the exchange rates of all the industrial countries were set free to float independently, exchange rates have been highly volatile and correlations between exchange rates, as well as between asset returns and exchange rates, have also changed dramatically. While a part of currency risks (idiosyncratic risks) get diversified away in an international portfolio invested in many currencies, systematic currency risks remain. Therefore, compared to a portfolios of investments purely in domestic assets, fluctuating exchange rates represent an additional risk factor for investors who diversify their portfolios internationally. Thus, it is important to study whether hedging the exchange rate risk is worthwhile and to what extent, as well as to investigate the relative effectiveness of alternative means for controlling currency risk.

Thus, currency hedging becomes an integral part of the overall international portfolio management decision. In practice, currency holdings are hardly optimized according to the diversification rules followed in domestic investments. This is surprising since modern capital markets and hedging instruments allow to separate the investments in particular markets from the holdings of the respective currencies: currencies can be treated as separate asset classes. Investors usually do not optimally hedge their exposures in foreign markets; instead, they simply use ad hoc rules (e.g. 70% hedging). Also, the currency hedging decision is usually taken after the portfolio positions in the underlying markets have been decided. Typically, the currency hedging decision is treated as subordinate to asset allocation. The first decision select the allocation of funds to international assets. Next, the investors decide if and how to hedge the exposure to foreign investments. This neglects cross hedging effects between markets and currencies and cannot produce a portfolio which jointly optimizes market and currency holdings, as it ignores interactions among changes in the underlying asset returns and exchange rates.

These observations point to the need for integrated simulation and optimization approaches that determine jointly portfolio structures and flexible hedging policies. This thesis will demonstrate the benefits of a unified consideration of market and currency risks in an integrated, total risk management decision framework for international portfolios.

Typical instruments for hedging the currency risk are the forward contracts which are regularly traded outside exchanges by banks and other financial institutions. An investor who knows that a certain amount of a foreign currency will be received at a certain time in the future (e.g., the revenues from liquidating the holdings in a foreign market) can fully hedge the foreign exchange risk by taking a short position in a currency forward contract i.e., selling the deterministic cashflow in the foreign currency at a predetermined (forward) exchange rate, to receive a deterministic cashflow in the base currency. However, the value of investments in foreign financial assets at the end of a holding period cannot be known with certainty as it is affected by their stochastic returns. Thus, the notion of full hedging is not attainable in practice. Moreover, as a currency forward contract fixes the exchange rate a priori, it protects against potential losses from a possible depreciation of the foreign currency on the one hand, but on the other hand it forgoes potential gains in the event of a currency appreciation. Clearly, the proper control of risk-reward tradeoffs in the context of active international portfolio management problems is a complex issue.

Currency options are alternative instruments for currency hedging purposes as they give the right, but not the obligation, to an investor to buy or sell an amount of foreign currency at a future date, at a fixed exercise price. Currency put options provide (at a cost) coverage against possible currency depreciations in order to protect the value of foreign investments. Their advantage over currency forward contracts is that they allow the possibility to benefit in the case of currency appreciation during the holding period. Combinations of multiple options may be also used to shape the payoff profile so as to provide desirable levels of downside risk coverage and upside potential.

We incorporate alternative decision choices involving currency forward contracts and/or options (either individual or specific combinations) in a stochastic programming framework for international portfolio management. Thus, we develop a flexible modelling framework that serves as a practical testbed to empirically assess the performance of alternative portfolio management strategies.

Moreover, quanto options provide the means to empirically control the market and currency risks of investments in foreign equities. These options are written on a foreign stock (or stock index) but have their cashflows (payoffs and price) expressed in a different reference currency by employing a fixed exchange rate (usually the forward rate for the term of the quanto). The use of these instruments in international portfolio management models is also examined in this thesis, and their effectiveness is empirically examined.

1.2 Research Questions

This thesis develops suitable modelling frameworks for addressing problems of managing optimally international portfolios and controlling the associated risks. The major aim is to investigate the relative effectiveness of alternative hedging instruments and strategies for controlling the primary risks that affect the performance of international portfolios. We study first suitable tactics for controlling exposure to risk factor separately, assessing the impact of hedging the particular factor in the performance of the portfolios. Then we analyze the additional value of controlling against all risk factors using derivatives, in an integrated manner. We find that the unified consideration of market and currency risks yields additional benefits in the performance of international portfolios. We demonstrate that a holistic view that incorporates suitable risk management means in a common portfolio optimization framework is an effective approach for managing the total risk exposure of international portfolios. This is the primary goal that this thesis set out to accomplish.

The study concerns the development, implementation and validation of scenario-based stochastic programming models for actively managing international investment portfolios of stock and bond indices in multiple currencies. Forward contracts and European options (including quantos) are incorporated in the portfolio optimization models as risk hedging instruments. The models address jointly the volatility of asset returns, the volatility of exchange rates, and their correlations. Integrated simulation and optimization procedures are employed to address these issues in a common framework. Appropriate scenario generation methods are used to generate jointly scenarios for asset returns and exchange rates, so as to capture their interdependencies. When multiple elements of uncertainty, such as asset returns and exchange rate fluctuations are considered simultaneously, capturing correlations among them is very important. The scenarios represent discrete, joint distributions of the random variables and constitute the primary inputs to the stochastic optimization models. The models determine jointly the portfolio positions (not only across the different markets, but also to specific assets within each market) and the levels of hedging with forward contracts, or via "fairly" priced derivative securities. The optimization models incorporate institutional considerations (no short sales, transaction costs) and determine a solution that is optimal, according to specific objectives, with respect to the postulated scenario set. Thus, several interrelated decisions that were previously examined separately, are cast in a common framework with consequent benefits to the investors.

This work considers tools to determine optimal diversification in international markets, and protection against adverse economic conditions by directly addressing the market and currency risks that could lead to significant losses in the portfolios. It investigates strategies that can hedge these risks. In today's marketplace, the modern financial manager must be able to use all the available tools to control a portfolio's total exposure to financial risks. The thesis develops models that integrally address multiple issues in international portfolio management, and proposes decision tactics to manage the risk exposures so as to achieve the desirable risk/return characteristics.

Effective techniques and practical decision support tools for managing portfolios of international investments are very important. Such tools are a primary concern for financial intermediaries (e.g., banks), institutional investors (e.g., insurance firms, pension fund and hedge-fund managers, global portfolio managers) and multinational firms that engage in financial activities in various countries. Moreover, international portfolios, optimally selected to minimize the downside risk can provide an effective way to mitigate the effects of extreme events and to generate healthy financial performance. This is particularly important in a period that has witnessed spectacular financial failures due to improper or ineffective financial risk management practices.

The problems faced in international investment management are complex as they involve many correlated sources of uncertainty (currency and market risk). Models that can effectively capture the stochastic evolution of the underlying financial processes and suitable stochastic optimization models inevitably become challenging and computationally demanding. The ability to optimize investment choices and to effectively manage risks in a global environment is essential for maintaining competitiveness, thus warranting the use of sophisticated mathematical tools to solve the complex planning problems. Stochastic programming models offer significant advantages over competing methods. This thesis aims to develop an effective modelling framework to address problems of international investment management in practical settings.

The problems of managing risk are more challenging than ever. Risk managers face a wide range of demands, from working with multiple variables, to finding technology solutions that enable comprehensive risk analysis that conforms institutional policies and meets regulatory requirements. This study's approach to risk management combines appropriate methodologies, technology and data analysis tools in an integrated manner to help investors make more informed financial decisions and effectively meet their objectives.

The thesis demonstrates the important role of optimization models in risk management. Mathematical programming techniques determine optimal solutions to complex portfolio problems with many constraints. We focus on minimizing downside risks of portfolios, through appropriate risk metrics. The downside risk of portfolios is what worries the investors.

Derivative securities are incorporated in the portfolio optimization models as risk hedging instruments. Pricing derivatives has largely depended upon efficient market assumptions; the traditional Black-Scholes framework assumes that asset returns are normally distributed. Market data of international index returns and exchange rate movements show that these random variables do not conform to normality or log-normality assumptions. They exhibit skewed distributions with heavy tails. These features must be properly accounted for. Hence, alternative valuation procedures are needed to derivatives and incorporate them in the portfolio management models. The valuation procedures must comply with fundamental financial principles (e.g., no-arbitrage).

This study provides tools to support the key function of risk management in international portfolios. The pricing of the options on international assets consistently with postulated scenarios for the underlying assets that represent uncertainty in stochastic programming models constitutes a novel contribution of this thesis. The participation of options in the portfolios follows from the solution of the optimization models that include options among the investment opportunity choices so as to minimize the downside risk of the international portfolios.

Options play an increasingly important role in risk management. Their asymmetric payoffs (individual options or appropriate combinations) provide the means to protect the value of holdings in an asset in the event of substantial variations in the market price of the underlying security. Currently, 22 countries worldwide have markets for exchange traded options on a range of contracts from stocks, indices, fixed income securities, interest rate securities and currencies. Given the growth of exchange traded options markets, there is little doubt that many international portfolio managers will include options in their portfolios. This makes clear the need to develop procedures that price and incorporate options in portfolios, essential in maintaining an integrated approach to risk management. We show how to use various types of options in combinations to benefit from, or alleviate potential adverse consequences of movements of the underlying international stock indices or exchange rates. We show how options allow an investor to limit his exposure to market and currency risks.

The increasing complexity of financial products, intensifying competition, and the need for more rapid decision making, increases the necessity for effective decision-support tools, implying an important role for optimization models in modern finance.

In summary, the research in this thesis concerns the following topics:

- The development of an integrated risk management framework where multiple risk factors are considered simultaneously. This is achieved by developing flexible optimization models for international portfolio management that capture these risk factors and jointly determine the optimal portfolio composition and the level of hedging against them.
- The adoption of appropriate methods for scenario generation that properly capture the uncertainty

of the random variables, without assuming explicitly a particular functional form for the distribution of these variables. The scenario generation methods must capture the empirically observed asymmetries and heavy tails of international index returns and exchange rates.

• The adoption of appropriate measures to control for risk of international portfolios when the distribution of asset returns is not normal. Asymmetric returns of the underlying assets and exchange rates, and the asymmetric nature of option payoffs leads us to adopt coherent risk measures for downside risk, appropriate for asymmetric distributions. We apply coherent risk measures for portfolio optimization.

• The pricing and the incorporation of derivatives (options on domestic and foreign stock indices, Quanto options on foreign stock indices and currency options) in scenario-based stochastic programming models. The valuation methods must account for asymmetric and heavy-tailed distributions of the underlying random variables. They must price the options consistently with the discrete scenarios of asset prices and exchange rates.

• The empirical investigation of alternative instruments and strategies in terms of their effectiveness to control risks. The alternative decision strategies consider the treatment of market and currency risk either separately or jointly, through appropriate instruments and hedging tactics. Their performance is empirically compared in terms of their risk-return profiles (efficient frontiers) in static tests, as well as in terms of their ex-post realized returns in backtesting simulations with real market data.

• The extension of the optimization models in a multistage stochastic programming setting. The models consider the dynamic management of portfolios through rebalancing in successive time periods. These models implement strategies that use derivatives to hedge both the market risk (stock options) as well as the currency risk (currency options), and specify jointly the appropriate investments in each market as well as the level of hedging.

1.3 Methodology

Uncertainty about future economic conditions plays a key role in portfolio management. The models developed in this thesis support optimal decision making in the face of uncertainty in dynamic setting. In this context, multiple risk factors are considered simultaneously and decisions about the optimal portfolio composition and hedging strategies against various risks have to be made. The paradigm of multistage stochastic programs with recourse is particularly suitable to address this problem. Stochastic programs posses several attractive features that make them applicable in international portfolio management problems. Uncertainty in input parameters of these models is represented by means of discrete distributions (scenarios) that depict the joint co-variation of the random variables. In multistage problems the progressive evolution of the random variables is expressed in terms of a scenario tree. The scenarios are not restricted to follow any specific distribution or stochastic process. Thus, any joint distribution of the random variables can be flexibly accommodated. Asymmetric distributions and heavy tails in the random variables (which is often the case in financial problems) can be readily incorporated in the portfolio optimization model. The choice of stochastic programs is made for several reasons:

• They can handle multi-asset problems. Using stochastic programs we can consider a large number of international assets in multiple markets.

• They can accommodate general distributions and dynamic aspects by means of scenario trees. We do not have to explicitly assume a specific stochastic process for the return of the assets; instead, we rely on the empirical distribution of these returns.

• They can address practical issues such as transaction costs, turnover constraints, limits on groups of assets, no short sales, etc. We can include in stochastic programs regulatory, institutional, market specific, or other constraints.

• They can flexibly use different risk measures. We have the choice to optimize the appropriate (for the specific problem) risk measure or a utility function.

Stochastic programming overcomes several limitations faced by continuous-time finance. Namely, restrictions on the number or assets, markets and options, the inability to accommodate general constraints, and restrictive distributional assumptions for the random variables. The scenario generation is a critical step for the entire modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution of the underlying random variables and is consistent with market observations and financial theory.

In this work we adopt scenario generation procedures that conform to fundamental financial principles. For example, if the outcomes of the random variables in a scenario set do not satisfy non-arbitrage conditions then the solutions of the stochastic programs will be biased and will imply unattainable spurious profits. The scenario generation methods that are used in this study produce representative scenarios based on historical observations of market data. We numerically test to ensure that the scenario sets comply to the fundamental no-arbitrage condition.

The ability to solve large-scale stochastic programming models with a large number of scenarios allows us to consider in a common framework multiple elements of risk and to deal simultaneously with many securities.

In order to incorporate options in stochastic programming models for risk management purposes, we first need to properly price these options. We adapt two different methods for pricing the options in accordance with the discrete distributions of a scenario tree. We validate the methods using market prices for the options and their underlying security. The proposed option pricing methods are motivated by the need to achieve internal consistency in stochastic programming models when options are incorporated. We analyze whether asymmetric distributions lead to option prices considerably different than the Black-Scholes prices, and to what extent nonzero skewness and excess kurtosis cause significant deviations. Moreover, the methods allow us to estimate directly the risk-neutral distribution of the underlying asset and to study its characteristics, thus explaining skewness premia effects.

1.4 Scientific Contributions

The contributions of this thesis are the following:

• The development of an integrated simulation and optimization framework for international portfolio management.

• The implementation of portfolio optimization models that jointly select the appropriate investments across currencies and determine the levels of hedging against risk exposures.

• The development of single, as well as multistage stochastic programming models for optimal selection of international portfolios in a dynamic setting. The models apply a coherent risk measure (CVaR) that is suitable for asymmetric distributions to minimize the excess shortfall.

• The adaptation of suitable methods for pricing derivatives in scenario-based stochastic programming setting. The resulting option prices are consistent with the postulated scenario set, while satisfying no arbitrage conditions. They are shown empirically to reproduce more closely than the Black-Scholes approach the market prices of options.

• The incorporation of derivatives in the stochastic programming models for risk hedging purposes.

• The development of an integrated risk management framework where both market and currency risks are considered simultaneously and trading strategies of derivative securities are used to hedge the risks.

• The empirical validation of the models through extensive numerical tests using real market data.

• The assessment of alternative risk hedging strategies in terms of their effectiveness to control risk while enabling the attainment of gains.

We elaborate on the novel contributions of this study.

We integrate a simulation and optimization framework for managing portfolios of international financial assets. Reliable models for managing financial risks, once they have been identified, in an integrated manner are still in their infancy. Our models capture the primary risk factors in international portfolios and jointly determine the selection of particular investments across multiple markets and the appropriate hedging strategies.

Previous studies considered the hedging decisions separately from investment decisions. We demonstrate that our holistic view that incorporates the instruments and investment decisions in a common portfolio optimization framework is an effective approach for managing the total risk exposure of international portfolios. This aim is a central theme of this dissertation. All our developments evolve around this central goal.

We employ optimization models for international portfolio management that minimize the excess shortfall risk of international portfolios as captured by the Conditional-Value-at-Risk (CVaR) measure. Due to the observed asymmetry of asset returns in international portfolios and the asymmetric option payoffs, the CVaR risk measure is more appropriate than alternative metrics such as the Mean-Absolute-Deviation (MAD) and variance.

We develop multistage stochastic programming models for managing international portfolios under uncertainty. Multi-stage models help decision makers gain useful insights and adopt more effective decisions. They shape decisions based on longer-term benefits and avoid myopic reactions to short-term market movements that may potentially prove risky. They determine appropriate dynamic contingency (recourse) decisions under changing economic conditions that are represented by means of scenario trees. As dynamic models consider longer planning horizons and account for portfolio rebalancing decisions at multiple time periods, they should reasonably be expected to outperform myopic models. Yet, comparative studies to establish the incremental benefits of multistage stochastic programs in comparison to single-period models can scantly be found in the published literature. Hence, it is still not clear whether the additional complexity in data processing, modeling and solution effort of multistage stochastic programs in comparison to myopic models is compensated by sufficient improvements in performance. We show through extensive numerical experiments that multistage models give performance improvements over their single-stage counterparts. Hence, we establish that multistage stochastic programming models are effective decision support tools for international portfolio management.

We adapt suitable methods for pricing and incorporating European type options in scenario-based stochastic programming models. The options are priced in accordance with the postulated scenario sets, thus yielding an internally consistent model framework. The pricing procedures comply with the no-arbitrage conditions. Using these methods, we can price options at any node of the scenario tree, thus allowing transactions with options at any decision stage. The option pricing procedures are not dependent on any explicit assumption regarding the distribution of the underlying security. In principle, they can be applied to any arbitrary discrete distribution (scenario tree) for the prices of the underlying. We validate through extensive empirical tests the proposed option pricing valuation procedures when the scenario sets for the prices of the underlying securities are generated by a moment-matching method.

We incorporate options in stochastic programming models for risk management purposes. These tools provide the means to investigate the performance of alternative tactics to mitigate market risk, including popular strategies that enforce specific combinations of stock index options. We find that the inclusion of options in the portfolio can materially reduce the downside risk. Portfolios that include options have return distributions with significantly lower tails and exhibit (more) positive skewness in contrast to the distributions of portfolios without options.

We extend a valuation procedure to price quantos; these are fixed exchange rate options on a foreign equity. They are used to treat jointly the market and currency risks of a position in a foreign equity index.

We analyze the hedging effectiveness of currency options in international portfolios. We examine the hedging role of currency options and we identify appropriate trading strategies of currency options by comparing empirically their performance against the use of currency forward contracts. Despite the advocacy of currency options for foreign exchange risk management, their incorporation in practical portfolio management models had remained largely unexplored. Empirical results indicate that the use of a single put currency option per currency leads to inferior performance compared to forward contracts. But appropriate trading strategies of currency options lead to superior results. The asymmetric nature of these options is suitable for hedging since currency options eliminate the risk from potential currency depreciations, while allowing for benefits from currency appreciations.

We develop an integrated risk management framework where the main risk factors are considered simultaneously using appropriate types of instruments (forward contracts and options). Considering both market and currency risks in a unified manner allows us to assess the sensitivity of the portfolio value to each risk factor, and hence assess the significance of each risk component to the overall risk of the international portfolio. The options schemes proposed in this study work best — and most efficiently — when we consider simultaneously both risk factors. Multistage stochastic programming models for international portfolio management generate optimal positions in international assets and optimal hedge levels. We use appropriate trading strategies to hedge against the risk factors either independently or jointly. We find that increasingly integrated views towards risk result in more effective risk management strategies, compared to unhedged positions. The more risk factors we hedge, the more stability in the performance of portfolios we get, and the greater the performance we attaint.

Our empirical results show that quantos provide particularly effective instruments for risk hedging purposes. This is due to the integrative nature of these instruments that cover both the market risk of the underlying security and the associated currency risk. Overall, we observe that progressively integrated views towards risk management are increasingly more effective. That is, incremental benefits in terms of reducing risk or generating cumulative profits can be gained as more risk factors are progressively controlled through appropriate hedging strategies. Hence, we demonstrate that integrated consideration of market and currency risks can yield substantial benefits for international investors. This result could possibly generalize to other portfolio management contexts that are governed by multiple risk factors.

All the models are tested empirically using market data. We compare the risk-return profiles (efficient frontiers) of alternative decision tactics in static tests. We also run backtesting experiments on a rolling horizon basis for more substantive comparisons. Observed values of asset prices and exchange rates are used to compute the realized returns of the optimal portfolios. This ex post evaluation of realized returns provides a more reliable performance assessment of alternative hedging strategies and instruments.

The primary novel contributions of this thesis concern:

- The application of multistage stochastic programming models to international financial management problems and its use as a testbed to empirically investigate the performance of various decision strategies.
- The incorporation of options in stochastic programming models for portfolio risk management.
- The development of an integrated modelling framework that can address in a unified a comprehensive manner a number of inter-related decisions in international portfolio management (i.e., optimal selection of a diversified portfolio and associated decisions for controlling market and currency risk).
- The extensive empirical assessment of alternative instruments and strategies for coping with market

and currency risk of international investments either separately, or jointly.

1.5 Thesis Overview

In the second chapter we review representative literature that pertains to the research threads considered in this thesis. we discuss works on the topics of international asset allocation, currency hedging, risk management, financial applications of stochastic programming, scenario generation approaches, and relevant option pricing methods. The development in this thesis touch on all these research domains. Of course, we do not exhaustively cover all major contributions in each of these topics. we focus on prior studies that form the foundation for the incremental contributions of this thesis.

The remaining chapters of the thesis gradually lead to the integrated framework of dynamic international portfolio management. We start with a single-stage model for international asset allocation where forward rates are used to (partly) hedge the currency risk, while the market risk is not explicitly addressed except via international diversification. Then we extend this model into a multistage setting. Next, we adopt methods for pricing and incorporating options in stochastic programming models. The next step uses stock options to hedge the exposure to market risk. We continue by introducing currency options to hedge currency risk. Finally, we develop an integrated decision framework where both kinds of options are incorporated in multistage stochastic programming models simultaneously, examining their effectiveness to control the total risk of international portfolios.

In chapter 3 we develop a simulation and optimization framework for multicurrency asset allocation problems in a static (single-period) setting. A principal component analysis approach is applied to generate scenarios depicting the discrete joint distributions of uncertain asset returns and exchange rates. We formulate and implement a stochastic programming model to optimize the conditional-value-at-risk (CVaR) metric of the portfolio's asset returns so as to minimize excess shortfall risk (i.e., the conditional expectation of portfolio losses in the tail of the distribution). The scenario-based optimization model includes currency hedging decisions based on the principle of selective hedging through forward currency exchanges. The model determines jointly the portfolio composition across currencies and the level of currency hedging for each market. We examine empirically the benefits of international diversification and the impact of hedging policies on the performance of international portfolios.

In chapter 4 we extend the international portfolio management model to a dynamic decision framework. We implement multistage stochastic programming models with a longer time horizon that consider successive portfolio rebalancing decisions at multiple time periods. The stochastic programming models capture decision dynamics, include an operational treatment of hedging decisions by means of implementable forward exchange contracts, and account for the effect of transaction costs. A moment-matching scenario generation procedure is adopted to represent uncertainty in asset returns and exchange rates in accordance with their observed empirical distributions. We test different hedging strategies concerning the permissible level of forward positions in each foreign market, and we examine empirically the impact of hedging policies on risk-return profiles of portfolios. We investigate the ex ante and ex post performance of the models for managing international portfolios of stock and bond indices. We find that multistage models improve both the ex ante and ex post performance of international portfolios over the single-period models.

Derivative securities are particularly well-suited for risk management purposes. By construction, options have highly asymmetric payoffs that can effectively cover against adverse price movements of the underlying security. In chapter 5 we adapt, implement and empirically validate appropriate methods to price options in accordance with discrete scenario sets that depict the distribution (and dynamic evolution) of asset prices. We confine our attention to European-type options that can be exercised only at the expiration time (maturity). We apply two different methods for pricing the options in accordance with the discrete distributions of a scenario tree. The proposed option pricing methods are motivated from the need to achieve internal consistency in stochastic programming models when options are incorporated, but their use is not confined to these models only. The proposed methods hold their own as promising option pricing methods. We demonstrate through empirical validation tests using market prices for the S&P500 stock index and options on this security that the proposed valuation procedures yield prices that are closer to the market prices of the options than those obtained by the Black-Scholes method, especially for deep out-of-the-money options. These procedures also allow us to investigate the impact of higher moments (skewness, kurtosis) of the discrete distribution of the underlying asset in the resulting option prices, as well as implied volatility smile effects that are typically observed in practice. Moreover, we are able to estimate the risk-neutral distribution of the underlying asset and to study its characteristics, thus explaining skewness premia effects.

In chapter 6 we focus on means for mitigating market risks. We extend the single-stage model of chapter 3 by introducing stock options in the portfolio for hedging against the market risk. Specifically, we consider two different types of European options on stock indices: unhedged (simple) and fully protected options (quantos). The first type of option is intended to hedge the market risk associated with the underlying stock index and does not account for changes in currency exchange rates. Quantos are options written on a foreign stock index that also protect against currency movements as they apply a prespecified exchange rate to convert the payoffs of the option to a different currency (i.e., the reference currency of the investor).

The introduction of options broadens the investment opportunity set and provides instruments geared towards risk control due to the asymmetric and nonlinear form of option payoffs. We suitably extend the portfolio optimization model so as to incorporate the options and we empirically investigate the impact that these options have on the performance of international portfolios of financial assets. The residual currency risk from other foreign holdings (e.g., in bond indices) can be covered with forward currency exchange contracts that are also included in the model. The goal is to control the portfolio's total risk exposure and to attain an effective balance between portfolio risk and expected return. In this chapter we confine our attention to a single-period model in order to simplify the notation and the presentation of the novel concepts.

In chapter 7 we investigate the use of currency options as means of controlling the currency risk of foreign investments in the context of international portfolios. Due to the asymmetric nature of their payoffs, options are particularly suitable instruments for risk management. Currency options can cover against losses from potential unfavorable changes in exchange rates, while preserving the upside potential. We adapt the multistage stochastic programming models developed in chapter 4 so as to incorporate decisions for optimal positions in currency options. We apply a valuation procedure presented in chapter 5 to price the currency options consistently with the postulated discrete distributions of asset prices and exchange rates. We investigate the effectiveness of decision strategies that employ currency options to control currency risk exposures in portfolios of international financial assets. To this end, we empirically compare such strategies against the alternative use of currency forward contracts as means of controlling currency risk. The goal is to identify appropriate means for risk management in international portfolios. Besides individual options, we also consider trading strategies that involve combinations of options and have specific payoff characteristics.

In chapter 8 we combine and extend the developments of all previous chapters in the context of an integrated risk management framework. We show that increasingly integrated views towards total risk management are more effective compared to consideration of constituent risk components in isolation. We implement multistage stochastic programming models that incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively. These models shape the overall portfolio risk profile accounting for both the market and the currency risks, thus contributing directly to the objective of total risk management. Empirical results indicate the superiority of this integrated risk management scheme. The simultaneous use of stock and currency options yields the best ex post performance of international portfolios. Moreover, the multistage model clearly outperforms the myopic (single-stage) variant of the model. The integrated modelling framework of this chapter provides a complete basis to comprehensively address all relevant decisions in international portfolio management in a unified manner.

Finally chapter 9 summarizes developments and main findings of the thesis.

Chapter 2

Literature Review

2.1 Introduction

This thesis studies the problem of international portfolio management. This problem is indeed complex, and involves several issues that are considered in a unified framework. Risk management of international portfolios is a major issue, since multiple risk factors are present in this context. Multistage stochastic programs with recourse are the particularly suitable tools to address this problem. Stochastic programs posses several attractive features that make them applicable in diverse practical problems.

With the accelerating trend towards international investments, the debate on optimal portfolio allocation and the associated market and currency risks becomes imperative. Asset managers aim to select investment portfolios that yield the maximum possible return, while at the same time ensuring an acceptable level of risk exposure. Risk derives from potential losses in portfolio value due to possible reductions in the market values of financial assets resulting from changes in equity prices, interest rates, foreign exchange rates, credit ratings of security issuers, etc. Diversification into multiple securities can practically eliminate idiosyncratic risk, that is, potential severe losses from any individual security. However, domestic diversification cannot mitigate systematic market risk. This is the risk associated with concurrent losses in most domestic securities due to high correlations between their returns. Since market risk differs from country to country, international diversification can reduce the overall risk exposure of investment portfolios.

Portfolios may benefit from international risk sharing in several ways: First, international investments can broaden the opportunity set, rewarding investors for higher return with the same risk, or lower risk for the same return. Second, international asset allocation may help portfolio managers diversify risk, offer more investment choices, or lower shortfall risk. Although, with the integration of international capital markets, institutional investors' holdings of foreign securities are increasing, actual investments are still strongly home-biased. Several explanations have been suggested for this phenomenon. One explanation is that foreign returns implicit in equities of domestic firms that have overseas operations may help international diversification. Another explanation is that due to government restrictions, transaction costs and taxes, diversification costs may exceed gains. However, none of these explanations is convincing.

While a part of currency and market risk (idiosyncratic risk) is diversified in an international portfolio by investing in many different currencies and assets, systematic risk would still remain. To manage the exposure to the multiple risk factors that affect the performance of international portfolios, a variety of approaches have been proposed. However, still there is no clear agreement, neither on the approaches nor on the hedging instruments and strategies. The risk management of international investments is an extremely important topic for banks and other financial institutions. Therefore, it is important to study the appropriate hedging strategies and instruments, and the extent to which investors should hedge. Below we review major studies on international asset allocation, and we identify the incremental contribution of our work in this direction.

Risk management is the discipline that provides tools to measure the risks and techniques to help us shape and make rational decisions about them. The first task is to measure the risks. Risk measurement identifies the risk factors that affect the performance of international portfolios. This involves the analysis of how separate risk factors interact and influence international portfolios. Then these individual risks are shaped into an overall risk profile for the portfolios. The definition of risk is a particularly difficult task. It can be defined as the uncertainty of the future outcome of a decision today. To measure the risk, a number of measures have been proposed in the literature. The standard deviation, also known as volatility, has been the most widely used measure. However, this measure relies on the assumption that the portfolio return distribution is symmetric and implies that the sensitivity of the investor is the same on the upside as on the downside. In order to take the asymmetry of the portfolio return distribution into account, the use of downside measures has been advocated. In this chapter we also review studies on risk management and risk measures, and we justify our choice of a measure to control the excess shortfall risk of international portfolios.

Stochastic programming is our approach to deal with uncertainty. This approach can deal simultaneously with all important aspects of international investments, such as the management of multiple risks, and the identification of the optimal portfolios. The stochastic programming method overcomes limitations of other techniques such as continuous-time finance, dynamic programming or optimal control. These methods provide significant insights about fundamental issues in investments and risk managements, and are very good approaches for theoretical contributions, but their practical use is limited by the many simplifying assumptions that are needed to derive the solutions.

In stochastic programs the uncertainty is treated using discrete random variables, that is, random variables that take on a countable set of values. With each of these values we associate a probability. Particularly, we assume that as time evolves, the random variables take one from a finite set of values, which is a discrete scenario. The set of these scenarios with the associated probabilities is a discrete distribution. These scenarios are then used as inputs to the stochastic programming models for managing international portfolios. Thus, the scenario generation procedure is a critical step for the optimal international asset allocation. We review the major applications of stochastic programming models to financial problems, as well as scenario generation procedures. The review justifies our choice to use stochastic programming models, and the choice of the particular scenario generation procedure that is adopted in this thesis.

Finally, options are used to hedge the exposure of international portfolios to market and currency risk. In this work we study the effectiveness of controlling these risk factors using derivatives. Alternative hedging strategies of various types of options are analyzed, and compared with each other. The incorporation of options in multistage stochastic programming models is a challenging task, since the options must be priced consistently with the postulated scenario set for the underlying assets, in a way that no arbitrage conditions are enforced. We review the major studies in pricing options (both stock and currency options).

2.2 International Asset Allocation

International diversification is practiced by institutional investors to improve the risk-return profiles of their portfolios. The inclusion of securities denominated in foreign currencies in the asset holdings can provide dual benefits: (1) the prospect for higher profit in the event of favorable performance of foreign markets and (2) the potential reduction in the portfolio's exposure to market risk. However, international investments introduce a new element of risk (currency risk).

International asset allocation is a major concern for global investors and multinational corporations, and a promising alternative for institutional investors. The merits of international diversification were originally shown by Grubel [83] and Levy and Sarnat [133]. Relatively high positive correlation among securities within an economy suggest the possibility that risk reduction might be facilitated by diversifying portfolios internationally. In their papers they discuss the potential gains from such diversification.

International diversification has now become an established practice for many investors and financial institutions. Compared to investments in domestic assets, fluctuating exchange rates represent an additional risk factor for investors who diversify their portfolios internationally. An argument against international diversification is that currency risks may offset the reduction in market risks. Indeed, currency fluctuations affect both the total return and the volatility of foreign currency-denominated investments. Does this mean that currency risk is so large that investors should avoid foreign investment? Not at all, for several reasons. First, market and currency risks are not additive as exchange rates are not perfectly correlated with domestic returns of assets. Second, the exchange risk of an investment may be hedged for major currencies by selling forward contracts or buying currency options. Third, the contribution of currency risk should be measured for the entire portfolio rather than for individual markets or securities since part of the risk gets diversified away by the mix of currencies represented in the portfolio (i.e., cross hedging).

When investors diversify their portfolios internationally, they face a numbers of questions: How important is the impact of international diversification? Does it compensate for the additional currency risk associated with foreign investments? Can the overall portfolio risk be significantly reduced by systematically hedging currency risk? Should the hedge ratio be less than one? What are the appropriate hedging instruments and strategies for an effective hedging scheme? These questions have to be appropriately addressed.

Empirical evidence suggests that currency hedging may improve the risk-return performance of international portfolios. For instance, Solnik [172] in his original derivation of the international capital asset pricing model shows that an optimal internationally diversified portfolio contains long and short positions in currency forward or future contracts or synthetic positions in the international cash market. Thus, currency hedging becomes an integral part of the overall international asset allocation decision. Eun and Resnick [68] indicate that stock portfolios perform better when fully hedged. Eun and Resnick [69] further consider international portfolios of stocks, bonds, and stocks and bonds. They show that when exchange rate risk is hedged with forward contracts, the risk-return relationship is improved over unhedged international investments for bond portfolios and stock and bond portfolios. Jorion [114] shows that international diversification of stocks and bonds is beneficial to portfolio returns, but the optimal hedging strategy is time-dependent. The above works show that if the investors would not control the uncertainty of the foreign currency exposure, the potential gains from international diversification may not be enough to justify the expense of an international investment. Due to the high correlations among the exchange rate changes in some periods, much of the exchange rate risk may remain nondiversifiable in a multi-currency portfolio. Therefore, it has been widely discussed that investors can conceivably eliminate much of the exchange rate risk using forward contracts. Using the unitary forward hedge ratio strategy in order to hedge the exchange rate risk, Eun and Resnick [69] show that such a strategy would reduce the volatility of the portfolio returns without a substantial reduction in average returns, in comparison to unhedged portfolios.

Perold and Schulman [155] advocate the view that currency hedging is a "free lunch" implying that 100% of foreign currency exposure should be fully hedged. Other researchers propose universal hedge ratios different that one. Black [26, 27] shows that under additional assumptions (such as the CAPM hold for many countries) the hedge ratios should be identical for all investors universally, and that investors should never fully hedge their foreign currency exposure, suggesting a 0.75 hedge ratio. The problem is how to identify this optimal hedge ratio, as it depends on a number of parameters such as the relative preferences of different nations or risk aversion, which cannot be objectively and easily determined. Gastineau [79] suggests a 0.50 universal hedge ratio. Black's universal hedge ratio uses arbitrary nonmarket weights in the definition of the market excess return on the market portfolio and its volatility. The universality of Black's hedge ratio follows from the assumptions that impose homogeneity on world investors. These assumptions require foreign investment to be in balance for all countries at all times.

Filatov and Rappoport [71] show that complete hedging is dominated by selective hedging, in which the hedge ratio may vary across currencies. They also show that the signs of covariances between exchange rates and domestic returns have historically been different across base currencies. Thus, complete hedging was optimal for a US investor for the period 1980-1989, while selective hedging was optimal for a non-US investor over the same period. Adler and Prasad [6] weaken some of the underlying assumptions and generalize Black's results by substituting universal hedges with regression hedges. They propose that investors use the minimum variance hedge ratios (regression coefficients) that result from regressing the world market portfolio, or another representative stock market index, on third currencies. Jensen's inequality guarantees that the hedge ratios will be the same for each national investor regardless of the numeraire currency. Larsen and Resnick [132] perform an ex ante study comparing unhedged international equity investments, unitary hedging, Black's universal hedge ratio, an arbitrary estimate of 0.77 of Black's universal hedge ratio, and the universal hedge ratios of Adler and Prasad. They find that a unitary hedging strategy works as well as, or better, than Black's or Adler and Prasad's universal hedging techniques under uncertainty. Froot [76] argues that, in the short run it certainly is possible to reduce variance through currency hedging. With an investment horizon of up to five years, a hedge ratio close to 100% is therefore recommended. But over the very long run, currency risk washes out. This suggests that a portfolio with a long horizon should follow a no-hedging policy. The no-hedging argument of Froot is based on a very long horizon (150 years). It assumes that the exchange rate reverts to its fundamental or purchasing power value over the long run. Although all these studies show that currency hedging improves the risk-return performance of international portfolios as well as reducing risk, they exhibit considerable disagreement on the strategies that the global investors should use to hedge the currency risk. Specifically, there is no consensus on how much to hedge or on optimal hedge ratios.

Glen and Jorion [80] consider dynamic hedging strategies. They report a temporal instability of the optimal policy, which means that the historical evidence may not give a consistent guideline about the optimal hedging strategy, and that large benefits could be obtained by following a state contingent tactical policy which varies the level and the structure of hedging depending on the investment opportunity set. They find that selective hedging is always better than either complete, or no hedging, or partial hedging for passive indices, but the results are statistically significant only for the hedged bond portfolio against the unhedged. Jorion [115] analyzes the effects of separating the investment and the currency hedging decisions. This is the case when the asset allocation is performed by one manager and the currency hedging decision is delegated to a currency overlay manager who treats the asset allocation as given ("partial optimization") and optimizes only hedging decisions on currencies. This is the approach typically used in practice. Jorion shows that conducting an asset allocation optimization for assets and currencies separately is clearly suboptimal. This conclusion motivates us to develop models that integrate the asset allocation with the hedging decisions. Abken and Shrikhande [3] confirm that the efficient frontiers for a US investor choosing stocks and bonds of seven countries is unstable across portfolios and across periods. They show that hedging is the best policy for equities over the period 1980-1985, while no hedging is optimal over the period 1986-1996. They find that the two policies are complementary in the case of bond portfolios. Solnik [173, 174] find that for a Swiss investor, average unhedged returns are very close to average fully hedged returns of stock and bond indices, over the period 1971-1996. He also finds that the standard deviation of hedged returns is lower than the standard deviation of unhedged returns (3-4 % for stock indices and 8-10 % for bond indices). The results of Solnik's studies indicate that, even if in the very long run hedging currency risk may be unimportant, in the short or medium term there is room to optimize investor-specific currency risk hedging decisions.

All these studies consider three major strategies: (a) Unitary hedging, which means that there is either complete or no hedging (i.e., the hedge ratio is either zero or one), (b) Partial hedging, where the hedge ratio can be different from zero or one, but uniform across countries and assets. Lately, Beltratti et al. [22] employed selective hedging, which is the more general approach, because the hedge ratio may vary across currencies and take any value between zero and one. Selective hedging was already identified by others as the more general strategy (encompassing the others as specific cases) and allows joint optimization of portfolio selection and currency hedging decisions. This is a starting point for the models we develop in this thesis.

Although there exist many empirical studies on the use of forward contracts to hedge the currency risk, the evidence for using options is not as much. Wong [188] examines the optimal hedging decision of a competitive exporting firm under hedgeable exchange rate risk and non-hedgeable price risk. The firm avails itself of a rich set of risk-sharing instruments including currency options and futures. He found that a long position in put options is the firm's optimal hedging strategy. Chang and Wong [46] study how a risk-averse multinational firm can employ derivative securities on related currencies to reduce its foreign risk exposure. They find that when the exchange rates are correlated, the firm would optimally use currency options for hedging purposes. Lien and Tse [134] compare the hedging effectiveness of currency options versus futures on the basis of lower partial moments (LPM). They find that currency futures are always a better hedging instrument than the currency options. The only situation in which options outperform futures occurs when the individual hedger is optimistic (with a large target return) and not too concerned about large losses. Steil [176] applies an expected utility analysis to derive optimal contingent claims for hedging foreign transaction exposure as well as optimal forward and option hedge alternatives. Using three utility functions, the quadratic, the negative exponential and the positive exponential, Steil shows that currency options have little, if any, useful role to play in the hedging of contingent foreign exchange transaction exposures. In all cases the optimal hedge consists of forward contracts. Hull and White [96] analyze the ways in which banks and other financial institutions can hedge their risks when they write non-exchange-traded foreign currency options. They find that Delta+Gamma hedging performs well when the options have a fairly constant implied standard deviation and a short time to maturity. Delta+Sigma hedging outperforms other hedging schemes when the options have a non-constant implied standard deviation and a long time to maturity.

Another issue addressed in the literature is that of the asymmetric nature of foreign exchange exposure due to the hedging strategies implemented by the companies. Booth [31] examines the role of transaction costs and the asymmetry produced in the firm's profit function in an attempt to provide a more realistic analysis of the use of hedging strategies, specifically currency options, that provide downside protection while allowing for upside potential. These asymmetric payoffs lead one to hypothesize that exchange rate exposure may display an asymmetric behavior. In his study, Booth suggests that hedging instruments with asymmetric payoffs, such as currency options, appear to be most useful.

Some of these studies apply currency options to hedge currency risk. The reported results are conflicting. Some of the aforementioned studies find that currency options constitute effective currency hedging instruments. In the majority of them, currency options are dominated by forward contracts. Our empirical results show that a currency hedging strategy consisting of a single put currency option leads to inferior performance compared to forward contracts. But properly constructed combinations of currency put options can lead to significant performance improvements. Thus, another purpose of this thesis is to examine the hedging role of currency options and to identify the appropriate trading strategies of these options. Another issue examined in this thesis is the proper application of currency options as hedging tools. Despite the widespread advocacy of using currency options in foreign exchange risk management, the question concerning their proper incorporation in portfolio optimization models has remained largely unexplored.

The overview presented above shows that there are many open questions in international portfolio management. The return of a foreign asset varies not only because of asset specific risk, but also because of unpredictable fluctuations in exchange rates. The empirical studies indicate that the effect of currency hedging on the performance of international portfolios is not universally clear. Several of the above studies reach very different conclusions. Thus, the optimal currency hedging strategy in international asset allocation is still an open question. The main conclusions of the studies mentioned above are the following. First, the merits of the hedging strategies remain an empirical issue and depend on factors such as the investor's opportunity set, the reference currency of the investor, the risk aversion and the time horizon, the timeframe of the particular study, the distribution of asset returns and interest and exchange rates, and the hedging policies and instruments that are used. Second, currency hedging is more significant, the more correlated the domestic asset returns are with exchange rates. Third, it is generally observed that there is an increase in local market correlations and that the volatility is contagious across markets.

The portfolio manager today is faced with a number of problems. A crucial is the identification of the risk factors influencing international portfolios. In particular, currency risk is a major source of uncertainty for the internationally diversified investor. Recent evidence signifies the importance of a joint consideration of currencies and asset price movements. What is needed is an integrated decision framework for both currencies and assets, but so far researchers and the practitioner community have largely relied on separate consideration of these risk factors. Typically, a large institution's portfolio is managed by multiple managers who follow the constraints imposed by the global investment committee. This committee decides the overall allocations to each geographical area and to each currency, and leaves limited freedom to the individual managers. Currency managers are then responsible for addressing the portfolio's currency exposures. Asset managers, in turn, apportion their designated allocation to various asset classes or individual assets, generally following in-house analysts' recommendations. As pointed out by Jorion [115] this overlay structure is inherently suboptimal because it ignores interactions between the assets in the portfolio and exchange rates (i.e., cross-hedging effects).

These observations point to the need for integrated simulation and optimization approaches that determine jointly portfolio structures and flexible hedging policies. Our models move exactly in this direction. The international portfolio management problem analyzed in this thesis, concerns the optimal allocation of funds to international assets within foreign markets, and the dynamic management of the portfolio through rebalancing in tactical periods. Essential features of our modelling tools include the representation of uncertainty in a manner that captures jointly the co-movements of the risk factors, the selection of the appropriate investments in each market, and the specification of currency hedging decisions for each market. All these decisions are considered in an integrated framework.

2.3 Risk Measures

Risk is an inevitable consequence of productive activity. This is especially true for financial activities where the results of investment decisions are observed or realized in the future, in unpredictable circumstances. Risk is the degree of uncertainty in attaining a certain level of portfolio return. It reflects the chance that the actual return of the portfolio may be very different than the expected return. Risk in international portfolios arises due to uncertainties in the return of the assets (market risk) and the exchange rates (currency risk). Investors can neither ignore nor insure themselves completely against these risks. They must be aware of their exposures to these risk factors and take them into consideration in their decision process so as to properly manage their total risk. This is not a simple task, even with the support of advanced mathematical techniques; poor risk management practices can lead to spectacular failures in the financial industry, as those witnessed during the last decade (e.g., Barings Bank, Long-term Capital Management, Orange County).

These observations point to the need for effective risk management techniques, that simultaneously take into consideration the important risk factors. It is crucial to control the different risk factors that affect the performance of international portfolios. The liberalization/integration of markets and their consequent interdependencies, increasing complexity of innovative financial instruments, intensifying competition, regulatory environments and the realization of severity of potential losses dictate the development of integrated portfolio risk management models that take into account all the above needs and control simultaneously the exposure of international portfolios to different risk factors. Derivative securities (forwards, stock and currency options) provide appropriate means to hedge the multiple risks. The inclusion of these instruments with asymmetric payoffs tilt the portfolio return distribution to the right. In order to control the overall portfolio risk, we need to employ risk measures that account for this asymmetry of portfolio returns.

The application of effective decision support tools for risk management must employ risk measures that properly reflect and quantify the risk exposure of an investor. Some examples of risk measures include variance, as in the traditional Markowitz MV model, the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CVaR) of portfolio losses. The risk management models must be able to flexibly incorporate different risk measures when this is required by regulators or management preferences. Regulators already mandate that financial institutions should control their holdings in certain ways and hold margin requirements for “risky” positions (e.g., The Basel Accord).

Optimization models for financial risk management often take the following form: an appropriate risk measure is optimized subject to operating constraints and a parametric constraint that a desirable performance measure (such as expected portfolio return) meets a prespecified target level.

Markowitz [139] proposes to express the risk of an asset's return by means of the deviation from the expected return (i.e., by the variance). So, for a portfolio of correlated assets the risk must be gauged via the covariances between all pairs of investments. The main innovation introduced by Markowitz is to measure the risk of a portfolio by taking into account the joint (multivariate) distribution of returns of all assets in the portfolio. In the mean-variance framework, the investor composes an efficient portfolio from a large universe of risky assets. Tobin [180] adds unlimited borrowing and lending at the riskless rate to the mean-variance model and shows that every investor holds a combination of just two portfolios: the riskless asset and a portfolio of risky assets. Sharpe [168], Litner [135], and Mossin [144] derive equilibrium models for the entire financial market based on the mean-variance framework of Markowitz and the two-fund separation result of Tobin, known as CAPM.

Sharpe [169] suggests the Mean Absolute Deviation (MaD) measure to quantify risk. The MAD is a constant multiple of the standard deviation, provided that the probability distribution of the random returns is normal. The advantage of this approach is that the resulting optimization problem can be transformed to a linear program. MAD models have been applied to various portfolio optimization problems; see, for example, [126, 117, 22, 129].

Markowitz describes the dependence structure of the random returns via the linear (Pearson) correlation coefficient between each pair of random returns. But the correlation coefficient is strictly related to the slope parameter of a linear regression of one random return on another, and measures only the linear components of the co-dependence of the random variables. This approach assumes that the risk-return characteristics of a portfolio are fully captured by the first two moments of the random returns, ignoring higher moments; it is appropriate when asset returns follow normal distributions. This framework is an initial step towards more realistic risk measures that are better suited when investment returns exhibit skewed, leptokurtic and/or heavy tailed distributions as it is the case in practice (e.g., Fama [70]). Moreover, if options are added to the investment opportunity set, then the portfolio return distribution becomes asymmetric. This is because derivative securities exhibit asymmetric payoff and, hence, cannot be adequately analyzed using risk measures that are geared towards symmetric return distributions. In order to deal with these problems, downside risk measures have been introduced (e.g., Bawa and Lindenberg [20], Fishburn [72], Harlow [87], Sortino and van der Meer [175]).

Recently (e.g., see Szegö [177] for a review) the study of extreme events — i.e., of the tails of the return distribution — has received due attention. The research on new risk measures is partly driven by new trends in regulatory requirements for financial institutions that dictate the use of sophisticated risk control models, and by the reaction of the reserach community to the introduction of inadequate or improper risk measures in the context of regulations. A complete state of the art review of risk measures is contained in the paper by Frittelli and Rosazza Giannin [75].

Over the last few years *Value-at-risk* (VaR) has become a standard metric for measuring risk for banks and other financial institutions. It's use has been popularized partly because tools for computing VaR have been made widely available (RiskMetrics [161]). VaR is a more natural measure of risk than other statistics, such as variance, semi-variance, or mean absolute deviation. VaR is a percentile based metric. It is usually defined as the maximal allowable loss with a certain confidence level $\alpha * 100\%$. Christoffersen et al. [48] show how to test that a VaR measure is properly specified. Regulators such as the Bank for International Settlements recommend the VaR-measure, together with expected return, as a basis to determine capital adequacy requirements.

Despite its popular use in risk measurement, VaR is not typically used in mathematical models for optimal portfolio selection. It is severely criticized for not being sub-additive, hence it is not a coherent risk measure in the sense of Artzner et al. [11]. This means that the risk of a portfolio can be larger than the sum of the risks of its constituent components when measured by VaR (see also Gaivoronski and Pflug [77]). While its calculation for a certain portfolio $\mathbf{x}$ reveals that the portfolio losses will exceed $\text{VaR}(\mathbf{x}, \alpha)$ with likelihood $(1 - \alpha) * 100\%$, it provides no information on the extent of the distribution's tail which may be quite long; in such cases, the losses may substantially exceed VaR and result in catastrophic outcomes. A reduction in VaR may stretch the tail beyond VaR (see, for example, Szego [177]). It may provide conflicting results at different confidence levels. Moreover, VaR is difficult to optimize. When the asset returns are specified in terms of discrete distributions (i.e., scenarios) the VaR function is nonsmooth and non-convex with respect to the portfolio positions and exhibits multiple local extrema. Efficient algorithms for solving problems with such objective functions are lacking.

Hence, substantial modelling and computational difficulties are faced in constructing large-scale structural models of trading risks for large, complex portfolios. Berkowitz and O'Brien [23] evaluate the performance of of risk measurement models of banks by examining the statistical accuracy of the VaR forecasts. Their results show that the VaR forecasts for six large commercial banks exceeded nominal coverage levels over a two-year period, and for some banks, VaRs were substantially removed from the lower range of trading Profit and Loss. Despite the detailed information employed in the bank models, their VaR forecasts did not outperform forecasts based on an ARMA + GARCH model.

Conditional value-at-risk (CVaR) is a related risk measure. It is usually defined as the conditional expectation of losses exceeding VaR at a given confidence level (VaR is also defined as a percentile of a loss function in this case). CVaR is a coherent risk measure in the sense of Artzner et al. [11]. It is also a continuous and convex function in the portfolio positions. Detailed analysis of the CVaR risk measure are provided in Rockafellar and Uryasev [162, 163], and Acerbi and Tasche [4, 178]. Alexander and Baptista [8] analyze the implications arising from imposing a CVaR constraint in an agent's portfolio selection problem, and compare them with those arising from the imposition of a VaR constraint. For a given confidence level, a CVaR constraint is tighter than a VaR constraint if the CVaR and VaR bounds coincide.

In this thesis, we focus on the development, implementation and empirical validation of stochastic programming models for international portfolio management that employ the conditional value-at-risk (CVaR) metric along with the lines suggested by Rockafellar and Uryasev [162, 163]. By optimizing

CVaR we mimimize the conditional expectation of portfolio losses in excess of a prespecified percentile of the return distribution, (i.e., we minimize the expected losses beyond VaR). Our motivation for applying CVaR models stems from the observation that returns of international assets and proportional changes of exchange rates are not normally distributed; they exhibit asymmetric distributions with fat tails. Moreover, we incorporate derivative securities in the optimization models. The non-linear payoff characteristic of options lead to asymmetric portfolio return distributions. CVaR is a coherent risk measure and is suitable for asymmetric distributions. Applications of the CVaR risk measure in financial planning problems are reported in and Uryasev [183], Andersson et al. [10], Krokhmal et al. [130] and Topaloglou et al. [181].

2.4 Stochastic Programming Applications in Finance

Many practical optimization problems involve uncertain parameters. When these uncertain parameters are treated as random variables with known probability distributions, stochastic optimization programs can be formulated. In stochastic programs uncertainty is typically represented by discrete distributions of the random variables; that is, random variables values from a discrete countable set. With each outcome from this set we associate a probability. In our problems, random variables represent the asset returns and the exchange rates. Particularly we assume that as time evolves in discrete increments, the random variables take values from a discrete set of outcomes, giving rise to distinct scenarios. When all scenarios are enumerated and appropriate decision variables are defined (satisfying non-anticipativity), we obtain a large-scale deterministic equivalent program.

In dynamic problems that involve decisions at multiple successive periods, the evolution in the outcomes of the random variables is depicted by a scenario tree, which is a critical input for the stochastic programming model. The root node of the scenario tree represents the current moment from which a number of branches extend, corresponding to possible discrete transitions of the variables from the current moment to the second decision time. Each of these branches ends at a successor node which itself has further branches, representing discrete outcomes of the random variables from the second to the third decision stage. This process is repeated until every decision time is represented by a collection of nodes in the tree. Once the scenario tree is constructed, the deterministic equivalent optimization program is formulated by associating a set of decision variables with each node of the tree, and by expressing the objective function and the constraints in terms of these decision variables and the corresponding postulated realizations of the random variables on the nodes of the scenario tree.

The paradigm of stochastic programming is applicable to risk management problems. The origins of stochastic programming can be traced to the early works of Dantzig [56] and Beale [21] in the 1950s. The applicability of stochastic programs to financial planning problems was first recognized by Crane [55], and Bradley and Crane [35]. Kusy and Ziemba [131] developed a multistage stochastic programming model for bank asset and liability management in the presence of uncertainty in deposits and withdrawals from accounts. Their results indicate the superiority of the stochastic programming model over a linear programming approximation based on the use of mean values for the random parameters. While stochastic programming models had been introduced several decades ago, computational technology has only recently allowed the solution of realistic size problems arising in practice. The field continues to develop with the advancement of available algorithms and computational power. It is a popular modelling framework for problems under uncertainty in a variety of disciplines including financial engineering.

Stochastic programs can accommodate different objective functions to capture the decision maker's risk taking preferences (e.g., utility functions, penalties on shortfalls and other risk measures, etc.) Moreover, they can effectively incorporate diverse managerial and regulatory requirements in their constraint sets, especially when such requirements are expressed in terms of linear constraints (inequalities or equalities) on the decision variables. Because of their flexibility, stochastic programs have attracted the attention of researchers and practitioners alike and are being increasingly applied in various problems. Applications of stochastic programming models to diverse practical problems are documented in the recent volume edited by Ziemba and Wallace [186]. Several representative financial applications of stochastic programming models can be found in collective volumes, Zenios [190], Luenberger et al. [137], Vladimirou et al. [184], Jarrow et al. [107], Ziemba and Mulvey [195], Birge et al. [25], Ziemba and Zenios [167].

Yet, only in recent years are stochastic programs gaining prominence in financial planning, following the development of efficient algorithms that exploit the special structures of large-scale stochastic programs. The advent of high performance parallel computing has enabled to tackle large-scale instances of stochastic programs for real-world problems (e.g., see Zenios [193] and Pflug [158]). Stochastic programs with recourse now enable us to solve planning problems under uncertainty that closely reflect dynamic risk-management decision problems.

Stochastic programming models are seeing an increasing use in financial applications in recent years. Numerous applications have been reported in the literature. For example, Wilkie [187], Mulvey [145], and Mulvey and Thorlacius [149] build forward-looking nonlinear factor models for pension plans. A plan's surplus is calculated as the value of assets minus the market value of liabilities. The problem of asset management for property insurance companies was modeled as a stochastic program by Cariño et al. [42, 43, 44] and is in use by a Japanese insurance company. Stochastic programming techniques for insurance firms are also reported in Mulvey [146], Mulvey et al. [148] and Høyland and Wallace [94]. The Russel-Yasuda-Kasai model [42, 43, 44] and the Towers-Perrin model [146, 148] were among the finalist for the Edelman Prize for best achievement in management science in 1993 and 1997, respectively.

An application of stochastic programming to short-term cashflow management problems is examined by Kallberg et al. [116]. Mulvey and Vladimirou [151, 150] formulate the asset allocation problem as a stochastic generalized network program and present computational results. They demonstrate that dynamic stochastic programs yield better results that single-period models when transaction costs are present. Other important applications of stochastic programs to asset-liability management problems are reported in Consigli and Dempster [50], Klaassen [122], Kouwenberg [128], Gondzio and Kouwenberg [81], among others. Kouwenberg and Zenios [129] review stochastic programming models for asset liability management.

Worzel et al. [189] and Zenios et al. [194] develop multistage stochastic programs with recourse to address portfolio management problems with fixed-income securities under uncertainty in interest rates. The models integrate the prescriptive stochastic programs with descriptive Monte Carlo simulation models of the term structure of interest rates. Hiller and Eckstein [90], Zenios [192], and Consiglio and Zenios [51] also apply stochastic programs to fixed-income portfolio management problems. For applications to credit risk management see Anderson et al. [9] and Jobst and Zenios [109].

A classification of asset-liability management techniques — and a comparison of the classical immunization approach with dynamic stochastic programs — is given in Hiller and Schaack [91]. Zenios [191] developed a multistage stochastic programming model for managing portfolios of mortgage-backed securities and suggested pricing procedures to generate scenarios of interest rates and holding-period returns of mortgage-backed securities. This model was tested and adopted by a money management firm. Nielsen and Zenios [153] present a dynamic stochastic programming model for funding the liability stream of single-premium deferred annuities (SPDA) using a portfolio of government bonds, mortgage-backed securities and derivatives products.

Multiperiod stochastic portfolio management models improve upon static decision strategies. The dynamic model adjusts the portfolio composition as conditions change. Cariño et al. [42] demonstrate the superiority of stochastic dynamic programming models over, for example, simple fixed-mix, or buy-and-hold strategies. A similar analysis is reported in Fleten et al. [73]. Dynamic strategies pinpoint the relationship between asset risks, liability risks, and goal achievement which ultimately maximizes the investor's wealth net of liabilities and penalty costs.

A drawback of multistage stochastic programs is the explicit consideration of recourse variables and constraints over the discrete outcomes of a scenario tree, which leads to very large-scale optimization programs that are computationally challenging to solve. The size of the resulting optimization programs grows exponentially with the number of decision stages. Consequently, the prudent selection of scenarios becomes a critical issue. On the one hand, the scenarios must effectively capture the stochastic evolution of the random variables, while on the other hand their number must be controlled in order to keep the size of the resulting optimization program within computationally tractable limits. However, stochastic programs have well-recognized structures that can be exploited in the design of appropriate solution algorithms. These algorithms are typically well-suited for execution on parallel, multi-processing computing systems (e.g., see, Vladimirou and Zenios [185] for a review of parallel algorithms for stochastic programming, and Zenios [193] for a discussion of the impact that high performance computing has had on Financial planning practices).

The ability to solve large-scale stochastic programming models with a large number of scenarios allows us to consider in a common framework multiple elements of risk and to deal simultaneously with many securities. Such models have been gaining acceptance as viable tools for addressing practical financial planning problems under uncertainty.

Stochastic programs assume a proper representation of the uncertain environment, dictating the inclusion of a representative set of scenarios. Obviously, the more scenarios in the scenario tree, the closer the approximation will be to the "true" stochastic problem corresponding to continuous processes (distributions) for the underlying random variables. However, the size of the optimization program — in terms of the number of constraints and variables — grows exponentially with the number of decision stages and the number of the underlying stochastic factors. An appropriate balance is needed between accuracy in the scenario set, for effectiveness of the solutions, and problem size so as to maintain computational tractability. Inevitably, effective sampling procedures become necessary.

We have chosen to adopt a stochastic programming framework for the problem of risk management in international portfolios that we study in this thesis, for the reasons highlighted in the introduction.. Stochastic programming models take an integrated view of the problem. We can easily incorporate transaction costs, multiple random variables (asset returns and exchange rates), regulatory and other market specific restrictions, that can be handled simultaneously in this framework.

Although multistage stochastic programming models have been used in financial problems, their advantages over single-stage models have not been extensively investigated. Through extensive empirical tests we show the superiority of multistage models over single-stage variants. In all cases, and especially when options are used to manage risks, the multistage models improve considerably the performance of international portfolios compared to their single-stage (myopic) counterparts.

2.5 Scenario Generation Approaches

A general way to describe risk is by using scenarios. Each scenario is a realization of the future value of all parameters that affect the performance of the portfolio under consideration. The collection of scenarios captures the range of likely variations in these parameters that could occur between the current time and the end of the planning horizon. These representations of uncertainty are of essence in risk management. Scenarios can capture the disparate sources of risk and enable measures to be developed that account for all sources of risk.

Scenario generation is a critical step in the modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution of the underlying financial primitives and is consistent with market observations and financial theory. Scenarios should conform to the prevalent theories that govern the underlying financial random variables. Scenarios should be derived from a "correct" theoretical model for the random variables, capturing at the same time the relevant past history. So, they should accurately approximate the theoretical model from which they are derived; a large number of scenarios may be necessary, using a fine discretization method. The scenarios should satisfy the no-arbitrage properties. This means that scenario-based estimates of future asset prices in a portfolio optimization model should not allow arbitrage opportunities (for details see Klaassen [121, 122]). The generation of scenario trees that include derivative securities requires special attention. If the prices of European call and put options with equal exercise prices do not satisfy the put-call parity, then there are arbitrage opportunities.

The origin of scenarios can be very diverse. They may be obtained from a discrete known distribution, be obtained in the course of a discretization/approximation of some continuous probability distribution which is estimated from historical data, from economic forecasting models, from bootstrapping historical market data, they can be augmented with subjective opinions of experts, etc. A review of scenario generation procedures is given in Dupačová et al. [65].

The first important step is to delineate the structure of the scenario tree, that is, the number of stages and the branching scheme. The stages correspond to points in time when it is possible to take additional decisions based on newly observed information. Such information can be obtained at specific dates (option expiration), or at regular intervals (every month, year, etc.).

Choosing a representative set of scenarios that covers the range of plausible outcomes of the random variables and captures their correlations is an important task. A number of scenario generation approaches have been reported in the literature. Mulvey and Vladimirou [150] and Beltratti et al. [22] generate scenarios by bootstrapping past market observations of asset returns. The main advantage of the bootstrapping approach is its simplicity. It assumes that past market conditions accurately portray the plausible joint outcomes of the random variables in the future. It assumes that past observations of asset returns are samples from independent and identical distributions. Bootstrapping captures co-movements of multiple random variables, but cannot represent temporal dependencies. Koskosides and Duarte [127] augment historical patterns with overlays of the investor's forward views of the market, by introducing forward expectation-based forecasts into the scenarios.

Statistical analysis of historical market data can also be used in the context of scenario generation. Market data are in the form of correlated, multivariate time series. The dimensionality of the random variables can be reduced with methods of multivariate statistics. Factor analysis is generally applied under the normality assumption. Principal component analysis, while also derived under the assumption of a multivariate normal distribution, can be applied for arbitrary empirical distributions. Both of these methods aim to explain the correlation structure of the multivariate random variables by a small number of uncorrelated factors or components. Samples of the factors (or principal components) are drawn from derived or empirical distributions, and are used to construct scenarios for the multivariate random variables in a way that preserves their correlation structure. Factor analysis has been used, for example, by Bertocchi et al. [24]. Principal component analysis has been used for scenario generation purposes by Mulvey and Vladimirou [151, 150]. In chapter 3, we also apply principal component analysis to generate scenarios for a single-stage portfolio management problem.

A moment-matching method generates scenarios so that principal moments of the random variables match specific target values. This method was suggested for single-stage problems by Keefer and Bodily [120], Keefer [119], and Smith [171]. Høyland and Wallace [95] extended the method to multi-period problems and generalized it so as to match arbitrary statistical properties of the random variables to pre-specified target values; they applied their approach to generate scenarios for an insurance problem [94]. The approach of Høyland et al. [93] — which we employ for most of the numerical experiments in this thesis — specializes the method of Høyland and Wallace [95] to generate scenarios so that the first four marginal moments and the correlations of the random variables match specific target values.

Hochreiter and Pflug [92] generate scenarios using a multi-dimensional facility location problem that minimizes a Wasserstein distance from a benchmark distribution. The benchmark can be the empirical distribution from historical data or some other assumed reference distribution. This method approximates effectively the benchmark distribution, including its tails. Pflug [156] illustrates how to optimally construct a scenario tree on the basis of a discretization of an underlying financial process by using a stochastic approximation technique

Another scenario generation alternative is to sample from (discretize) continuous distributions or stochastic processes for the underlying financial variables. The assumed distributions are typically calibrated using empirical market data. The accuracy of the resulting scenario sets depends on the coarseness of the discretization. Finer discretizations reflect more accurately the continuous distributions, but lead to a large number of scenarios and very large-scale stochastic programs. A balance is needed between the accuracy of the discretization and computational tractability of the resulting stochastic program. To this end, variance reduction techniques (e.g., antithetic sampling) are often adopted. Dupačová et al. [66] and Heitsch and Römisch [88] have recently introduced a systematic approach to reduce the size of large scenario trees. Model-based scenario generation methods for asset returns have been applied to problems in the insurance industry. See, for example, the Towers-Perrin model by Mulvey and his colleagues [146, 148, 149], and the Russel-Yasuda-Kasai model of Ziemba and his collaborators [42, 43]. Other models for generating asset returns are discussed in Brennan et al. [37].

Lattice structures are the model of choice for depicting the stochastic evolution of interest rates. These models are appropriate, and have seen wide use, in problems involving fixed-income securities. The lattice generating parameters are determined on the basis of current market prices and estimates of volatility from historical data (e.g., see, Black et al. [28], Hull and White [98]). Zenios [192] discusses simulation techniques to generate scenarios of holding-period returns for fixed-income portfolio models based on an interest rate lattice. Mulvey and Zenios [152] show how correlations of fixed-income instruments can be captured in the context of interest rate lattices, by estimating risk-adjusted premiums. Scenario generation procedures based on interest rate lattices have been used in a series of fixed-income portfolio management studies by Zenios and his collaborators [189, **?**, 153, **?**, 51]. Prepayment models have been mapped to lattice structures of interest rates in order to address portfolio management problems with mortgage-backed securities [191]. Jobst and Zenios [109] integrate credit risk models with interest rate lattices in stochastic programming models for managing portfolios of corporate bonds.

Recent studies in financial risk management are increasingly stressing the need to capture and cope with the effects of extreme events. This trend has been reinforced after several dramatic financial failures in recent years, and the realization that rare and unpredictable events (e.g., crises in some emerging markets, the September 11, 2001 incidents) can have severe and far-reaching financial consequences across many markets. These realizations have brought to the forefront the need for a better understanding of extreme events and have stimulated active research towards methods for modeling such events. Extreme event models typically involve the use of heavy-tailed distributions (e.g., α-stable distributions) or suitable Monte Carlo simulation methods. We do not attempt to review here the growing line of literature on methods for modeling extreme events. A good starting point is the recent book edited by Rachev [159].

In this thesis we apply the moment-matching method of Høyland et al. [93] to generate scenarios for multi-currency asset returns and exchange rates. With this procedure, the scenarios are generated so that the first four marginal moments and the correlations of the domestic returns of the assets and the exchange rates, over the scenario sets, match specific target values. We use as targets estimates of the corresponding statistics from market data, so as to generate scenarios that comply with historical observations.

Empirical analysis of market data revealed that the instruments we use in the model validation tests of this thesis (international stock and bond indices), as well as currency exchange fluctuations, exhibit skewed distributions with excess kurtosis. This renders normal and lognormal distributions that are commonly used in portfolio analysis inapplicable for the international portfolio management problems. This was the primary motivation for the adoption of the moment-matching scenario generation method. This procedure enables the generation of scenario sets that closely reflect the multivariate empirical distributions of the random variables implied by past market data; no specific distribution function is explicitly specified in this regard. We numerically test the generated scenario sets to ensure that they are arbitrage free. Moreover, the asymmetric form of the international portfolios return distributions makes the traditional mean-variance framework inappropriate for our analysis. The asymmetry of the return distributions is made even more pronounced with the inclusion of derivative securities in the portfolios investment opportunity set. As a result, we apply in the portfolio management models the conditional value-at-risk (CVaR) risk measure that is suitable for asymmetric distributions as it is concerned with the excess shortfall (tail) risk. We elaborate on the modeling choices and their justification in subsequent chapters of the thesis.

We should stress that the modeling framework we develop in this thesis is not restricted either to the specific choice of a scenario generation procedure or to the specific risk measure. We have found that these modeling choices have served well our analysis objectives. However, the stochastic programming models presented in this thesis can accommodate alternative scenario generation methods; they can also be easily adapted to use other suitable risk measures

2.6 Option Pricing Methods

We empirically study fundamental questions on the performance of hedging strategies for risk management in international portfolios. Tho this end, we incorporate derivative securities in the stochastic programming models. Specifically, we introduce to multistage stochastic programs currency forward contracts, currency options, and options on stock indices as means to control currency and market risks, respectively, to which international portfolios are exposed. To do so, we first need to devise appropriate valuation procedures so as to price options in accordance with the postulated discrete distributions (scenarios) for the underlying instruments, in order to have an internally consistent modelling framework. We review hare the relevant literature on option valuation methods.

Options are instruments with payoffs and prices dependent on the stochastic evolution of associated underlying financial variables. The pricing and incorporation of options in stochastic programming models is a challenging task, since the options must be valued in a way that is consistent with the discrete scenarios assumed by the stochastic programming model.

The development of effective option valuation methods has been a subject of very active and productive research over the last thirty years. Since Osborne [154], the standard view has been that stock prices follow a geometric Brownian motion. Based on this assumption, Black and Scholes [29] derived explicit formulas for pricing both put and call options. Merton [142] extended the B-S formulas to account for the effect of dividends. Several empirical studies (see, for example, Rubinstein [165, 166]) showed that the B-S model misprices deep in-the money and out-of-the money options. The volatility estimates in the Black-Scholes formula, implied by observed market data for options and their underlying securities, differ across exercise prices and maturities, and form "smile" patterns. A volatility smile describes implied volatilities that are convex and monotonically decreasing functions of exercise prices. In contrast, the assumption of a geometric Brownian motion implies constant volatility relative to exercise prices and maturities. Moreover, if extreme events are more frequent than the normal distribution implies, then the B-S formula underprices out-of-the-money put options. This is a consequence of the fact that risk-neutral probability densities implied by market prices of options are typically negatively skewed and leptokurtic, unlike the positively skewed lognormal distribution assumed in the Black-Scholes approach.

The critical issued in option pricing is the modeling of the stochastic evolution of asset prices over time. This is a key issue for depicting uncertain asset price movements in portfolio management models and also for pricing options on the assets. Several approaches have been proposed. For instance, Merton [143] suggested that the underlying asset returns are generated by a mixture of both continuous and jump processes. Standard jump models have primarily been used to match volatility smiles and smirks; this they can do fairly well for a single maturity, but less well for multiple maturities. As discussed by Bates [18] the standard assumption of independent and identically distributed returns in jump models implies that these models converge towards BS option prices at longer maturities; yet, there are still pronounced volatility smiles and smirks at those maturities.

Cox et al. [54] proposed a numerical valuation model with a binomial tree to model the evolution of stock prices. This model assumes a perfect market for a contingent claim. A hedging portfolio containing stocks and bonds is rebalanced to have the same payoff as that of the target stock option at each time period for each random outcome of the underlying asset.

Boyle [33] suggested Monte Carlo simulations as an alternative to the binomial model for pricing options. This approach has the advantage that its convergence rate is independent of the number of state variables, while that of the binomial model is exponential in the number of state variables. Monte Carlo simulations generate paths for the price of the underlying asset until maturity. The price of a European option is computed as the discounted expected payoffs over all path, under the risk-neutral measure (Boyle et al. [34]). Monte Carlo simulations have also been used to price American-style options (Broadie and Glasserman [39]).

Another approach is to use finite difference approximations to price options. Dempster and Hutton [59] and Dempster et al. [60] used linear programming to solve the finite difference approximations to price American-style put options. If a closed-form solution for the pricing equation cannot be derived for an option and historical prices are available, researchers turn to the neural network approach. A neural network can be trained to (re)produce prices using a specified set of inputs. The model can then be used for out-of-sample pricing (e.g., Hutchinson et al. [99]). The neural network approach has the potential to generate prices for complex options that are traded on competitive markets.

Hull and White [97] introduced a stochastic volatility model. They showed that when volatility is stochastic, but uncorrelated with the underlying asset price, the price of a European option is the BS price obtained by using the average volatility over the lifetime of the option. However, this framework requires a market price of volatility risk. In other words, a second factor is introduced, requiring the option to satisfy a bivariate stochastic differential equation. Bates [16] and [17] extended the jump-diffusion process for the underlying asset returns to incorporate stochastic volatilities. Heston [89] showed that a closed-form solution for a European call can be derived as an integral of the future security price density which can be calculated by an inverse Fourier transform. This method may also be applied when the correlation between the increments of the driving Brownian motion of the underlying asset and the volatility is nonzero.

The major strength of stochastic volatility models is their qualitative consistency with the stochastic, but typically mean-reverting evolution of implicit standard deviations. The option pricing implications of standard stochastic volatility models are fairly close to a BS model with ad hoc updates of the volatility estimate, when the volatility process is calibrated using parameter values judged plausible given the time series properties of asset volatility or option returns. Consequently, standard stochastic volatility models with plausible parameters cannot easily match observed volatility smiles and smirks, while even unconstrained stochastic volatility models price options better after jumps are added.

Bakshi et al. [13] and Das and Sundaram [57] analyzed the extent to which models under either stochastic volatility or random jumps are capable of resolving the well-known limitations related to the BS model. They showed that neither class of models provides an adequate explanation of the biases found in the empirical literature for the BS model. Both papers found that stochastic volatility models seem to behave slightly better than models with jumbs jumps.

An alternative approach for dealing with nonconstant volatility was suggested by Rubinstein [166], Jackwerth and Rubinstein [101], and in a series of related papers by Derman and Kani [61], Derman et al. [62] and Dupire [67]. Instead of imposing a parametric functional form for volatility, they developed a binomial or trinomial lattices in order to approximate the whole structure of market prices. In this way, perfect fit with observed option prices is achieved. This procedure captures, by construction, the most salient characteristics of the data. In particular, the implied tree employed in the numerical estimation correctly reproduces the volatility smile. The most popular models within this family use recombining binomial trees implied by the smile from observed prices of European options. Rubinstein showed how to compute the implied probability distribution using quadratic programming. Jackwerth and Rubinstein generalized this approach using nonlinear programming to minimize four different objective functions.

Other studies considered the inference of the risk-neutral probability measure. The objective time series models and risk premia, imply an associated "risk-neutral" probability measure that can be used to price any derivative as the expected discounted value of its future payoffs. While the objective and risk-neutral probability measures are related, and may share some common parameters, they are identical only in the case of zero risk premia for all relevant risks. Current models attribute to risk premia any discrepancies between objective properties of asset returns, and risk-neutral probability measures inferred from the volatility smirk. The fundamental theorem of asset pricing states that in the absence of arbitrage opportunities in option prices, there exists some pricing kernel that can rthe investigations examinations by Ait-Sahalia and Lo [7], Jackwerth [100], and Rosenberg and Engle [164] into general characteristics of the pricing kernel.

Jarrow and Rudd [106] proposed a semi-parametric model that approximates the state price density of the underlying asset returns using empirical counterparts of the implied moments. Corrado and Su [52, 53] applied a related way to cope with the smile effect in market prices of options. It is well understood that volatility smiles are a consequence of empirical violations of the normality assumption in the BS model. In other words, skewness and kurtosis in the option-implied distributions of asset returns are the source of volatility smiles. This is, of course, closely related to stochastic volatility models which can nicely explain the behavior of option prices in terms of the underlying distribution of returns. In particular, the correlation between the Brownian motion associated with the underlying asset and the volatility affects skewness of returns, while the volatility of volatility is directly related with kurtosis. Following this reasoning, Corrado and Su suggested to approximate the probability density function of asset returns by means of Gram-Charlier series expansion. The first two moments of the approximate distribution remain the same as those of the normal distribution, but third and fourth moments are introduced as the higher order terms of the expansion. The Black-Scholes pricing model is then augmented in an intuitive way by introducing additional contributions due to the third and fourth moments. The series is truncated after the fourth term, noting that for practical purposes the first four moments of the underlying distribution should capture most of the effect on option prices. The model is simple to implement because it is specified as the sum of three parts: a Black-Scholes option price, plus separate adjustments for nonnormal skewness and kurtosis. The skewness and kurtosis coefficients are the historical skewness and kurtosis of the underlying asset.

At present, empirical options research seems substantially focused on stock index options, and the volatility smirk evidence of negatively skewed risk-neutral distributions. This empirical focus is certainly reasonable. Equity is the riskiest portion of most investors' portfolios, and options have prominence as instruments for managing market risk — as it is evident in the sharply increased demand for put options on stock indices since the crash of 1987. Put options and associated option trading strategies offer downside protection in equity investments.

For currency options, the standard valuation models are still those developed by Garman and Kohlhagen [78] and by Grabbe [82] Both approaches assume normal distributions for exchange rates, an assumption that is not usually supported by empirical data. A number of stylized facts have been established by empirical studies of exchange rate returns. The unconditional distribution of exchange rate returns is leptokurtic, as the distribution tends to exhibit a sharper peak and fatter tails relative to the normal distribution; see for example, Friedman and Vandersteel [74] and Booth and Glassman [32]. Shastri and Wethyavivorn [170] showed that a mixed jump-diffusion process is better suited to model the exchange rate process. Adams and Wyatt [5] extended Merton's [142] stochastic interest rate model to the pricing of currency options. Chesney and Scott [47] and Melino and Turnbull [141] investigated the consequences of stochastic volatility for pricing spot foreign currency options, and found that the stochastic volatility model gives a significant improvement over the standard model. Bakshi and Chen [14] provide several interesting results for the pricing of foreign contingent claims within a stochastic volatility framework. Doffou and Hilliard [63] investigated the effects of stochastic interest rates and jumps in the spot exchange rate on the pricing of currency futures, forwards and currency options. They concluded that the jump process is able to correct for fat tails and for non-normal levels of skewness in the distribution of returns. Finally, Bollen and Rasiel [30] compared currency option valuation models based on regime-switching, GARCH, and jump-diffusion processes in the Over-The-Counter market. They showed that the three models provide significant improvement over a fixed-smile model such as Black-Scholes [29], and that the jump diffusion gives the tightest fit to the data.

The question we need to address in out attempt to incorporate options in stochastic programming models for portfolio management, is how we should price options when the distribution of the underlying asset is approximated by a set of discrete scenarios. We cannot use one of the popular pricing formulas to price these options (e.g., the Black-Scholes formula) because the discrete distribution of our postulated scenarios for the underlying assets typically does not satisfy the distributional assumptions of these formulas (lognormal, in the BS framework). Pricing using existing methods would lead to arbitrage opportunities, because of the inconsistency of the pricing method and the discrete distribution (in terms of scenarios) of the underlying asset. Thus, we need to develop a suitable method to price the options in accordance with the postulated scenarios of asset prices, while also conforming to fundamental financial principles.

A contribution this work, is the adaptation of two alternative methods for pricing and incorporating options in scenario-based multistage stochastic programming models. The resulting option prices are consistent with the postulated scenario sets, while satisfying the fundamental no arbitrage conditions. Using these methods we can price options at any node in the scenario tree, thus allowing to consider transactions in options at any decision stage.

2.7 A Synthesis

This overview of the literature is by no means exhaustive. It has primarily focused on works from which we draw for the developments presented in this dissertation. The research reviewed comes from two distinct, yet related, scientific disciplines; namely, finance and operations research. The financial literature covered economic models for international asset allocation, asset pricing methods and arbitrage principles. The operations research literature covered specially stochastic programming models and, particularly, their applications to financial problems. Financial modeling and risk management have been particularly fertile application domains for stochastic programming as is evident by the rapidly growing body of literature of these topics.

The research of this thesis spans both scientific disciplines. It aims to bring stochastic programming models to bear on the challenging and practically important problem of international portfolio management. By bringing these scientific domains, this dissertation develops a framework to analyze in a unified manner the decisions involved in managing international portfolios (i.e., optimal portfolio selection across markets and controlling market and currency risks). The variants of the stochastic programming modes provide an appropriate testbed to empirical assess the performance of alternative risk management instruments and decision tactics. We demonstrate through extensive numerical tests that the stochastic programming models are not merely tools for experimental analysis, but can also be effective decision support tools in practical settings.

Chapter 3

Single-period Models for International Asset Allocation

3.1 Introduction

In this chapter[1] we develop a simulation and optimization framework for multicurrency asset allocation problems. Here we confine our attention to models with a single-period planning horizon. Thus, we address problems concerning a single portfolio selection decision. The portfolio is structured from a set of stock and bond indices denominated in multiple currencies. The models internalize currency hedging decisions and determine the fraction of each investment in a foreign asset that is hedged via a forward currency exchange contract. The models employ a selective hedging feature that encompasses common alternative hedging strategies. Hence, the allocation of funds in multiple markets, the selection of specific securities within each market and the level of currency hedging for each investment are addressed in a unified approach.

The simulation applies a selective sampling approach in the context of principal component analysis so as to generate discrete scenarios depicting the joint distribution of asset returns and exchange rates. We then develop and implement models that use the scenarios as inputs and optimize the *conditional-value-at-risk* (CVaR) metric.

We asses the effectiveness of the scenario generation procedure and the stability of the model‘s results by means of out-of-sample simulations. We also investigate empirically the benefits of international diversification and the impact of common hedging strategies on the risk-return profiles of portfolios. We also compare the performance of the CVaR model against that of a model that applies the *mean absolute deviation* (MAD) risk measure.

Asset managers aim to select investment portfolios that yield the maximum possible return, while at the same time ensuring an acceptable level of risk exposure. Risk derives from potential losses in portfolio value due to possible reductions in the market values of financial assets resulting from changes in equity prices, interest rates, foreign exchange rates, credit ratings of security issuers, etc.

[1]This chapter is based on the paper "CVaR Models with Selective Hedging for International Asset Allocation" by Topaloglou, Vladimirou and Zenios [181].

Diversification into multiple securities can practically eliminate idiosyncratic risk, that is, potential severe losses from any individual security. However, domestic diversification can not mitigate systematic market risk. This is the risk associated with concurrent losses in most domestic securities due to high correlations between their returns. Since market risk differs from country to country, international diversification can reduce the overall risk exposure of investment portfolios.

International diversification is practiced by institutional investors to improve the risk-return profiles of their portfolios. The inclusion of securities denominated in foreign currencies in the asset holdings can provide dual benefits: (1) the prospect for higher profit in the event of favorable performance of foreign markets and (2) the potential reduction in the portfolio's exposure to market risk. However, international investments introduce a new element of risk (currency risk). The volatility of return from a foreign asset depends not only on the differential change of its price within any given period (domestic return) but also on the variation of the foreign exchange rate to the reference currency, as well as on the correlation between the two. Exchange rates between currencies are correlated with domestic returns of assets in the respective countries — in particular with the returns of interest-sensitive securities (e.g. bonds). The effects of exchange rate changes on the overall risk profile of international portfolios are discussed in Eun and Resnick [68]. As fluctuating exchange rates can mitigate the potential gains from international diversification, holistic risk management approaches are needed that account for all risk factors affecting the performance of international portfolios. This thesis provides such a holistic treatment of the problem while previous studies did not.

Currency risk is typically hedged via forward contracts. However, the portfolio selection and hedging decisions are often considered separately. An extensive body of literature has studied the merits of hedging exchange rate risk. See, for example, Perold and Schulman [155], Eun and Resnick [68], Jorion [114], Black [27], Filatov and Rappoport [71], Glen and Jorion [80], Abken and Shrikhande [3], Solnik [174], Beltratti et al. [22]. This literature presents somewhat different views as to the optimal course of action for international portfolio management depending on the focus of each study with regard to factors such as the investment opportunity set, the risk aversion preference and time horizon of the decision maker, the reference currency of the investor, the investment strategy (passive vs active), the distribution of asset returns and exchange rates in the time frame of the study (i.e. the historical data used in calibrating the distributions), and the hedging strategies that were compared. The overall conclusions from these studies are: (a) the relative merits of hedging strategies remain mostly an empirical issue and depend on the factors mentioned above, (b) currency hedging becomes more important for foreign investments whose domestic returns exhibit considerable correlation with the exchange rate to the reference currency. These observations point to the need for integrated simulation and optimization approaches that determine jointly portfolio structures and flexible hedging policies. Our models move exactly in this direction.

With the exception of Beltratti et al. [22] who internalized selective hedging decisions within a portfolio optimization model, in all the other studies cited in the literature review (section) the hedging policy was prespecified at the portfolio selection stage. That is the degree of hedging the currency risk of foreign asset holdings (i.e., no hedging, complete hedging or partial hedging with an identical hedge ratio in all foreign markets) was prespecified when the asset allocation strategy was decided. The performance of presented alternative hedging strategies was then compared ex-post in order to identify the most effective choice. Unitary hedging concerns the selection of either no hedging or complete hedging of the currency risk associated with all foreign asset holdings. In partial hedging, the hedge ratio can be different from zero or one, but it is common across all foreign markets. This approach follows from the theory proposed by Black [27] that suggests the existence of a universal hedge ratio that is optimal for all investors. Selective hedging is the more general approach as it permits the hedge ratio to be different across markets and to take any value between zero and one.

We adopt the approach of Beltratti et al. [22] as we apply scenario-based optimization models that simultaneously determine the portfolio composition and the appropriate hedging level for each position in a foreign asset. The models prescribe optimal selective hedging policies by means of forward currency exchanges. Our models encompass all three hedging strategies mentioned above. They can yield as a special case solutions that imply a uniform hedge ratio across markets. Any value of the hedge ratio between zero and one is allowable, including the two extreme values that correspond to no hedging and complete hedging, respectively. Our aim is to assess the performance of the optimization models as risk management tools in selecting internationally diversified portfolios.

Risk management entails the exercise of control over some statistical characteristic(s) of the uncertain portfolio return. The aim is to avoid portfolios that may likely be susceptible to severe losses. We focus on the development, implementation, and testing of a model that employs the conditional value-at-risk (CVaR) metric [162, 163]. By optimizing CVaR we maximize the conditional expectation of total portfolio returns below a prespecified low percentile of the distribution, thus we minimize the expected losses in severe circumstances. Our motivation for applying a CVaR model stems from the observation that returns of international assets and proportional changes of exchange rates exhibit asymmetric distributions with fatter than normal tails. Empirical evidence supporting this assertion is given in section 3.5.2.

This study extends the work of Beltrati et al. [22] in several directions. We employ the CVaR risk metric that accounts for asymmetric return distributions. Due to the observed asymmetry of asset returns in the international asset allocation problem, CVaR is a more appropriate metric than alternative risk measures that are geared towards symmetric distributions (see, e.g., Jobst and Zenios [109]). We also apply a more rigorous scenario generation method than Beltrati et al. [22] who relied on bootstrapping of historical data. We test the effectiveness of the scenario generation procedure and the stability of the model's results in out-of-sample simulations. Moreover, we contrast the performance of the CVaR model against that of a mean absolute deviation (MAD) model. We conduct backtests using historical market data to investigate empirically the ex post performance of the models in selecting international portfolios of stock and bond indices.

Essential to the application of the optimization models is an effective representation of the random returns of the assets. We devise a sampling procedure based on principal component analysis to jointly generate scenarios of domestic holding period returns for international assets, as well as spot exchange rates at the end of the holding period. On the basis of these — and the currently quoted spot and forward exchange rates — we compute corresponding scenarios of asset returns, in terms of a reference currency, for hedged and unhedged positions in each asset. Our sampling procedure is superior to random sampling in terms of approximating the statistical properties of historical data sets. It also leads to more stable risk-return efficient frontiers as we demonstrate with out-of-sample simulations.

The scenarios of asset returns and their associated probabilities constitute the necessary inputs to the optimization models that determine portfolio compositions. The parametric optimization models trade off expected portfolio return against the relevant risk measure. We thus trace the efficient risk-return frontiers for the respective risk measures. Comparisons of these frontiers enable a relative assessment of alternative models. These static evaluations compare potential performance profiles at a single point in time.

We also carried out backtesting experiments, whereby the models were repeatedly applied in several successive time periods and the attained returns of their selected portfolios were determined ex post on the basis of observed market data. The results of backtests provide a more reliable basis for comparative assessment of the models as they reflect realized performance over longer time periods. Although in static tests the MAD and CVaR models often select portfolios that trace almost indistinguishable ex ante risk-return frontiers, in backtests the CVaR model attains superior performance, and the CVaR-optimized portfolios yield higher growth rates and lower volatility than the MAD-optimized portfolios.

The rest of the chapter is organized as follows. In section 4.6 we discuss the incorporation of the CVar risk measure in stochastic programming models. In section 3.3 we discuss the effects of exchange rate fluctuations on the overall risk of international portfolios, and we elaborate on the international asset allocation model. In section 3.4 we present our scenario generation method based on principal component analysis. In section 3.5 we examine the statistical characteristics of the historical data set, we describe our computational tests, and we discuss the empirical results. Finally, section 3.6 discusses overall conclusions from the application of the model.

The contributions of our study are: the development of a CVaR model for optimal selection of international portfolios incorporating currency hedging decisions within the portfolio selection model; the empirical comparison, using historical market data, of the CVaR model with a MAD model; the development of a scenario generation method based on principal component analysis for depicting hedged and unhedged returns of international securities. The empirical results indicate that this integrated simulation and optimization framework can provide an effective decision support tool in international asset allocation.

3.2 Risk Management Models

Consider a set of investment opportunities indexed by $i = 1, 2, \ldots, n$. At the end of a certain holding period the assets generate returns $\tilde{\mathbf{r}} = (\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n)^\top$. The returns are unknown at the beginning of the holding period — i.e. at the time of the portfolio selection — and are treated as random variables. Denote their mean value by $\bar{\mathbf{r}} = \mathcal{E}(\tilde{\mathbf{r}}) = (\bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n)^\top$. At the beginning of the holding period the investor wishes to apportion his budget to these assets by deciding on a specific allocation $\mathbf{x} = (x_1, x_2, \ldots, x_n)^\top$, such that $x_i \geq 0$ (i.e., short sales are disallowed) and $\sum_{i=1}^n x_i = 1$ (budget constraint). Using the conformable vector $\mathbf{1} = (1, 1, \ldots, 1)^\top$ of ones, we express the basic portfolio constraints in vector notation as

$$X = \{ \mathbf{x} : \mathbf{x}^\top \mathbf{1} = 1 , \mathbf{x} \geq 0 \}.$$

We use boldface characters to denote vectors, and $\sim$ to denote random variables.

For portfolios that involve both hedged and unhedged international positions the investment opportunity set is partitioned into these two categories. This implies a corresponding partitioning of the vectors $\tilde{\mathbf{r}}, \bar{\mathbf{r}}, \mathbf{x}$. Thus, these vectors are composed of a concatenation of two subvectors corresponding to the hedged and unhedged asset positions, respectively. Assets denominated in the investor's base currency can be assigned to either of the two subvestors. Total asset returns are expressed in terms of the base currency.

The uncertain return of the portfolio at the end of the holding period is $R(\mathbf{x}, \tilde{\mathbf{r}}) = \mathbf{x}^\top \tilde{\mathbf{r}} = \sum_{i=1}^n x_i \tilde{r}_i$. This is a random variable with a distribution function, say F, i.e. $F(\mathbf{x}, u) = P\{R(\mathbf{x}, \tilde{\mathbf{r}}) \leq u\}$. Of course the distribution function F depends on the portfolio composition $\mathbf{x}$. The expected return of the portfolio is $\mathcal{E}(R(\mathbf{x}, \tilde{\mathbf{r}})) = R(\mathbf{x}, \bar{\mathbf{r}}) = \mathbf{x}^\top \bar{\mathbf{r}}$. Suppose the uncertain returns of the assets, $\tilde{\mathbf{r}}$, are represented by a finite set of discrete scenarios $\Omega = \{s : s = 1, 2, \ldots, S\}$, whereby the returns under a particular scenario $s \in \Omega$ take the values $\mathbf{r}_s = (r_{1s}, r_{2s}, \ldots, r_{ns})^\top$ with associated probability $p_s > 0$, $\sum_{s=1}^S p_s = 1$. The mean return of the assets is $\bar{\mathbf{r}} = \sum_{s=1}^S p_s \mathbf{r}_s$. The portfolio return under a particular realization of asset returns $\mathbf{r}_s$ (i.e., scenario $s \in \Omega$) is denoted $R(\mathbf{x}, \mathbf{r}_s) = \mathbf{x}^\top \mathbf{r}_s = \sum_{i=1}^n x_i r_{is}$. The expected portfolio return is expressed as $R(\mathbf{x}, \bar{\mathbf{r}}) = \sum_{s=1}^S p_s R(\mathbf{x}, \mathbf{r}_s) = \mathbf{x}^\top \bar{\mathbf{r}} = \sum_{i=1}^n x_i \bar{r}_i$.

Suppose φ is some risk measure. Then for a certain minimal expected portfolio return μ, the φ-efficient portfolio is obtained from the solution of the following problem:

$$\begin{array}{ll} \text{Minimize}_{\mathbf{x} \in X} & \varphi(\mathbf{x}^\top \tilde{\mathbf{r}}) \\ \text{s.t.} & \mathbf{x}^\top \bar{\mathbf{r}} \geq \mu \end{array} \tag{3.1}$$

The curve that depicts the dependence of the optimal value of this parametric program on the required minimal expected portfolio return μ is the φ-efficient frontier. This is a generalization of the classical concept of the mean-variance efficient frontier to an arbitrary risk measure φ. The choice of the risk measure generally depends on the preferences of the decision maker or, in some cases, on regulatory specifications. Matters of computational tractability also affect this choice.

Value-at-risk (VaR) is a percentile based metric that has become an industry standard for risk measurement purposes [161]. It is usually defined as the maximal allowable loss with a certain confidence level $\alpha * 100\%$. Here we define VaR equivalently, in terms of returns, as the minimal portfolio return for a prespecified confidence level $\alpha * 100\%$. Thus,

$$\text{VaR}(\mathbf{x}, \alpha) = \min\{u : F(\mathbf{x}, u) \geq 1 - \alpha\} = \min\{u : P\{R(\mathbf{x}, \tilde{\mathbf{r}}) \leq u\} \geq 1 - \alpha\}. \tag{3.2}$$

$\text{VaR}(\mathbf{x},\alpha)$ is the $(1-\alpha)*100\%$ percentile of the distribution of portfolio return.

Despite its popular use in risk measurement, VaR is not typically used in mathematical models for optimal portfolio selection. While its calculation for a certain portfolio $\mathbf{x}$ reveals that the portfolio return will be below $\text{VaR}(\mathbf{x},\alpha)$ with likelihood $(1-\alpha)*100\%$, it provides no information on the extent of the distribution's tail which may be quite long; in such cases, the portfolio return may take substantially lower values than VaR and result in severe losses. VaR lacks a theoretical property for coherent risk measures [11], namely, sub-additivity. Moreover, VaR is difficult to optimize. When the asset returns are specified in terms of scenarios the VaR function is nonsmooth and non-convex with respect to the portfolio positions $\mathbf{x}$ and exhibits multiple local extrema. Efficient algorithms for solving problems with such objective functions are lacking.

Conditional value-at-risk (CVaR) is a related risk measure. It is usually defined as the conditional expectation of losses exceeding VaR at a given confidence level (VaR is also defined as a percentile of a loss function in this case). Here, we define CVaR equivalently as the conditional expectation of portfolio returns below the VaR return. As introduced by Rockafellar and Uryasev [162], for continuous distributions, CVaR is defined as

$$\text{CVaR}(\mathbf{x},\alpha) = \mathcal{E}\left[R(\mathbf{x},\tilde{\mathbf{r}}) \mid R(\mathbf{x},\tilde{\mathbf{r}}) \leq \text{VaR}(\mathbf{x},\alpha)\right]. \tag{3.3}$$

Hence, this definition of CVaR that is applicable to continuous distributions measures the expected value of the $(1-\alpha)*100\%$ lowest returns for portfolio $\mathbf{x}$ (i.e., the conditional expectation of portfolio returns below $\text{VaR}(\mathbf{x},\alpha)$).

For discrete distributions, the formula in (4.12) gives a nonconvex function in portfolio positions $\mathbf{x}$, and is a non-subadditive risk measure. A definition of CVaR for general distributions (including discrete distributions) has been introduced by Rockafellar and Uryasev [163]:

$$\text{CVaR}(\mathbf{x},\alpha) = \left(1 - \frac{\sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\leq z\}} p_s}{1-\alpha}\right) z \;+\; \frac{1}{1-\alpha} \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\leq z\}} p_s R(\mathbf{x},\mathbf{r}_s), \tag{3.4}$$

where $z = \text{VaR}(\mathbf{x},\alpha)$. As we consider discrete distributions (i.e., scenarios) in this paper, we will utilize this alternative definition of CVaR. Note that CVaR as defined for discrete distributions in (4.13) may not be equal to the conditional expectation of portfolio returns below $\text{VaR}(\mathbf{x},\alpha)$. This definition of CVaR for discrete distributions measures only *approximately* the conditional portfolio returns below the respective $\text{VaR}(\mathbf{x},\alpha)$ value.

As indicated by Pflug [157] and Rockafellar and Uryasev [163], by contrast to VaR, CVaR is a coherent risk measure in the sense of Artzner et al. [11]. CVaR quantifies the expected portfolio return in a low percentile of the distribution. Hence, it can be used to exercise some control on the lower tail of the return distribution and thus, it is a suitable risk measure for skewed distributions. As it was shown by Rockafellar and Uryasev [163], when the uncertain asset returns are represented by a discrete distribution CVaR can be optimized by linear programming (LP). We follow their approach in the derivation below.

Lets define for every scenario $s \in \Omega$ an auxiliary variable

$$y_s^+ \;=\; \max\left[0, z - R(\mathbf{x},\mathbf{r}_s)\right],$$

which is equal to zero when the portfolio return for the particular scenario exceeds $\mathrm{VaR}(\mathbf{x},\alpha)$, and is equal to the return shortfall in relation to VaR when the portfolio return is below $\mathrm{VaR}(\mathbf{x},\alpha)$. Using these auxiliary variables we have

$$\begin{aligned}\sum_{s\in\Omega} p_s y_s^+ &= \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s y_s^+ + \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)>z\}} p_s y_s^+ \\ &= \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s(z - R(\mathbf{x},\mathbf{r}_s)) \\ &= z \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s - \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s R(\mathbf{x},\mathbf{r}_s) \\ &= z(1-\alpha) - \left(\left(1-\alpha - \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s\right) z + \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s R(\mathbf{x},\mathbf{r}_s)\right).\end{aligned}$$

Dividing both sides of the equation by $(1-\alpha)$ and rearranging terms we get

$$z - \frac{\sum_{s\in\Omega} p_s y_s^+}{1-\alpha} = \left(1 - \frac{\sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s}{1-\alpha}\right) z + \frac{1}{1-\alpha} \sum_{\{s\in\Omega|R(\mathbf{x},\mathbf{r}_s)\le z\}} p_s R(\mathbf{x},\mathbf{r}_s). \tag{3.5}$$

From equations (4.13) and (4.14) we observe that the right hand side term of (4.14) is $\mathrm{CVaR}(\mathbf{x},\alpha)$. Therefore, the conditional value-at-risk of portfolio return can be optimized using a linear program with the left hand side expression of (4.14) as the objective function. The resulting LP that trades off the optimal CVaR-measure of portfolio return at a prespecified confidence level $\alpha * 100\%$ against the expected portfolio return μ is written as

$$\begin{array}{lll}\text{Maximize} & z - \frac{1}{1-\alpha}\sum_{s=1}^{S} p_s y_s^+ & \\ \text{s.t.} & \mathbf{x}\in X, \quad z\in\Re & \\ & \mathbf{x}^\top \bar{\mathbf{r}} \ge \mu & \\ & y_s^+ \ge z - \mathbf{x}^\top \mathbf{r}_s & s=1,2,\ldots,S \\ & y_s^+ \ge 0 & s=1,2,\ldots,S\end{array} \tag{3.6}$$

Solving the parametric program (3.6) for different values of the expected portfolio return μ yields the CVAR-efficient frontier. For each expected return target μ, the optimal value of program (3.6) is the corresponding $\mathrm{CVaR}(\mathbf{x},\alpha)$. The value of the free variable z at the optimal solution of (3.6) is the corresponding $\mathrm{VaR}(\mathbf{x},\alpha)$ value. A formal proof is provided in Rockafellar and Uryasev [163] (Theorem 14 and Corollary 15).

Efficient LP solvers for large-scale programs make possible the optimization of CVaR in a variety of portfolio management problems. When there is a relatively small number of variables, appropriate nonsmooth optimization algorithms — such as the Variable Metric Algorithm employed by Rockafellar and Uryasev [163] — can be even more efficient as they can forgo the explicit definition of the auxiliary variables y_s^+, $s=1,2,\ldots,S$ and work directly with the subgradients.

Program (3.6) optimizes the CVaR risk measure for portfolio return and simultaneously determines the corresponding VaR value (z). As defined in (4.13), in terms of portfolio return, CVaR is a lower bound for VaR (i.e., $\text{CVaR}(\mathbf{x}, \alpha) \leq \text{VaR}(\mathbf{x}, \alpha)$). Hence, by maximizing CVaR program (3.6) should be expected to yield larger values for VaR as well. As we illustrate in the empirical results of section 3.5.3 the bound of CVaR on VaR is not necessarily tight. Still, the extent to which optimizing CVaR also yields near-optimal results from the VaR perspective in practical settings remains an unresolved empirical question due to the computational complexity of optimizing VaR in such cases.

Computational issues aside, there is an ongoing debate among academics and practitioners whether VaR or CVaR is the most appropriate metric for risk management. VaR is the industry standard for *risk measurement*. On the other hand, CVaR has achieved popularity as a suitable risk measure in the insurance industry and is gradually gaining acceptance in the financial community. Its appeal lies not only in its theoretical properties of coherence, but also in its ease of implementation in portfolio optimization models and its ability to reduce the tail of the distribution, thus exercising risk management control [109]. Several recent studies have applied CVaR for portfolio selection in various applications (e.g., [162, 163, 9]).

In the mean absolute deviation framework of Konno and Yamazaki [126] risk is defined as the mean absolute deviation of portfolo return from its expected value:

$$\text{MAD}(\mathbf{x}) = \mathcal{E}\left[\,|R(\mathbf{x}, \tilde{\mathbf{r}}) - R(\mathbf{x}, \bar{\mathbf{r}})|\,\right].$$

When the uncertain asset returns are represented in terms of a discrete scenario set the MAD metric becomes:

$$\text{MAD}(\mathbf{x}) = \sum_{s=1}^{S} p_s \, |\mathbf{x}^\top \mathbf{r}_s - \mathbf{x}^\top \bar{\mathbf{r}}|.$$

In this case MAD can be optimized by the following linear program:

$$\begin{array}{lll} \text{Minimize} & \sum_{s=1}^{S} p_s y_s & \\ \text{s.t.} & \mathbf{x} \in X & \\ & \mathbf{x}^\top \bar{\mathbf{r}} \geq \mu & \\ & y_s \geq \mathbf{x}^\top (\mathbf{r}_s - \bar{\mathbf{r}}) & s = 1, 2, \ldots, S \\ & y_s \geq \mathbf{x}^\top (\bar{\mathbf{r}} - \mathbf{r}_s) & s = 1, 2, \ldots, S \\ & y_s \geq 0 & s = 1, 2, \ldots, S \end{array} \tag{3.7}$$

The auxiliary variables y_s are introduced to linearize the absolute value expression, akin to the approach followed earlier to linearize the piecewise linear function $\max\left[0, z - R(\mathbf{x}, \mathbf{r}_s)\right]$ in the CVaR case. Again, by solving the parametric program (3.7) for various values of expected portfolio return μ we can construct the MAD-efficient frontier. MAD models have been applied to various portfolio optimization problems; see, for example, [126, 117, 22, 129].

Several relevant optimization models for risk management and portfolio selection, as well as related financial application studies, can be found in the collections edited by Zenios [**?**], Ziemba and Mulvey [195], and in the review paper of Kouwenberg and Zenios [129].

3.3 The International Asset Allocation Model

In this section we discuss the effect of the exchange rate fluctuations on the overall risk of international portfolios, and the hedging strategies that may be incorporated in the model. Then we present the precise formulation of the international asset allocation model that is used in the computational tests.

3.3.1 Effects of Currency Risk

Currency risk is the impact on the total return of an international portfolio due to the volatility of the exchange rates of foreign currencies to the investor's reference currency, i.e. the detrimental effect to an investor due to potential depreciation of foreign currencies in which investments are held. First, we examine the effect of exchange rate fluctuations on the return of the international portfolio measured in terms of the base (reference) currency of the investor of a foreign asset portfolio. Lets call the return the "base return" and denote it $\tilde{r}_b$. This return is of course a random variable because it depends on the uncertain asset returns and uncertain exchange rates. In the simple case where we invest only in one foreign market c, the "base return" from an unhedged investment in this foreign market is

$$\tilde{r}_b = (1 + \tilde{r}_{cd})\frac{\widetilde{e_c}}{e_c} - 1 \; = \; (1 + \tilde{r}_{cd})(1 + r\hat{e}_c) - 1 = \tilde{r}_{cd} + r\hat{e}_c + \tilde{r}_{cd} r\hat{e}_c$$

where $\tilde{r}_{cd}$ is the domestic rate of return in currency c and $r\hat{e}_c$ is the rate of appreciation of the currency c against the base currency (again a random variable). In order to simplify the analysis of the effects of exchange rate fluctuations on the risk of the portfolio, Eun and Resnick [68]) approximate the base rate of return $\tilde{r}_b$ by

$$\tilde{r}_b \approx \tilde{r}_{cd} + r\hat{e}_c \tag{3.9}$$

Thus, the cross-product $\tilde{r}_{cd} r\hat{e}_c$ is neglected as it is very small in magnitude. Based on equation (3.9), the variance of the base rate of return can be approximated by

$$var(\tilde{r}_b) \approx var(\tilde{r}_{cd}) + var(r\hat{e}_c) + 2cov(\tilde{r}_{cd}, r\hat{e}_c) \tag{3.10}$$

This analysis can be extended into a portfolio context. The variance of the portfolio return in terms of base reference currency $\tilde{r}_p$ can be written as

$$var(\tilde{r}_p) = \sum_{c_i} \sum_{c_j} x_{c_i} x_{c_j} cov(\tilde{r}_{c_i b}, \tilde{r}_{c_j b}), \tag{3.11}$$

where $\tilde{r}_{c_i b}, \tilde{r}_{c_j b}$ are the rate of returns in the base currency of investments in markets c_i, c_j respectively, and x_{c_i}, x_{c_j} are the fractions of wealth invested in markets c_i and c_j. Note that $\tilde{r}_p$ refers to the return of a portfolio while $\tilde{r}_b$ denoted the return of a single foreign investment, but viewed from the perspective of the base currency. Noting from equation (3.9) that

$$cov(\tilde{r}_{c_i b}, \tilde{r}_{c_j b}) \approx cov(\tilde{r}_{c_i d}, \tilde{r}_{c_j d}) + cov(r\hat{e}_{c_i}, r\hat{e}_{c_j}) + cov(\tilde{r}_{c_i d}, r\hat{e_{c_j d}}) + cov(\tilde{r}_{c_j d}, r\hat{e}_{c_i}), \tag{3.12}$$

and using equation (3.11), the variance of the portfolio return in the base currency can be approximated by

$$\begin{aligned} var(\tilde{r}_p) &\approx \sum_{c_i}\sum_{c_j} x_{c_i}x_{c_j}\Big\{cov(\tilde{r}_{c_i d},\tilde{r}_{c_j d}) + \sum_{c_i}\sum_{c_j} cov(r\hat{e}_{c_i}, r\hat{e}_{c_j}) \\ &+ 2\sum_{c_i}\sum_{c_j} cov(\tilde{r}_{c_i d}, r\hat{e}_{c_j})\Big\} \end{aligned} \tag{3.13}$$

It is clear from equation (3.13) that the overall portfolio risk depends on three factors. It depends on the covariances among the local market returns, on the covariances among the exchange rate changes, and on the covariances among the local stock returns and the exchange rate changes. Thus fluctuations in the exchange rates contribute to the overall portfolio risk via the last two factors.

We can see from equation (3.13) that the exchange rates contribute to the variance of a portfolios return not only through their own variance, but also through their covariance with the local market returns. Eun and Resnick [68]) report that the exchange rates between major currencies exhibit nearly as much volatility as their respective stock markets. They also indicate that the covariance between the local stock market returns and the exchange rate changes is positive for the major markets. In fact, the correlation coefficient between the two random variables is significantly different from zero at least at the ten percent level for each country. The exchange rate movements are thus found to reinforce, rather than offset, the risk from fluctuations in the value of holdings in foreign stock markets.

Currency exchange rates exhibit nonstationary fluctuations. Currencies are subject to periods of excessive volatility and the behavior of different currencies relative to each other may change dramatically through time. Estimating currency risks and correlations with market risk is one of the most challenging risk measurement activities.

Currency hedging decisions considered in this study concern the position in currency forward contracts. A foreign exchange forward contract is an agreement between two parties (the investor or and a bank) to buy (or sell) a specific amount of foreign currency at a prespecified future date at an exchange rate predetermined at the time of the agreement. Forwards are a simple, cost-effective way to alter the variability in the returns of foreign asset holdings.

Assume that the investor sells the expected foreign currency proceeds forward. This amounts to exchanging the uncertain return in the base currency $(1+\tilde{r}_{cd})(1+r\hat{e}_c)-1$ for the return $(1+\tilde{r}_{cd})(1+\bar{f}_c)-1$ where $\bar{f}_c$ is the forward exchange rate depreciation (i.e., $\frac{f_c}{e_c}$). Thus, using forward contracts the hedged rate of return in the base currency is given by

$$\tilde{r}_h = (1+\tilde{r}_{cd})\frac{f_c}{e_c} - 1 \;=\; (1+\tilde{r}_{cd})(1+\bar{f}_c) - 1 = \tilde{r}_{cd} + \bar{f}_c + \tilde{r}_{cd}\bar{f}_c$$

Again, the cross-product $\tilde{r}_{cd}\bar{f}_c$ is neglected as it is very small in magnitude. Thus the effect of exchange rate changes on the risk of a foreign stock market investment can be offset by means of a forward exchange contract in a full hedging setting. Of course this is only an approximation. In practice, the amount of the transaction is usually prespecified. But the ending value a foreign investment is random, and hence the notion of full hedging is only an approximation.

3.3.2 The Asset Allocation Model

We apply a single-stage stochastic program to model an international portfolio management problem. We depict the uncertainty in the local returns of assets (i.e., in their respective currencies) and in the differential changes of exchange rates by means of a discrete distribution (scenario set). The problem of portfolio restructuring is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. Without loss of generality, no direct exchanges between foreign currencies are executed – either in the spot or in the forward market – in order to simplify the formulation of the model and to reduce its data needs, as well as the number of decision variables. All currency exchanges are executed with respect to the base currency. Thus, to reposition his investments from one market (currency) to another, the investor must first convert to base currency the proceeds of foreign asset sales in the market in which he reduces his presence and then purchase the foreign currency in which he wishes to increase his investments. The current spot exchange rates of foreign currencies to US dollar apply in the currency exchange transactions. At the end of the holding period (one month) we compute the scenario-dependent value of each investment using its projected price under the respective scenario. The US dollar-equivalent value is determined by applying the corresponding estimate of the appropriate spot exchange rate to US dollar at the end of the period in case of unhedged positions, or using the forward rate for hedged positions.

The investor's portfolio is exposed to market risk in the domestic and foreign markets, as well as to currency exchange risk. To hedge the currency risk, the investor may enter into currency exchange contracts in the forward market. Note that the amount of the forward currency transaction per currency in order to hedge the respective exposure to the foreign currency risk must be scenario invariant at $t = 0$. But this feature is not incorporated in this model. As we said before, this is an approximation for hedging decisions. The respective forward currency exchanges should reflect deterministic contracts (in terms of the amount), drawn on the basis of the currently quoted forward exchange rates. These forward currency exchange contracts reflect the hedging decisions.

The model does not contain variables for deterministic forward contracts but applies an approximation. We assume that the investor can fully hedge his exposure to foreign investments. The selective hedging is achieved using two variables; the first one expresses totaly unhedged investments in foreign markets while the second expresses totaly hedged investments in foreign markets.

The scenario generation procedure (described in the next section) yields discrete distributions for the asset returns (hedged and unhedged) and for the exchange rates.

We use the following notation:

Definitions of sets:

C_0 set of currencies (markets), including the base (reference) currency,

C_0 set of foreign currencies,

I_c set of asset classes denominated in currency $c \in C_0$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term bond index in each country)

Ω the set of scenarios

Input Parameters (Data):

μ	a prespecified target expected return of the revised portfolio,
α	the prespecified confidence level (percentile) for the CVaR specification,
e_c	current spot exchange rate for currency $c \in C$, (in units of base currency to one unit of the foreign currency),
f_c	currently quoted forward exchange rate for currency $c \in C$
r^s_{ich}	hedged return of asset $i \in I_c$ of currency $c \in C_0$ under scenario $s \in \Omega$
r^s_{icu}	unhedged return of asset $i \in I_c$ of currency $c \in C_0$ under scenario $s \in \Omega$
e^s_c	spot exchange rate of currency $c \in C$ at the end of the horizon under scenario $s \in \Omega$
p_s	occurrence probability of scenario $s \in \Omega$
T	the time horizon (in our case 1 month)

Decision Variables:

w^h_{ic}	weight of asset class $i \in I_c$ of currency $c \in C_0$ in the portfolio, fully hedged
w^u_{ic}	weight of asset class $i \in I_c$ of currency $c \in C_0$ in the portfolio, unhedged

Auxiliary variables:

y_s	auxiliary variables used to linearize the nondifferentiable function in the definition of CVaR. Also auxiliary variables for MAD
z	the VaR value of a portfolio (at a prespecified confidence level, percentile α),
R_s	holding-period return of the international portfolio under scenario $s \in \Omega$,
$\bar{R}$	expected holding-period return of the international portfolio.

We formulate the international portfolio management model as follows:

$$\begin{aligned}
\texttt{maximize} \quad & z - \frac{1}{1-\alpha}\sum_{s\in\Omega} p_s y_s && \text{(3.15a)}\\
\texttt{s.t.} \quad & R_s = \sum_{c\in C_0}\sum_{i\in I_c}(w^h_{ic} r^s_{ich} + w^u_{ic} r^s_{icu}), \quad \forall s\in\Omega && \text{(3.15b)}\\
& \bar{R} = \sum_{s\in\Omega} p_s R_s && \text{(3.15c)}\\
& \bar{R} \geq \mu && \text{(3.15d)}\\
& y_s \geq z - R_s\,, \quad \forall s\in\Omega && \text{(3.15e)}\\
& y_s \geq 0\,, \quad \forall s\in\Omega && \text{(3.15f)}\\
& \sum_{c\in C_0}\sum_{i\in I_c}(w^h_{ic} + w^u_{ic}) = 1, && \text{(3.15g)}\\
& w^h_{ic} \geq 0 \quad \forall i\in I_c\,, \quad \forall c\in C_0 && \text{(3.15h)}\\
& w^u_{ic} \geq 0 \quad \forall i\in I_c\,, \quad \forall c\in C_0 && \text{(3.15i)}
\end{aligned}$$

This formulation minimizes the CVaR (3.15a) of the international portfolio losses at the end of the horizon, while constraining the expected portfolio return.

We additionally consider the alternative risk measure of the mean absolute deviation (MAD). In this case we minimize the expected absolute deviation of portfolio returns over all scenarios from the expected portfolio return. The resulting linear program in this case is:

$$\begin{aligned}
\texttt{minimize} \quad & \sum_{s\in\Omega} p_s y_s && \text{(3.16a)}\\
\texttt{s.t.} \quad & y_n \geq \bar{R} - R_s, \quad \forall\, s\in\Omega && \text{(3.16b)}\\
& y_s \geq R_s - \bar{R}, \quad \forall\, s\in\Omega && \text{(3.16c)}\\
& y_s \geq 0, \quad \forall\, s\in\Omega && \text{(3.16d)}\\
& && \text{(3.16e)}
\end{aligned}$$

and subject additionally to constraints (3.15b), (3.15c), (3.15d), (3.15g), (3.15h) and (3.15i).

Equation (3.15b) defines the total portfolio return under scenario s. The model may selectively choose the optimal level of hedging for each asset and for each market separately. Equation (3.15c) defines the expected return of the portfolio at the end of the horizon, while equation (3.15d) constrains the expected return by the target return μ. Constraints (3.15e), and (3.15f) are the definitional constraints for determining `CVaR`, while constraints (3.16b) and (3.16c) are the definitional constraints for determining MAD. Equation (3.15g) insures that the weights of all investments (hedged and unhedged) sum to one; that is, budget constraints ensuring full investment of available budget. Finally, equations (3.15h) and (3.15i) insure that the weights on assets purchased are nonnegative, thus disallowing short sales.

3.4 Scenario Generation

A central issue in any portfolio selection model is a depiction of the uncertain returns of the investment alternatives. Usually this issue is addressed by defining either the expected values and the covariance matrix of the asset returns, or a set of possible realizations (scenarios) of the random returns. The former approach applies in the mean-variance setting. In the latter case, the scenarios of plausible asset returns can be generated by a model, can be obtained from experts' opinions, or by bootstrapping observed past returns; see the review by Kouwenberg and Zenios [129].

Past market observations usually play some role in scenario generation procedures either by being bootstrapped as plausible scenarios, or by providing the basis for the calibration of statistical models from which scenarios are then sampled. We use past observations of asset returns and exchange rates in our scenario generation. However, as we point out in section 3.5, empirical evidence indicates that these random variables do not follow a multinormal, or log-normal, distribution; they, in fact, exhibit asymmetries. As the distribution of asset returns is unknown, we make no assumption regarding either their joint or their marginal distributions. We resort to a sampling procedure based on principal component analysis. Thus we derive a relatively small number of uncorrelated factors that capture to a great degree the overall variability exhibited in past observations of the random variables. By combining samples from empirical marginal distributions of these uncorrelated random factors we can obtain scenarios of asset returns and exchange rates through a simple transformation. In this manner, we generate the required scenarios of exchange rates and domestic returns for the assets with statistical properties similar to those of the historical observations. Our scenario generation procedure is described next.

3.4.1 Principal Component Analysis

Principal component analysis (PCA) is geared to reduce the dimensionality of a multivariate forecasting problem and to overcome the difficulty posed by the correlation of the random variables, while it preserves the covariation structure in the derived samples. This is achieved with a linear transformation to a new set of variables (principal components, PCs) which are uncorrelated and ordered so that a reduced set of them captures most of the variability exhibited by all the original variables. The theory of PCA is covered in Jollife [111].

Let the relevant random variables be $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_m)^\top$. Their mean values are $\bar{\mathbf{u}} = (\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m)^\top$ and their covariance matrix is $\mathbf{Q}$. In our case, the random variables are the domestic returns of the assets and the proportional changes of spot exchange rates to the base currency over a certain time horizon. Consider a linear transformation

$$\tilde{v}_j = \mathbf{c}_j^\top \tilde{\mathbf{u}} , \quad j = 1, 2 \ldots, m ; \qquad \tilde{\mathbf{v}} = \mathbf{C}\, \tilde{\mathbf{u}} , \tag{3.17}$$

where $\mathbf{C}$ is an $(m \times m)$ matrix with rows $\mathbf{c}_j^\top$. The mean of random variable $\tilde{v}_j$ is $\bar{v}_j = \mathbf{c}_j^\top \bar{\mathbf{u}}$, while its variance is given by

$$\sigma_j^2 = \mathrm{Var}(\mathbf{c}_j^\top \tilde{\mathbf{u}}) = \mathbf{c}_j^\top \mathbf{Q} \mathbf{c}_j . \tag{3.18}$$

Similarly, the covariance σ_{ij} of the random variables $\tilde{v}_i,\ \tilde{v}_j$ is given by

$$\sigma_{ij} \;=\; \text{Covar}(\mathbf{c}_i^\top \tilde{\mathbf{u}},\ \mathbf{c}_j^\top \tilde{\mathbf{u}}) \;=\; \mathbf{c}_i^\top \mathbf{Q} \mathbf{c}_j\,.$$

The aim is to derive random variables $\tilde{\mathbf{v}}$ which are uncorrelated. We want $\sigma_{ij} = 0$, for $i \neq j$; thus, $\mathbf{c}_i,\ \mathbf{c}_j$ must be $\mathbf{Q}$-orthogonal. The eigenvectors of the covariance matrix $\mathbf{Q}$ satisfy the orthogonality requirement. Therefore, the coefficient vectors $\mathbf{c}_j^\top$ in (3.17) are the eigenvectors from the normalized solutions of the eigenproblems

$$\mathbf{Q}\,\mathbf{c}_j \;=\; \lambda_j\,\mathbf{c}_j\,, \qquad\qquad j = 1, 2, \ldots, m\,, \tag{3.19}$$

where λ_j is the j^{th} eigenvalue of $\mathbf{Q}$. The resulting principal components $\tilde{\mathbf{v}}$ obtained by (3.17) are uncorrelated. In view of (3.18) and (3.19) the variance of $\tilde{v}_j$ is $\sigma_j^2 = \mathbf{c}_j^\top \mathbf{Q} \mathbf{c}_j = \lambda_j$; the last equation follows from the normalization of the eigenvectors (i.e., $\mathbf{c}_j^\top \mathbf{c}_j = 1,\ j = 1, 2, \ldots, m$).

Additionally, we want to capture as much of the variability in the original random variables $\tilde{\mathbf{u}}$ as possible with few principal components. The explanatory power of each PC is associated with its variance. Consequently, the principal components $\tilde{v}_j$ are sorted in descending order of variance (eigenvalue λ_j). A measure of the original variables' variability explained by the set of first k PCs ($k < m$) is given by the ratio $\sum_{j=1}^{k} \lambda_j / \sum_{j=1}^{m} \lambda_j$. Hence, the degree of variability in the original variables captured by a retained set of the first k PCs is controlled by selecting $k = \min\{\kappa\ :\ \sum_{j=1}^{\kappa} \lambda_j / \sum_{j=1}^{m} \lambda_j \geq \delta\}$; $\delta \approx 1$ is a desired accuracy level.

Once we determine the set of PCs to retain alternative sampling techniques can be applied. If the initial random variables $\tilde{\mathbf{u}}$ follow a multinormal distribution then the PCs, $\tilde{\mathbf{v}}$, will be univariate normal with the parameters indicated above. If one assumes that the PCs have approximately normal distributions then random samples can be drawn independently from the assumed distribution of each PC and can be combined to construct a set of scenarios for the PCs. Applying the inverse transformation of (3.17) on each PC scenario yields a corresponding scenario for the initial random variables. A scenario $\mathbf{u}_s$ for the initial random variables $\tilde{\mathbf{u}}$ is typically computed by

$$\mathbf{u}_s \;=\; \mathbf{C}_k^{-1}\,\mathbf{v}_s^k\,, \tag{3.20}$$

where $\mathbf{C}_k^{-1}$ is the matrix of the first k columns of $\mathbf{C}^{-1}$, and $\mathbf{v}_s^k$ denotes the vector of values for the k retained PCs under scenario s. So, in (3.20) only the subset of retained PCs is used, while the remaining PCs are ignored.

3.4.2 Sampling Procedure

Here we make no distributional assumptions, as empirical evidence indicates that the random variables we consider in this study can not be well approximated by normal distributions. Instead, we devise an alternative empirical sampling procedure. We substitute the values of past observations of the random variables $\tilde{\mathbf{u}}$ in the definition of PCs (eqn. (3.17)) and obtain corresponding values for the PCs; these are termed *principal component scores* and reflect the historical implied values of the PCs.

We assume that the distribution of each PC is completely described by its historical scores; hence, an empirical distribution is constructed for each PC. We sample from the empirical distribution of each retained PC and combine these samples to generate representative joint scenarios $\mathbf{v}_s^k$ of the PCs, which can then be applied in (3.20) to obtain corresponding scenarios $\mathbf{u}_s$ for the random variables $\tilde{\mathbf{u}}$. When a sufficient number of PCs is retained and representative samples per PC are taken, the resulting scenario set for $\tilde{\mathbf{u}}$ has very similar statistical characteristics with those of the historical observations of these random variables.

Because only a small number of samples per PC can be afforded — so as to limit the size of the joint scenario set for computational tractability — we do not sample randomly the PC values. Instead, we obtain representative sample sets of small size. The procedure works as follows. First, we determine for each PC the minimum, the maximum, and the mean value from its principal component scores. For the j^{th} PC let us denote these values as $v_j^{\min}$, $v_j^{\max}$ and $\bar{v}_j$, respectively. We specify for each of the retained PCs a required number of samples; say that for the j^{th} PC this number is n_j. Next, for each of the retained PCs we divide the range $[v_j^{\min}, \bar{v}_j]$ into $n_j/2$ intervals of equal width; similarly, we partition the range $[\bar{v}_j, v_j^{\max}]$ into $n_j/2$ equal segments. In both cases the positioning of the segments starts from the extreme points $v_j^{\min}$ and $v_j^{\max}$, respectively, and proceeds towards the mean. We assign to the midpoint of each of the n_j segments the entire probability mass associated with its respective interval, as determined from the empirical distribution of the principal component scores. Note that when n_j is even, the mean value $\bar{v}_j$ of the PC is at the boundary of the two adjacent intervals in the middle of the distribution. When n_j is odd, $\bar{v}_j$ lies in the middle interval which, in this case, is composed of two subintervals, of potentially unequal width.

Each scenario of PC values, $\mathbf{v}_s^k$, is constructed from a specific combination of samples from the k retained PCs. As the PCs are uncorrelated, the probability of a scenario is simply the product of the marginal probabilities of its constituent PC samples. The total number of scenarios is $N = \prod_{j=1}^{k} n_j$, arising from all possible combinations of PC samples.

This sampling procedure has some clear advantages. The number of samples per PC can be directly controlled. We take more samples for the first PCs that have higher explanatory power and reduce the number of samples for subsequent PCs. The differential partitioning scheme of the PC's empirical distributions aims at a more effective approximation of potentially skewed distributions; note also that the samples for each PC are not equiprobable. With a small number of samples we are able to obtain a representative approximation of each PC's distribution.

Another novelty in our scenario generation procedure is that we do not completely ignore the PCs with low variance from which we do not sample. Instead of (3.20), we compute the corresponding scenario $\mathbf{u}_s$ for the initial random variables by

$$\mathbf{u}_s = \mathbf{C}_k^{-1} \mathbf{v}_s^k + \mathbf{C}_{m-k}^{-1} \bar{\mathbf{v}}^{m-k}. \tag{3.21}$$

$\mathbf{C}_k^{-1}$ is again the matrix of the first k columns of $\mathbf{C}^{-1}$ that correspond to the retained PCs, while $\mathbf{C}_{m-k}^{-1}$ is the matrix of the remaining $(m-k)$ columns of $\mathbf{C}^{-1}$. Similarly, $\mathbf{v}_s^k$ denotes the values of the k retained PCs in scenario s, while $\bar{\mathbf{v}}^{m-k}$ denotes the mean values of the remaining $(m-k)$ PCs, as computed from their historical scores. The second term in the right hand side of (3.21) is a constant. However, we have found that its inclusion in the generation of the scenarios $\mathbf{u}_s$ yields an estimate of the mean in the resulting scenario set (i.e., $\sum_{s=1}^{\Omega} p_s \mathbf{u}_s$) that approximates more closely the mean obtained directly from historical observations.

Through empirical tests we validated that our selective sampling procedure leads to superior approximations of the statistical properties of historical data sets for variables $\tilde{\mathbf{u}}$ in comparison to random sampling. In turn, as we illustrate in section 3.5, scenario sets generated with this sampling procedure lead to more stable and reliable identification of risk-return efficient frontiers in contrast to random samples.

Ideally one wishes to associate some economic interpretation with the PCs, such as the aggregate effect of identifiable economic factors on individual market variables. For example, applications of PCA on term structures of interest rates have revealed that three PCs are usually sufficient to capture with high accuracy the total variability in term structures. These three PCs have been associated with shift, twist and butterfly movements of term structures. However, identifying an economic interpretation for the PCs is not always possible. Here we employ PCs only to reduce the dimensionality of the random variables so as to facilitate the scenario generation procedure, without attempting to deduct economic interpretations.

3.4.3 Hedged and Unhedged Asset Returns

This study considers an asset allocation problem concerned with hedged and unhedged investments in stock and bond indices denominated in multiple currencies. The observable market data are the asset values (index levels) and the currency exchange rates, both spot and forward. From past observations of the index levels we compute domestic returns for the indices over specific time intervals; in this study we use monthly time periods. So, the monthly returns of the indices, expressed in their respective domestic terms, and the monthly proportional changes in the spot exchange rates of the foreign currencies to a base currency, are the relevant random variables. These constitute the random vector $\tilde{\mathbf{u}}$ in the earlier discussion of the PCA approach. Applying PCA on historical values of these variables we obtain scenarios for the following quantities:

- r_{icd}^s estimated monthly return for asset $i \in I_c$ of currency $c \in C_0$, in domestic terms, under scenario $s \in \Omega$,
- e_c^s estimated spot exchange rate of the base currency to 1 currency $c \in C$ at the end of the monthly holding period under scenario $s \in \Omega$.

At any point in time we also know:

- e_c the currently quoted spot exchange rate of the base currency to 1 currency $c \in C$
- f_c the currently quoted one-month forward exchange rate of the base currency to 1 currency $c \in C$.

Using these data we compute the monthly return of an *unhedged* position in asset i of currency c, in terms of the base currency, for each scenario $s \in \Omega$ as follows

$$r_{icu}^s = \frac{e_c}{e_c^s}(1 + r_{icd}^s) - 1 . \tag{3.22}$$

This computation takes into account the equivalent foreign value of a base currency transfer at the current spot exchange rate e_c, the growth factor $(1 + r^s_{icd})$ of the investment in the foreign asset, and the conversion of the final proceeds back to the base currency at the end of the holding period using the spot exchange rate e^s_c applicable at that time. Similarly, the monthly return of a *hedged* position in asset i of currency c, in base-currency terms, for each scenario $s \in \Omega$ is computed by

$$r^s_{ich} \;=\; \frac{e_c}{f_c}(1 + r^s_{icd}) \;-\; 1\,. \tag{3.23}$$

The difference in (3.23) is that the final proceeds from a foreign investment i are converted back to the base currency using the known forward exchange rate f_c. The scenarios of hedged and unhedged returns are fed as inputs to the optimization models discussed in section 3.3.2.

Here we assume that the total proceeds at the end of the holding period from an allocation in a "completely hedged" foreign investment, which are uncertain (scenario dependent), can be converted back to the base currency with the known forward exchange rate f_c. In fact, a scenario-invariant amount (say the expected value of such proceeds) should be specified in a forward exchange contract, while any residual amounts from the scenario dependent proceeds are converted back with the spot exchange rates e^s_c prevailing at the end of the period. To accurately capture the value of variable forward transfers of foreign currency to the base currency we could resort to the use of quantos, but that would complicate the model. The simplifying approximation we use here is rather commonplace in the literature. As discussed in Eun and Resnick [68] the error from this approximation should be very small, especially if we assume that forward exchange rates are a fair estimate of the future spot exchange rates.

Modeling extensions to handle accurately the forward exchange contracts are possible with the use of more advanced optimization models. In chapter 4 we develop stochastic programs to capture decision dynamics and we generalize the models so as to account for transaction costs in a multiperiod portfolio management setting.

3.4.4 Bayes-Stein Estimation Corrections

As the PCA procedure is calibrated based on a limited number of recent market observations, the statistical characteristics of a scenario set carry a residual estimation risk. As indicated by Jorion [112] and Eun and Resnick [68], the mean-return vector of international assets exhibits intertemporal instability, while the variance-covariance matrix of international asset returns demonstrates greater stability through time. Thus, the expected-return vector is more prone to estimation error. The mean-return vector has a major influence on the results of portfolio optimization models and, consequently, its accurate estimation is of primary importance. The Bayes-Stein approach for determining the expected asset returns aims to mitigate the effects of estimation risk. It yields a uniform improvement on the classical sample mean as it relies on a more general estimation model. The approach was formalized by Jorion [112, 113]; see also Eun and Resnick [68].

A revised estimate $\hat{\mathbf{r}}$ for the mean-return vector is computed from

$$\hat{\mathbf{r}} \;=\; (1-\vartheta)\bar{\rho} \;+\; \vartheta\mathbf{1}\rho_0\,, \tag{3.24}$$

where $\bar{\rho}$ is the sample mean-return vector, determined from historical observations of asset returns, ρ_0 denotes the mean return of the minimum-variance portfolio based on the same historical observations, $\mathbf{1}$ is the vector of ones, and ϑ represents the estimated shrinkage factor for shrinking the elements of $\bar{\rho}$ toward ρ_0. The shrinkage parameter is estimated by

$$\vartheta \;=\; \frac{(m+2)(T-1)}{(m+2)(T-1) \,+\, (\bar{\rho}-\rho_0\mathbf{1})^{\top} T V^{-1}(T-m-2)(\bar{\rho}-\rho_0\mathbf{1})}\,, \tag{3.25}$$

where T is the length of the time series of sample observations, m is the number of random variables (i.e., hedged and unhedged asset returns), and V is their sample variance-covariance matrix computed on the basis of the historical observations.

In related work, Jobson and Korkie [108] derived a variation of equation (3.24), in which the grand mean of the elements of $\bar{\rho}$ is substituted for the shrinkage target ρ_0 and the shrinkage factor ϑ is estimated differently.

In order to attain the revised mean-return estimate $\hat{\mathbf{r}}$ of equation (3.24) the asset returns under each scenario $s \in \Omega$ must be modified by adding to their initial values $\mathbf{r}^s$ the correction term $(\hat{\mathbf{r}} - \bar{\mathbf{r}})$. This results in an update of the scenarios of returns for the unhedged as well as the hedged asset positions.

We apply the Bayes-Stein procedure in all subsequent experiments to revise the values of asset returns in the postulated scenarios before we solve the portfolio optimization models. We first solve the minimum-variance problem — calibrated on the basis of market observations over a prespecified historical period — to determine the shrinkage target return ρ_0. We then apply equations (3.25) and (3.24) to determine the revised estimate of the mean-return vector on the basis of which we update the values of asset returns under all scenarios. We then proceed to solve the portfolio optimization model using the revised scenarios.

3.5 Empirical Analysis

3.5.1 Data Sources

We consider portfolios composed of positions in stock indices and bond indices of short term (1–3 years) and long term (7–10 years) maturity ranges in the United States (US), United Kingdom (UK), Germany (GR) and Japan (JPN). The following investment instruments are considered:

USS	US stock index
UKS	UK stock index
GRS	German stock index
JPS	Japanese stock index
US1	US government bond index (1–3 years maturity)
US7	US government bond index (7–10 years maturity)
UK1	UK government bond index (1–3 years maturity)
UK7	UK government bond index (7–10 years maturity)
GR1	German government bond index (1–3 years maturity)
GR7	German government bond index (7–10 years maturity)
JP1	Japanese government bond index (1–3 years maturity)
JP7	Japanese government bond index (7–10 years maturity)

The values of the stock indices were obtained from the Morgan Stanley Capital International database. The values of the bond indices and the exchange rates were obtained from Datastream. The collected time series involve monthly data for the period from April 1988 through May 2001.

3.5.2 Statistical Characteristics of Historical Data

First we analyze the statistical characteristics of the data covering the period 01/1990–08/2000 that were used in static tests for the determination of risk-return efficient frontiers on September 2000. As we can see from Table 3.1, both the domestic returns of the indices and the proportional changes of exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. Jarque-Bera tests on these data indicated that normality and log-normality hypotheses can not be accepted for the majority of them.[2] This clearly influenced our choice of the scenario generation procedure; it was also a primary motivation for our decision to consider the CVaR risk metric that is suitable for skewed distributions. Even though the skewness and kurtosis values approach somewhat those of the normal distribution when the asset returns are expressed in terms of the base currency, still the normality hypothesis does not hold.

We applied PCA using the 128 monthly observations over the period 01/1990–08/2000. We retained 7 principal components that explain about 97% of the total variability. We took 4, 4, 4, 3, 3, 3, 3 samples, respectively, for the retained PCs for a total of 5184 scenarios. Using this scenario set we considered the asset allocation problem in the beginning of September 2000. We generated the efficient risk-return frontiers with the CVaR and the MAD models for different combinations of the investment opportunity set: US assets only, all assets without hedging, all assets with complete hedging, and all assets with selective hedging. Based on the results of these tests we are able to

[2] The Jarque-Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jarque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

Statistical Characteristics of Monthly Domestic Returns of Assets					
Asset Class	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera Statistic
USS	1.519%	3.900%	-0.465	4.271	11.769
UKS	1.164%	4.166%	-0.233	3.285	1.391
GRS	1.213%	5.773%	-0.511	4.503	15.738
JPS	-0.133%	6.336%	0.022	3.609	1.546
US1	0.537%	0.473%	-0.144	2.801	0.727
US7	0.688%	1.646%	-0.047	3.276	0.299
UK1	0.723%	0.710%	1.330	7.209	121.156
UK7	0.913%	1.932%	0.108	3.482	1.157
GR1	0.537%	0.458%	0.655	5.319	34.052
GR7	0.670%	1.390%	-0.863	4.482	25.421
JP1	0.327%	0.522%	0.492	4.147	10.891
JP7	0.608%	1.731%	-0.514	5.149	27.039
Statistical Characteristics of Monthly Proportional Spot Exchange Rate Changes					
Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera Statistic
UStoUK	-0.074%	0.081%	-1.084	6.790	189.330
UStoGR	-0.167%	0.088%	-0.398	3.908	18.842
UStoJP	0.303%	0.133%	1.123	6.904	35.966

Table 3.1: Statistical characteristics of historical monthly data for domestic returns of assets and proportional changes of spot exchange rates over the period 01/1990–08/2000.

address several questions relating to the management of international portfolios.

3.5.3 Investigation Issues

International Diversification Benefits

We examine the effects of international diversification from the perspective of a US investor. Figure 3.1 contrasts the efficient frontiers of portfolios composed of US assets only against those of internationally diversified portfolios. We observe that international diversification improves the risk-return profiles of the portfolios regardless of the risk metric and regardless of whether hedging is employed or not, although the selective hedging strategy clearly exhibits the best performance. The risk-return efficient frontiers of international portfolios clearly dominate the efficient frontiers of portfolios composed solely of US assets. The same behavior is observed whether we use the CVaR or the MAD risk metric. The benefits of international diversification are verified through backtests discussed later in this section.

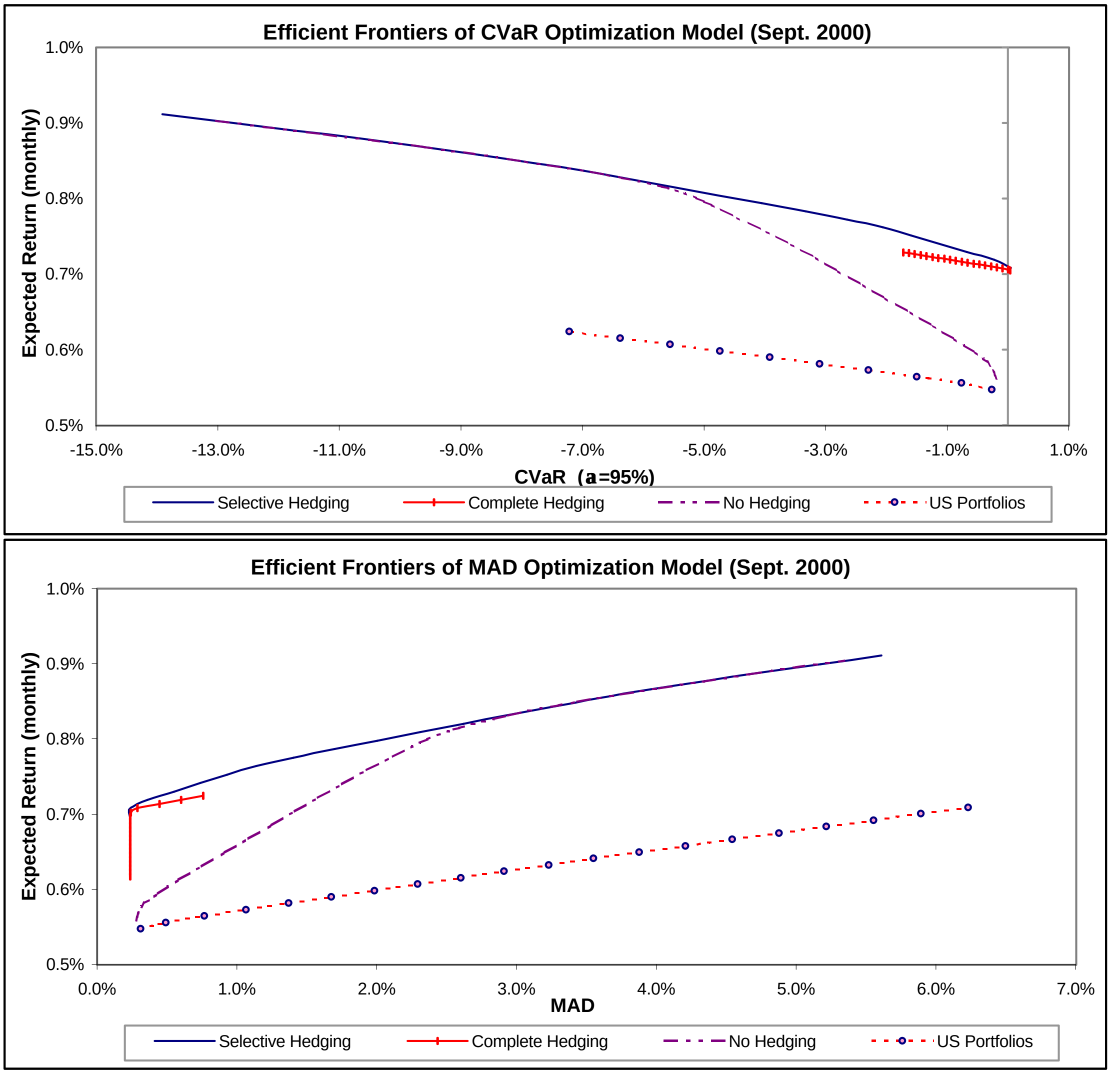


Figure 3.1: Efficient frontiers of CVaR and MAD optimization models for US and internationally diversified portfolios of stocks and bonds (with alternative hedging strategies).

Assessment of Hedging Strategies

Figure 3.1 depicts efficient frontiers of international stock and bond portfolios on September 2000 constructed with the CVAR and MAD models, respectively, using three alternative hedging policies (no-, complete- and selective-hedging). Both models indicate that at the low-risk end of the spectrum completely hedged portfolios are preferable to unhedged portfolios as they yield dominant risk-return profiles over the overlapping range of their efficient frontiers. However, completely hedged portfolios can reach only a limited range of expected return. The efficient frontiers of the no-hedging strategy extend into a range of higher expected returns (and risk) than are attainable by complete hedging. Hence, more aggressive return targets are reached only with riskier unhedged investments; increasing target returns necessitate increasing exposure to currency risk.

In these static tests, selective hedging is the superior strategy as it leads to efficient frontiers that envelope those of the other two hedging strategies. This observation obviously holds for both optimization models as the selective hedging strategy encompasses both of the other two hedging alternatives. We note that in these tests both the CVaR and the MAD models lead to consistent assessments regarding the order of preference of the alternative hedging strategies at any level of target return.

Figure 3.2 illustrates the compositions of several selectively hedged international portfolios corresponding to different points of the CVaR-efficient frontier (for confidence level $\alpha = 95\%$) on September 2000. The postfix $\_u$ or $\_h$ is used on the asset symbols listed in the legend of the graph to denote unhedged, respectively hedged, positions in the corresponding assets. In the risk neutral case the expected return of the portfolio is maximized without any consideration on risk. Hence, the entire budget is allocated to the asset with the highest expected return (in this case the unhedged position in the German stock index). In the minimum risk case, the risk measure is optimized without any constraint on the target expected return. The minimum risk portfolio involves almost exclusively a hedged allocation in the short-term Japanese bond index (JP1$_h$). Greater levels of diversification are exhibited in efficient portfolios with intermediate levels of expected return (and risk) which include both hedged and unhedged positions in multiple international assets. Note that the hedge ratio varies across countries (e.g., Japanese assets are hedged while German assets are unhedged). The hedged proportions also vary between points on the efficient frontier that correspond to different levels of expected return and risk. This points to the advantageous flexibility of the selective hedging approach in comparison to the more restrictive unitary and partial hedging policies.

Effectiveness of Scenario Generation and Model Stability

We carried out several tests to validate the effectiveness of our scenario generation procedure and the stability of the models' results. Using the PCA results from the data of the period 01/1990–08/2000, and following our selective sampling approach, we generated a larger set of 33,075 scenarios by taking 7, 7, 5, 5, 3, 3, 3 samples from the 7 retained PCs, respectively. We also generated sets of 5184 scenarios by taking random samples from the PCs. In each of these scenario sets we took 4, 4, 4,

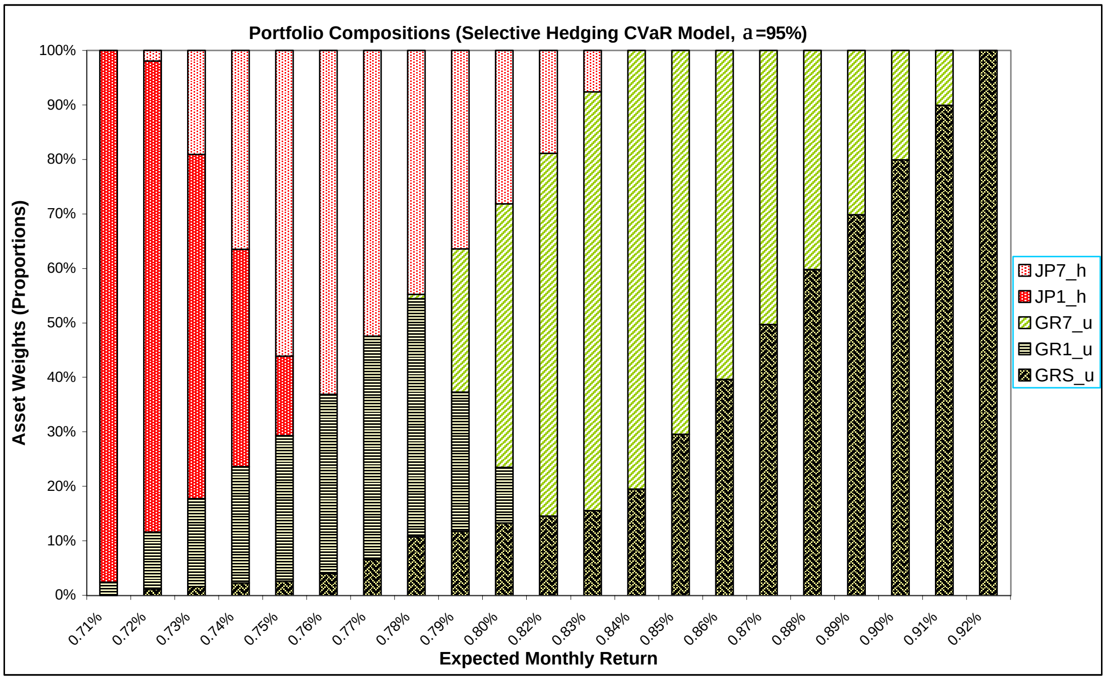


Figure 3.2: Compositions of selectively-hedged international portfolios from the CVaR-efficient frontier of September 2000 (at confidence level α=95%).

3, 3, 3, 3 samples from the corresponding PCs, similar to the scenario set that we had constructed initially. However, in these sets we took random samples rather than employing our selective sampling procedure. We produced the efficient frontiers by employing each of these scenario sets in the two optimization models.

The efficient frontiers for the corresponding scenario sets are plotted in the upper graphs of Figures 3.3 and 3.4 for the CVaR and MAD models, respectively. We note that the efficient frontiers for the 5184-scenario set and the 33,075-scenario set that were generated with the selective sampling procedure are very close for both risk metrics. This illustrates the consistency of the selective sampling approach. We also observe that the random scenario sets imply dominant efficient frontiers in both models. This, however, is misleading as we demonstrate in the middle graphs of Figures 3.3 and 3.4. We took the "efficient" portfolios obtained with the various scenario sets and simulated their performance over the large set of 33,075 scenarios which played the role of an out-of-sample scenario set. For each portfolio we determined the expected return and risk measure (95%-CVaR or MAD) over the out-of-sample simulation. The results of these simulations are plotted in the middle graphs of Figures 3.3 and 3.4 for the CVaR and the MAD metric, respectively. Again, we observe that the simulated frontiers of the initial 5184-scenario set practically retrace the efficient (optimal) frontiers of the 33,075-scenario set in both the CVaR and the MAD models. This is a strong indication of the effectiveness and consistency of our selective sampling procedure, as the resulting frontiers remain quite stable. On the contrary, the simulated frontiers for the random scenario sets are far from efficient with respect to the out-of-sample scenarios.

Finally, we generated 18 different scenario sets ranging in size from 25,000 to 60,000 scenarios. These scenario sets were produced on the basis of the selective sampling procedure by varying the number of samples for each PC. The optimal portfolios of the 33,075-scenario models were then simulated on each of these out-of-sample scenario sets and their corresponding risk-return characteristics were recorded. The results of these simulations are depicted in the bottom graphs of Figures 3.3 and 3.4. Again we observe that the resulting risk-return curves of the simulations remain rather stable; they remain within a narrow band around the efficient frontiers optimized on the 33,075-scenario set, both in the CVaR and in the MAD model. The width of this band collapses at the minimum risk end of the frontiers, indicating that the minimum risk portfolio is practically invariant with respect to sample. All out-of-sample tests indicate that the selective sampling procedure is effective and the results of the models are stable.

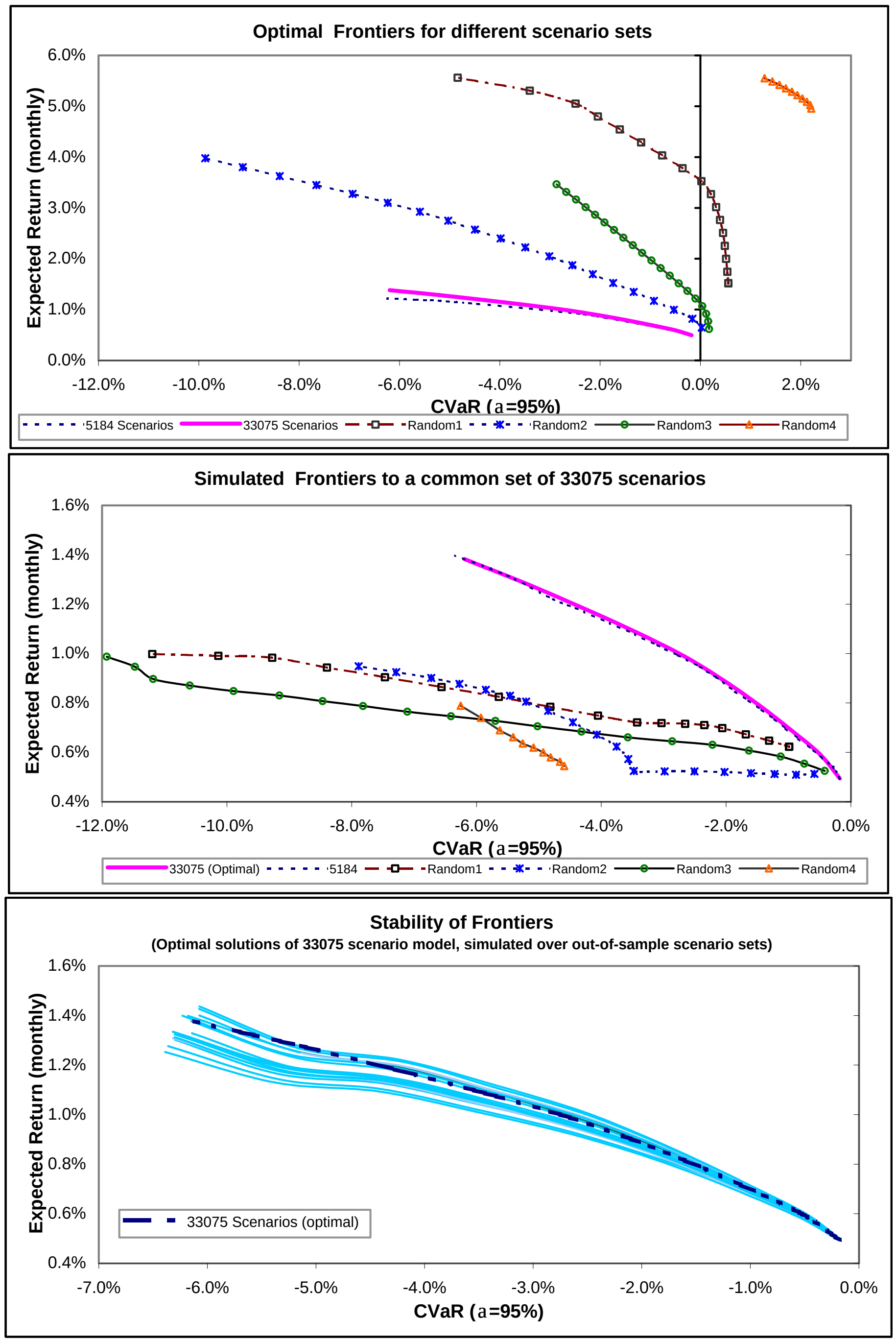


Figure 3.3: Risk-return frontiers of portfolios generated with the CVaR model for several in-sample and out-of-sample test cases.

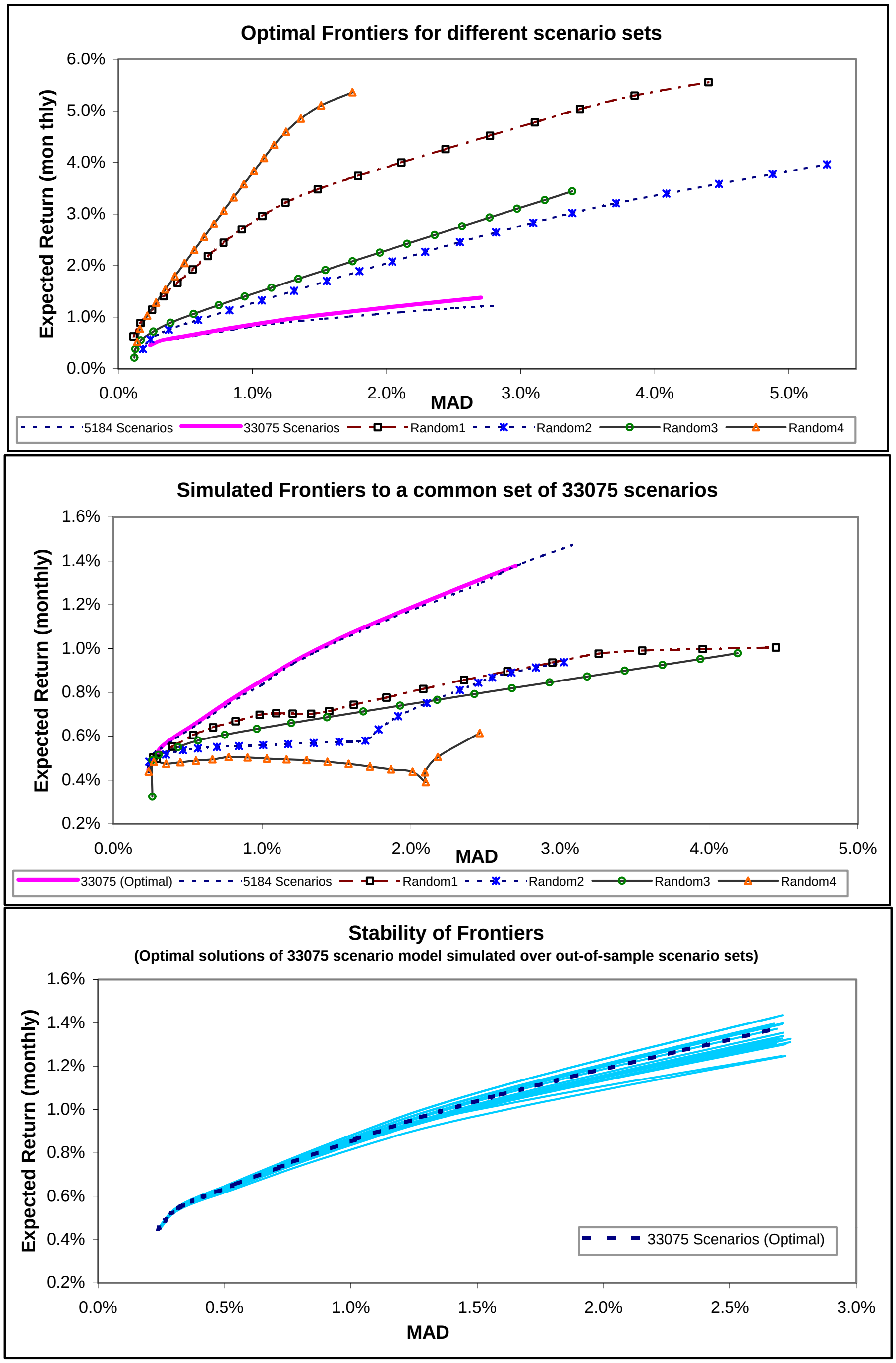


Figure 3.4: Risk-return frontiers of portfolios generated with the MAD model for several in-sample and out-of-sample test cases.

Tightness of CVaR bound on VaR

We now examine empirically the relation between CVaR and VaR. In Figure 3.5 we plot the efficient frontiers for the CVaR-optimal solutions at two different confidence levels, α =95% and 99%. In the same figure we also plot the corresponding VaR estimates for these solutions, denoted as VaR(CVaR*); these curves depict the VaR values of the CVaR-optimized portfolios at the respective confidence levels. As expected, CVaR provides a lower bound for VaR (recall that they are both expressed in terms of portfolio return) at the corresponding confidence level α. However, note that this bound is not tight, although the difference between CVaR and VaR is reduced at increasing confidence levels. It should be kept in mind that the frontiers of VaR against expected return for the CVaR-optimal portfolios need not be efficient from the VaR perspective; the exact VaR-efficient frontiers are not available.

Similar results are reported by Jobst and Zenios [109] for portfolios of corporate bonds. In fact, the differences of their CVaR-efficient frontiers from the VaR(CVaR*) curves are more prononounced because the returns of corporate-bond portfolios are more skewed (due to credit and default risk) than those of international indices and foreign exchange rates.

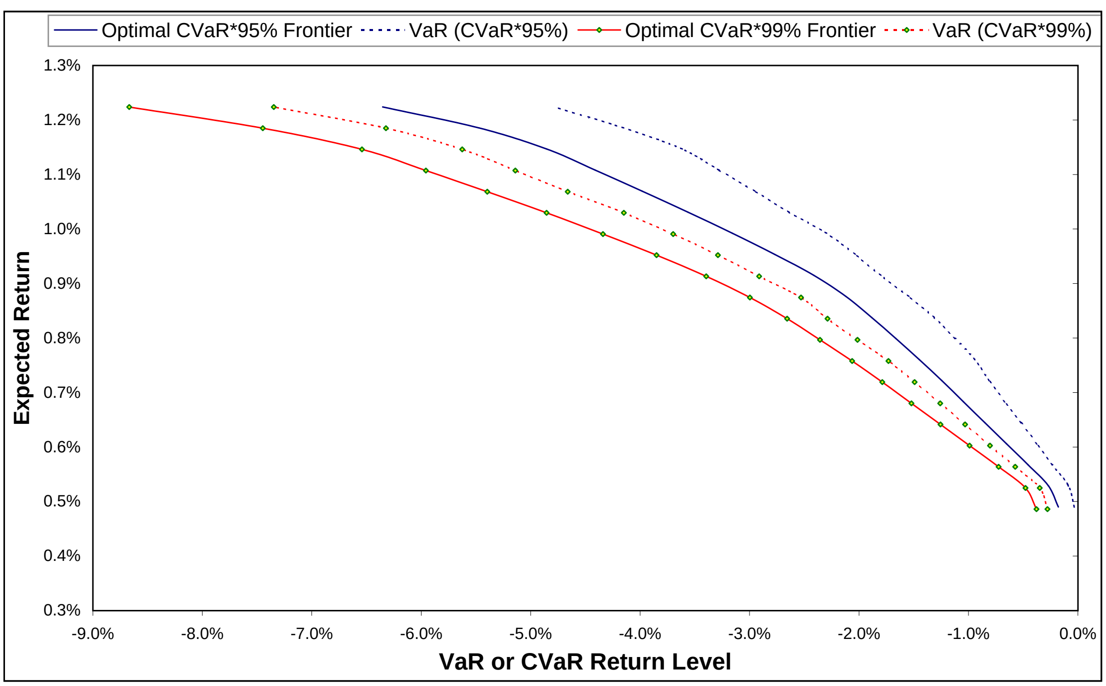


Figure 3.5: Comparison of CVaR*-optimal and VaR frontiers at confidence levels α=95% and 99%.

Dynamic Tests: Comparative Performance of Risk Management Metrics

So far we have observed that in the static tests the CVaR and the MAD models exhibit practically the same behavior. For example, in the asset allocation problem of September 2000 they rank the same way, in order of relative preference, the alternative hedging strategies at all levels of target return. Moreover, the two models yield almost indistinguishable frontiers for the asset selection problem of September 2000. That is, the frontier of CVaR for the MAD-optimized portfolios is almost indistinguishable from the CVaR-efficient frontier for the same scenario set. Conversely, the frontier of MAD for the CVaR-optimized portfolios is almost identical to the MAD-efficient frontier. The results of the two models also exhibit quite similar stability in out-of-sample simulations. However, a definitive comparison between the CVaR and the MAD models can not be reliably made based on static tests alone.

Thus, we resort to backtesting experiments on a rolling horizon basis for a more substantive comparison between the two models. The rolling horizon simulations cover the 37-month period from 04/1998 to 04/2001. At each month, we use the historical data from the previous 10 years (120 monthly observations) to calibrate the PCA procedure and to generate 5184 scenarios by the selective sampling procedure described in sections 3.4.1–3.4.3. The asset returns of these scenarios are updated according to the Bayes-Stein approach of section 3.4.4. We then solve the resulting optimization model and record the optimal portfolio. The clock is advanced one month and the realized return of the portfolio is determined from the actual market values of the assets and the observed exchange rates. The same procedure is then repeated for the next time period and the ex post realized returns are compounded. We ran such backtesting experiments for both the CVaR and the MAD models using various values of target monthly return μ.

The results for the CVaR model are depicted in Figure 3.6. The minimum risk portfolio ($\mu = 0.0\%$) attains a stable growth path representing a 0.6% geometric mean of monthly returns (7.2% annual) over the test period. Increasing ex post returns are achieved with increasing levels of target return μ, obviously at the expense of increasing volatility. The MAD model generates growth paths with similar patterns to those of the CVaR model. However, as illustrated in Figure 3.7, its ex post performance is somewhat less successful in comparison to that of the CVaR model. The CVaR model consistently outperforms the MAD model, especially for the low risk strategies. As the value of the target return parameter μ is increased the two models behave more similarly. This is due to the fact that as μ increases, meeting the requested target return level becomes the governing factor over the minimization of the respective risk metric.

The superior performance of the CVaR model over the MAD model in our test cases is evident in Figure 3.8. This figure depicts the geometric mean against the standard deviation of ex post realized monthly returns over the 37-month test period for all simulation experiments. Clearly, the CVaR model outperforms the MAD model at all levels of target return μ by generating steeper and more stable growth paths (i.e., higher returns with lower volatility). The difference in the performance of the two models is gradually reduced with increasing values of the parameter μ. On the same graph we

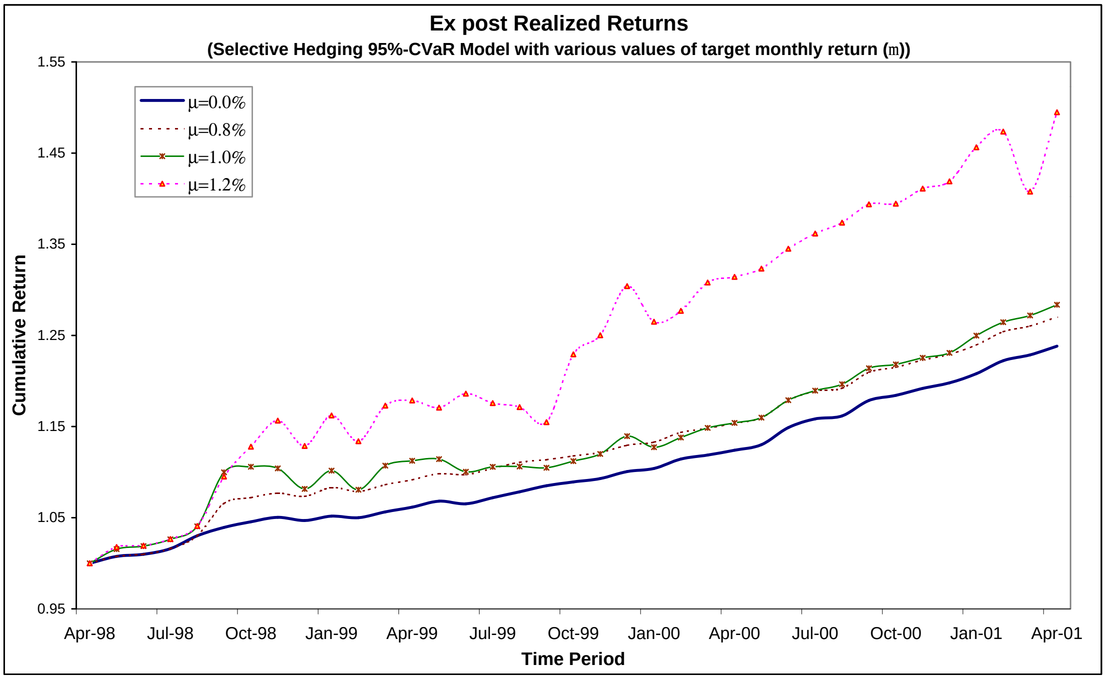


Figure 3.6: Ex-post realized returns with the selective hedging CVaR model (at confidence level $\alpha = 95\%$) over the period 04/1998–04/2001 for different values of target monthly return (μ).

also plot the ex post performance of individual assets over the same test period. Both optimization models outperform all individual assets, with the CVaR model being the most successful in realizing effective growth performance while limiting risk (as evidenced by the low volatility levels).

Finally, we revisit the question regarding the potential of international diversification. We conducted backtesting experiments with the CVaR model (at confidence level $\alpha = 95\%$) allowing portfolios of US assets only and contrasted the results with corresponding experiments that allow selectively hedged international portfolios. The results are summarized in Figure 3.9 for the minimum risk case (i.e., $\mu = 0.0\%$). While the selected US and international portfolios perform similarly for the first year of the simulation, after that the international portfolios clearly outperform the US portfolios.

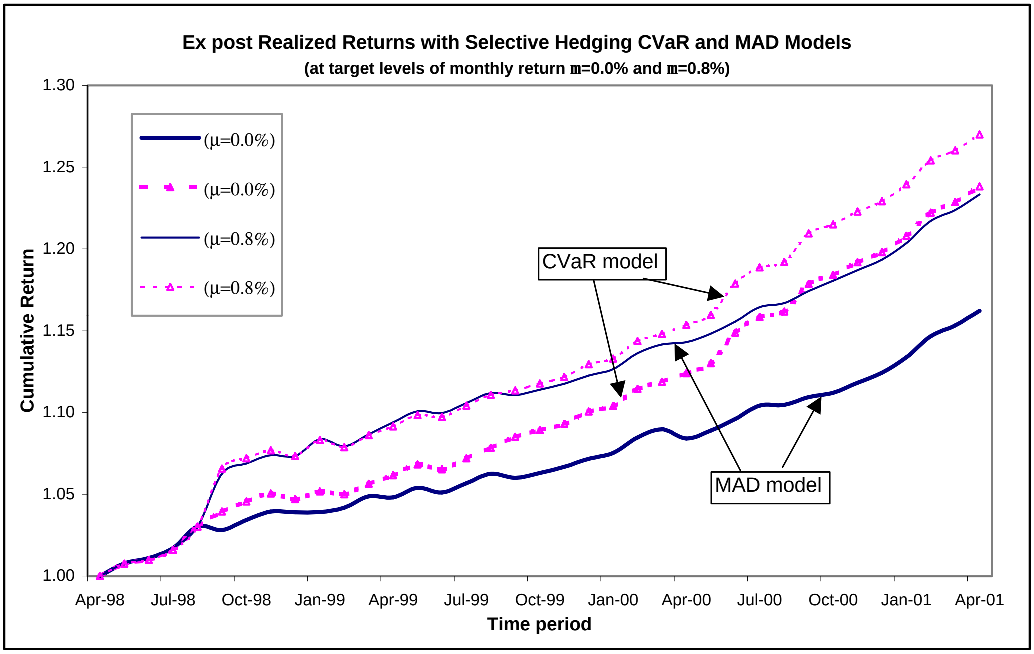


Figure 3.7: Ex-post realized return paths with the selective hedging CVaR and MAD models over the period 04/1998–04/2001 (for target monthly return values μ=0.0% and 0.8%).

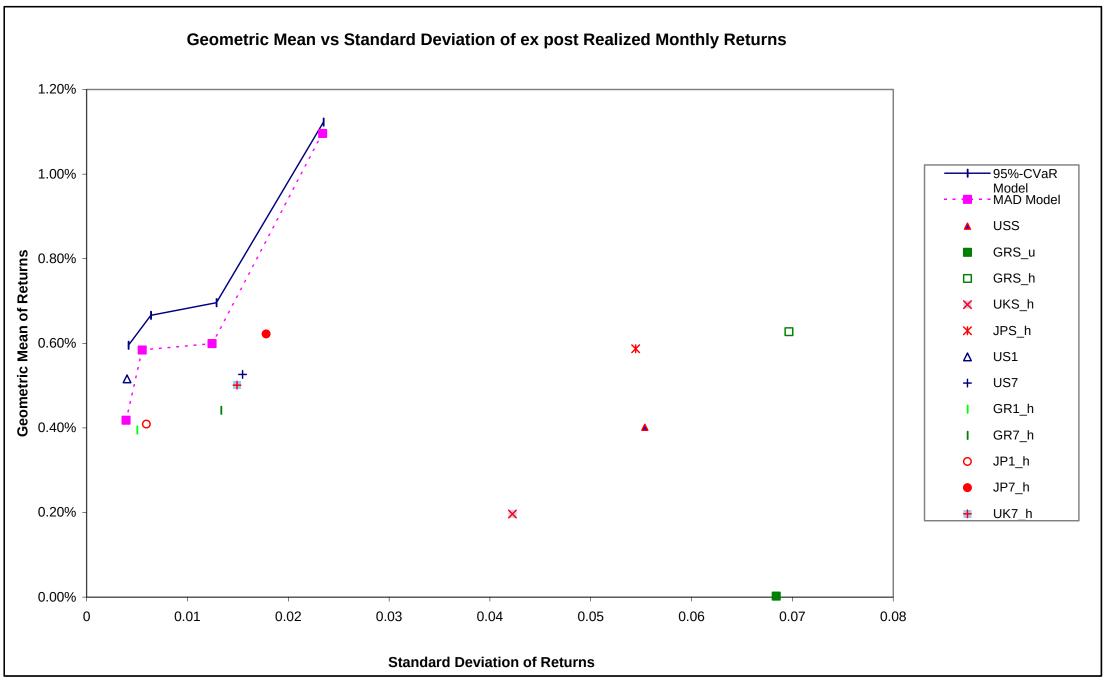


Figure 3.8: Comparative ex post performance — in terms of the realized geometric mean and standard deviation of monthly returns — of individual assets, and international portfolios selected by the CVaR and MAD models over the period 04/1998–04/2001.

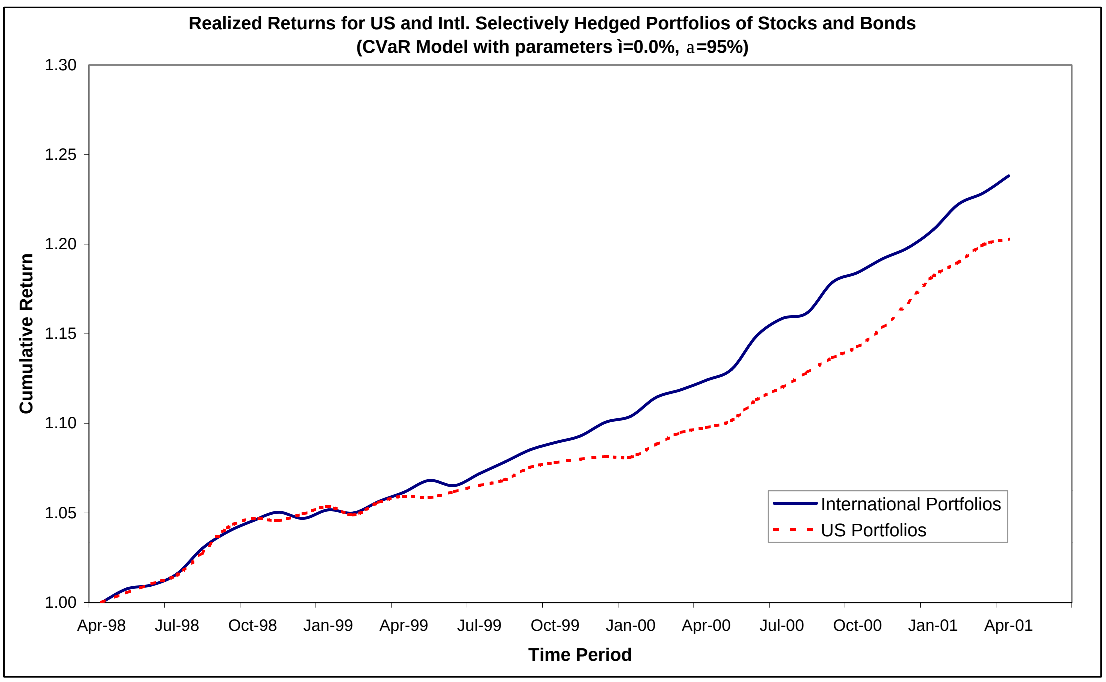


Figure 3.9: Ex-post realized return paths for US and selectively hedged international portfolios of stocks and bonds obtained with the CVaR model (with parameters α=95% and μ=0.0%) over the backtest period 04/1998–04/2001.

3.6 Conclusions

We developed an integrated simulation and optimization framework for international asset allocation and we demonstrated its effectiveness as a decision support tool through a series of empirical tests. This framework involves both a scenario generation approach for depicting the uncertainty in asset returns and exchanges rates, and suitable portfolio optimization models for risk management.

The scenario generation procedure employs a selective sampling approach with PCA to capture the joint variation of domestic asset returns and exchange rates (i.e., market and currency risk). It is combined with the Bayes-Stein estimation correction to counter estimation risk for the mean returns. The consistency and effectiveness of the scenario generation method, and its superiority over random sampling, were established by means of out-of-sample simulations. For sufficiently large scenario sets the efficient portfolios at any level of target return and, consequently, the risk-return efficient frontiers, remain quite stable with respect to sample.

The risk management optimization model uses the CVaR metric that is suitable for asymmetric return distributions. This choice is consistent with the asymmetric return distributions exhibited in historical data of stock and bond indices in multiple countries. Our model internalizes selective hedging decisions within the portfolio selection context to yield flexible investment recommendations. We verified empirically earlier findings regarding the value of the selective hedging strategy and we demonstrated the benefits of international diversification in improving the risk-return performance of investment portfolios.

We compared the CVaR model with the MAD model both in static tests as well as in dynamic backtesting experiments. We observe that in static tests both models behave quite similarly; they exhibit similar stability with respect to scenario samples and they trace almost indistinguishable risk-return profiles. However, when the models were repeatedly applied over successive time periods in the context of backtesting experiments with real market data, the CVaR model clearly outperformed the MAD model especially for low-risk portfolios. In the backtesting simulations the CVaR model produced more effective ex post realized return paths, both in terms of higher growth rates and lower volatility.

Chapter 4

A Dynamic Stochastic Programming Model for International Portfolio Management

In the previous chapter we developed a single-period optimization model for international asset allocation and we demonstrated its potential as a decision support tool. We demonstrated the benefits of international diversification in improving the risk-return performance of investment portfolios and we verified empirically earlier findings regarding the value of the selective hedging strategy. We found that the optimal currency hedge ratios are different for each country and thus demonstrated that a "universal" hedge ratio is suboptimal.

In this chapter we develop international portfolio management models in a multiperiod decision framework. We implement stochastic programming models with a longer time horizon that consider successive portfolio rebalancing decisions at multiple time periods. The stochastic programming models capture decision dynamics, include an operational treatment of hedging decisions by means of implementable forward exchange contracts, and account for the effect of transaction costs.

We test different hedging strategies concerning the permissible level of forward positions in each foreign market and we examine empirically the impact of hedging policies on risk-return profiles of portfolios. We investigate the ex ante and ex post performance of the models for managing international portfolios of stock and bond indices. We find that multistage models improve both the ex ante and ex post performance of international portfolios over the single-period models.

4.1 Introduction

In this chapter we extend the work of the previous chapter in several directions. First, we develop a general multistage stochastic programming model for international portfolio management that considers portfolio rebalancing decisions at multiple points in time. The model accounts for the effects of portfolio (re)structuring decisions over a multi-period horizon, as well as for the flexibility of decision dynamics through subsequent portfolio rebalancing. Second, we introduce linear (proportional) transactions costs for purchases and sales of financial assets, as well as for currency exchange transactions. Third, we employ a more effective procedure to represent the uncertainty of the random variables, namely the asset returns and the exchange rates, so as to capture their empirical distributions. Fourth, we operationalize the currency hedging decisions by incorporating in the model explicit decision variables that correspond to currency forward exchange contracts. These contracts are allowed to take different values for each currency, thus enabling a selective hedging approach. With these extensions, the model captures with a high degree of realism the decision framework in practical settings.

The single-stage model of the previous chapter takes a myopic view of the problem, in the sense that it optimizes a single portfolio structuring decision at a certain point in time. The model has a single-period horizon and cannot account for longer-term effects. However, portfolio managers are usually concerned with the management of their portfolios in a dynamic setting that involves multiple portfolio rebalancing decisions as more information is progressively gathered regarding the economic environment. This is manifested in a sequence of transactions on asset holdings that successively restructure the portfolio in response to changing perceptions (i.e., estimates of distributions) on economic conditions. More flexible models that take a longer-term view of the problem and account for decision dynamics become indispensable tools for portfolio management. This chapter addresses exactly the development of suitable decision support tools for managing international portfolios of financial assets in a dynamic setting.

Under some conditions, myopic portfolio management models can provide acceptable approximations and serve as decision support tools in practice. As we demonstrated in the previous chapter, these models take as inputs scenarios that capture the short-term co-movements of asset returns and exchange rates and determine portfolios that trade off expected return against a reduction in downside risk over a single-period horizon — a horizon of one month was used in the previous chapter. In some cases, solving a sequence of myopic models based on the maximization of expected utility of portfolio returns results in the same solutions as solving a corresponding multi-period, dynamic portfolio management problem (see, Mossin [144], Hakansson [84] and [85]). This result holds under some critical assumptions: absence of transaction costs and taxes, absence of exogenous cashflows (i.e., cash infusions or liabilities) during the planning horizon, specific forms of utility functions and, most importantly, inter-temporal independence of stochastic asset returns (see, Hakansson [85]). These are rather restrictive assumptions that do not usually hold in practice.

Transaction costs do play a role in international portfolio management and their effect must be properly considered in portfolio optimization models. Their omission can lead to high portfolio turnover with a consequent reduction in gains and a potential increase in risk exposure. As we indicate in section 4.2.2, the returns of several international indices and the movements of exchange rates exhibit not only high volatility but also asymmetric distributions with fat tails. Unlike the typical assumption of symmetric distributions of asset returns that is often made in portfolio management studies, in this study we explicitly account for skewness and excess kurtosis in the distributions of the random variables. Consideration of these features is important in our models that aim to control the downside risk of portfolio returns. We employ the conditional value-at-risk (CVaR) metric in the objective function in order to minimize the expected losses in the left tail of the portfolio's return distribution. This objective does not fall in the class of utility functions for which myopic models can yield equivalent solutions to dynamic models. Hence, it is important to develop models that explicitly account for decision dynamics.

The decision framework considered in this study for the international portfolio management problem can be summarized as follows. A decision maker is concerned with the management of a set of financial assets (stock and bond indices in this case) denominated in multiple currencies so as to generate profits while at the same time controlling his downside risk exposure. The problem has a multi-period planning horizon and rebalancing decisions are allowed at periodic intervals as new information on market conditions becomes available. Rebalancing decisions involve repositioning of holdings through sales and purchases of assets, as well as currency exchange transactions in the spot market. Currency forward contracts can also be employed to (partly) hedge the currency risk of foreign investments. The forward contracts have a term (maturity) of a single period within the planning horizon.

In this framework, the decision maker starts with a certain portfolio and has full knowledge of the current market conditions (i.e., asset prices and exchange rates) at the time he has to make his initial decision. Thus, the initial asset holdings in the various currencies as well as the entire portfolio can be accurately valued. The decision maker must assess the potential movements of the asset prices (and consequently their returns) and the exchange rates that affect the total value and risk exposure of his portfolio. His perceptions for such market movements must be expressed in terms of a joint probability distribution of the random variables over a certain time interval. His decision to rebalance the portfolio holdings and to enter into currency forward exchange contracts so as to achieve a better risk-return tradeoff should be based on this probability distribution. But as the decision maker has a longer time horizon, he wants to evaluate both the effects of his portfolio restructuring decision in subsequent periods, as well as the impact that the flexibility for subsequent rebalancing decisions can have on his immediate decision. Those later portfolio rebalancing decisions will of course be influenced by the portfolio composition available at that time, the prevailing market conditions at that time, as well as the perception for subsequent potential movements of the random variables. A multi-period decision model becomes necessary to account for the evolution of the stochastic asset prices and exchanges rates at discrete (and usually equidistant) decision points during the planning horizon.

The paradigm of multistage stochastic programs with recourse is particularly suitable to address this problem. Stochastic programs posses several attractive features that make them applicable in diverse practical problems. Uncertainty in input parameters of these models is represented by means of discrete distributions (scenarios) that depict the joint co-variation of the random variables. In multistage problems the progressive evolution of the random variables is expressed in terms of a scenario tree, as we discuss in the next section. The scenarios are not restricted to follow any particular distribution or stochastic process. Thus, any joint distribution of the random variables can be flexibly accommodated. Asymmetric distributions and heavy tails in the random variables can be readily incorporated in the portfolio optimization model. Many alternative scenario generation procedures have been proposed for stochastic programming models. These include, for example, bootstrapping historical data, fitting assumed distributions to historical data and then sampling or discretizing these distributions, discretizing stochastic processes in terms of lattices, econometric models calibrated with historical data, moment matching methods, employing expert opinions regarding possible outcomes of the random variables, etc.

For example, Koskosides and Duarte [127] (their model also provides the flexibility to overlay investor's forward views on the historical patterns, introducing forward expectation-based biases into the scenarios), Mulvey and Vladimirou [150] and Consiglio, Laurent and Zenios used bootstrapping to generate scenarios, a method which samples from historical data. The first who suggested the moment matching approach for static problems were Smith [171], Keefer and Bodily [120], and Keefer [119]. Høyland and Wallace [95] and Høyland et al. [93] extended and refined the idea for multi-period problems. Hochreiter and Pflug [92] generate scenarios using a multidimensional facility location problem that minimizes a Wasserstein distance measure from a target distribution. Mulvey and Zenios [147], and Zenios [192], discuss simulation techniques to generate scenarios of returns for fixed income portfolio models, based on an underlying interest rate lattice. Lattice models for scenario generation that integrate interest rate and credit risk are given in Jobst and Zenios [109]. The scenario generation procedures are usually tailored to the needs of a specific application. Dupačová, Consigli and Wallace [65] review alternative scenario generation methods for stochastic programs. Indeed, this flexibility in the representation of uncertainty in input parameters is a major advantage of stochastic programming models.

Model-based scenario generation methods for asset returns are popular in the insurance industry, see for example the Russel Yasuda model by Cariño et al. [42, 43] and the Tower Perrin model by Mulvey [146] and [149]. Perhaps the most complete scenario generation system developed by the company Towers Perrin for pension management problems (Mulvey [146]). The economic forecasting system consists of a linked set of modules that generate scenarios for different economic factors and asset returns. At the highest level of the system, the Treasury yield curve is modelled by a two-factor model based on Brennan and Schwartz [36].

Stochastic programs can accommodate different objective functions to capture the decision maker's risk taking preferences (e.g., utility functions, penalties on shortfalls and other risk measures, etc.) Moreover, they can effectively incorporate diverse managerial and regulatory requirements in their constraint sets, especially when such requirements are expressed in terms of linear constraints (inequalities or equalities) on the decision variables. Because of their flexibility, stochastic programs have attracted the attention of researchers and practitioners alike and are being applied in various problems. Applications of stochastic programming models to diverse practical problems are documented in the volume edited by Ziemba and Wallace [186].

The Financial optimization models were developed for portfolio management. Bradley and Crane [35] and Kusy and Ziemba [131] made the seminal contributions in this direction. In financial modeling under uncertainty, in particular, stochastic programming models are seeing an increasing use in recent years. Numerous applications have been reported in the literature, for example Wilkie [187], Mulvey [145], and Mulvey and Thorlacius [149] build forward-looking non-linear factor models for pension plans. A plan's surplus is calculated as the value of assets minus the market value of liabilities. Cariño et al. [42, 43] as well as Mulvey [147] et al. [146, 148] applied stochastic programming techiques for insurance firms. For applications to the asset allocation see Mulvey and Vladimirou [151, 150]. For applications to fixed-income securities see Zenios et al. [194], Zenios [192], Consiglio and Zenios [51], Zenios, Holmer, McKendall, Vassiadou-Zeniou [194] and Worzel et al. [189], Hiller and Eckstein [90] and Dupačová et al. [64]. For applications to asset/liability management see Consigli and Dempster [50], Cariño and Ziemba [44], Kouwenberg [128] and Gondzio and Kouwenberg [81], Klaassen [122], and for applications to credit risk see Jobst and Zenios [109].

Several representative financial applications of stochastic programming models can be found in collective volumes, such as Kouwenberg and Zenios [129], Zenios [190], Mulvey and Ziemba [195]. Vladimirou, Zenios and Wets [184], Birge, Edirisinghe and Ziemba [25], Ziemba/ and Zenios [167], and Zenios [45].

A drawback of multistage stochastic programming models is that the explicit consideration of recourse variables and constraints over the discrete outcomes of a scenario tree leads to very large-scale optimization programs that are computationally challenging to solve. The size of the resulting optimization programs grows exponentially with the number of decision stages. Consequently, the prudent selection of scenarios becomes a critical issue. On the one hand, the scenarios must effectively capture the stochastic evolution of the random variables, while on the other hand their number must be controlled in order to keep the size of the resulting optimization program within computationally tractable limits. However, stochastic programs have well-recognized structures that can be exploited in the design of appropriate solution algorithms. These algorithms are typically well-suited for execution on parallel, multi-processing computing systems (e.g., see, Vladimirou and Zenios [185] for a review of parallel algorithms for Stochastic Programming).

The review of specialized algorithms for large-scale stochastic programs is beyond the scope of this study. However, we note that significant progress in the development and implementation of efficient algorithms that capitalize on the special structures of stochastic programs, as well as advancements of modern computing capabilities, now enable the application of large-scale stochastic programs to many practical problems. Moreover, flexible mathematical modeling systems facilitate the formulation and deployment of stochastic programming models. In this thesis, multistage stochastic programming models are brought to bear on important practical problems of risk management for international portfolios. As we demonstrate through extensive numerical experiments, even commercially available, general purpose modeling systems and optimization solvers can be sufficient to address realistic instances of these models.

Of course, multistage stochastic programs are larger and more complex than myopic models and thus more difficult to solve. However, they provide definitive advantages. They consider a longer time horizon that encompasses decisions (portfolio revisions) at multiple points in time. Hence, the consequences of portfolio compositions in subsequent periods are directly considered. Similarly, the potential for portfolio rebalancing decisions and the evolution of uncertainty in later periods influence the decisions of earlier periods. The impact of transaction costs and the potential benefits of diversification are also more properly accounted for in multi-period decision models. All these interdependencies are encompassed in an integrated framework in multistage stochastic programs.

Multi-stage models help decision makers gain useful insights and adopt more effective decisions. They shape decisions based on longer-term benefits and avoid myopic reactions to short-term market movements that may potentially prove risky. They determine appropriate dynamic contingency (recourse) decisions under changing economic decisions (i.e., those reflected in the scenario tree). As dynamic models consider longer planning horizons and account for portfolio rebalancing decisions at multiple time periods, they should reasonably be expected to outperform myopic models. Yet, comparative studies to establish the incremental benefits of multistage stochastic programs in comparison to single-period models can scantly be found in the published literature. Hence, it is still not clear whether the additional complexity in data processing, modeling and solution effort of multistage stochastic programs in comparison to myopic models is compensated by sufficient improvements in performance.

Besides the development and implementation of a dynamic stochastic programming model for the important problem of international portfolio management, another key contribution of this study is the empirical investigation of the model's efficacy as a practical decision support tool. We demonstrate through empirical tests the superiority of the dynamic model over its myopic counterpart. This same observation is confirmed in later chapters that present more complex stochastic programming models that incorporate options for risk hedging purposes. Again, the dynamic models outperform the static, single-period models.

To summarize, in this chapter we formulate a multistage stochastic programming model for managing international portfolios of stock and bond indices in a dynamic setting. We depict uncertainty in asset returns and exchange rates in terms of a scenario tree. We employ a moment-matching procedure to generate the scenarios that reflect the potential outcomes of the random variables. The model determines jointly the optimal portfolio composition — that is, not only the allocation of funds across currencies, but also the selection of specific asset holdings in each currency — as well as the appropriate positions in forward currency exchanges to hedge currency risk exposures, at each stage of the planning horizon. The model determines sequences of portfolio rebalancing decisions as the scenarios evolve over time. The objective function minimizes the conditional value-at risk (CVaR) metric of portfolio losses over a multi-period horizon — that is, it minimizes the expected portfolio losses beyond a critical percentile of the total portfolio return distribution. Equivalently, the objective function minimizes the expected value in the lower tail (up to a prespecified percentile) of the terminal portfolio value at the end of the planning horizon. A target portfolio return over the planning horizon is imposed by means of a parametric constraint.

We use the dynamic stochastic programming model as a basis to empirically compare the effectiveness of alternative policies for hedging currency risks through forward currency exchange contracts.

Thus, we investigate the impact of such strategies on ex ante risk-return profiles of international portfolios of stock and bond indices, as well as their ex post performance in backtesting experiments that involve successive application of the models over several time periods using real market data. Moreover, we compare the performance of two-stage stochastic programming models with that of their myopic counterparts.

The rest of the chapter is organized as follows. In section 4.2 we describe the representation of uncertainty in the multistage optimization model. We illustrate the construction of a scenario tree that depicts the potential evolution in the values of the random variables. We present the statistical characteristics of the random variables based on historical data, and we discuss the scenario generation procedure that is based on principles of moment matching. In section 4.3 we formulate the stochastic programming model for international portfolio management and discuss its features. In section 4.4 we describe the computational tests and we present the empirical results. In section 4.5 we discuss the conclusions and findings of this study. In the appendix we derive the CVaR functional that is used as the objective function in the portfolio management model of this chapter, as well as in the models of later chapters.

4.2 Representation of Uncertainty

The representation of uncertainty in input parameters of the portfolio optimization model is a critical step of the modelling process. In stochastic programs, uncertainty is typically specified in terms of scenario sets that reflect discrete joint distributions of the random inputs. In multistage problems this construct gives rise to a scenario tree, such as the sample tree shown in Figure 4.1. The scenario tree depicts the evolution in the values of the random variables in each period of the planning horizon. The random variables in the dynamic international portfolio management problem are the asset returns during each period in the planning horizon, as well as the spot and forward currency exchange rates at each stage after the first period.

4.2.1 The Scenario Tree

We describe now the structure of the scenario, or event, tree and define notation that is used in the model formulation. The planning horizon is divided to periods $t = 0, 1, ..., T$ that correspond to the times at which portfolio rebalancing decisions can be made. The scenario tree has a depth equal to the number of periods (equivalently, decision stages); in this study we use monthly decision periods. The root node of the tree (denoted as $n = 0$) corresponds to the initial stage at the present time ($t = 0$). All input data associated with the root node are known with certainty (i.e., they are deterministic). The scenario tree branches out from the root node to depict the potential progressive outcomes in the values of the random variables at each subsequent period. The branches emanating from the root node reflect the possible outcomes of the random variables during the first period ($t = 1$). Similarly, the forward branches emanating from any subsequent node of the tree represent the discrete, conditional distribution of the random variables during the respective time period. This distribution is conditional on the state associated with the node from which the branches emanate. Each node of the tree reflects a possible state at the corresponding time period, and captures a joint realization of the random variables at that time.

Each complete scenario distinguishes a sequence of joint realizations of the random variables during all periods of the planning horizon. Thus, it has a one-to-one correspondence with a terminal (leaf) node of the scenario tree. The constituent joint realizations of the random variables during each period are identified by the nodes on the unique path from the root node to the corresponding leaf node associated with the scenario (such as the highlighted path in Figure 4.1). The scenario tree need not be symmetric, and of course it is rarely binomial as shown in Figure 4.1 simply for illustration purposes.

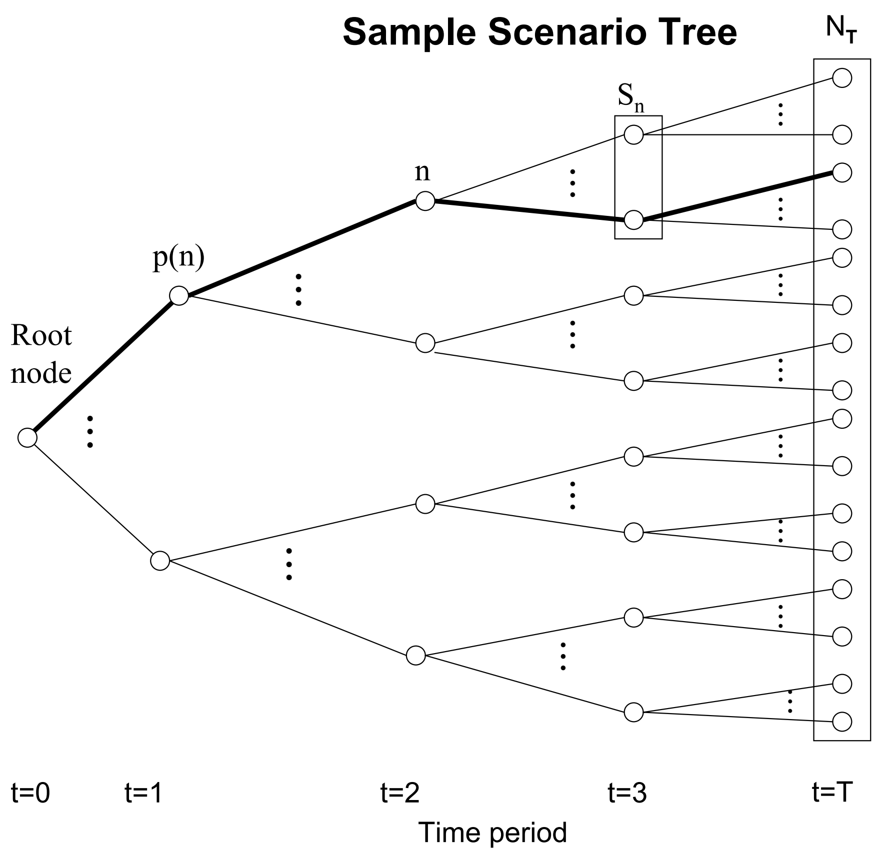


Figure 4.1: General form of a scenario tree

We define the following notation:

$\boldsymbol{N}$	is the set of nodes of the scenario tree,
$n \in \boldsymbol{N}$	is a typical node of the scenario tree ($n = 0$ denotes the root node at $t = 0$),
$\boldsymbol{N_t} \subset \boldsymbol{N}$	is the set of distinct nodes of the tree at time period $t = 0, 1, ..., T$,
$\boldsymbol{N_T} \subset \boldsymbol{N}$	is the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios,
$p(n) \in \boldsymbol{N}$	is the unique predecessor node of node $n \in \boldsymbol{N}$,
$S_n \subset \boldsymbol{N}$	is the set of immediate successor nodes of node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n.
p_t^n	is the conditional probability for the outcome associated with the transition from the predecessor node $p(n)$ to node $n \in \boldsymbol{N}$,
p_n	is the probability of the state associated with node $n \in \boldsymbol{N}$.

The probability of occurrence, p_n, of a certain node (state) $n \in \boldsymbol{N}$ is determined by multiplying the conditional probabilities of the outcomes on the path from the root to the specific node; thus, by compounding the conditional probabilities of the constituent outcomes that lead to the specific state. Of course, the probabilities of all distinct nodes at any decision stage must sum to one (i.e., $\sum_{n \in \boldsymbol{N_t}} p_n = 1, t = 0, 1, ..., T$). Also, the probability of a node is equal to the sum of the probabilities of its immediate successor nodes (i.e., $p_n = \sum_{m \in \boldsymbol{S_n}} p_m, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$).

The scenario tree is a logical structure representing the evolution of the multi-variate random variables (asset returns and exchange rates) over time. At each state (node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$) of the scenario tree, the joint realizations of the random variables in the next time period are depicted by the branches of the tree emanating from that node. Each such joint realization is associated with an immediate successor node of node n. A portfolio rebalancing problem is considered at every node of the scenario tree, except for the leaf nodes at the last period of the planning horizon (i.e., $\forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$). At the leaf nodes ($n \in \boldsymbol{N_T}$) we need only compute the corresponding terminal value of the portfolio under the respective scenarios.

We define distinct (recourse) decision variables and associated constraints to represent the portfolio rebalancing problem that must be addressed at every node of the scenario tree (except for the leaf nodes). Obviously, the size of the resulting multistage stochastic program grows substantially with the number of nodes on the scenario tree. Thus, in multistage problems attention must be paid to limit the branching factor from each node — and, consequently, the total number of scenarios — in order to keep the size of the optimization program within computationally tractable limits. Scenario reduction techniques have been recently proposed for this purpose (see, Dupačová et al. [66]).

Stochastic programs must conform with the logical requirement for non-anticipativity of the decision variables. That is, scenarios that share common information history (outcomes) up to a particular time period — i.e., have common subpath of the scenario tree up to that period — must yield identical decisions up to that period. The condition of non-anticipativity is explicitly enforced in our model formulation as decision variables are defined for each node — instead of each path — of the scenario tree.

4.2.2 Scenario Generation

Having explained the general structure of a scenario tree and its use on the formulation of a multistage stochastic program, we discuss now a scenario generation procedure to produce the discrete outcomes of the random variables that are mapped to the scenario tree structure. Stochastic programs are not restricted to any particular distributional forms of the random variables. They can accommodate discrete distributions that are generated by diverse alternative methods and can capture important features, such as skewness and heavy tails. This flexibility of stochastic programs to accommodate diverse distributional characteristics of the random variables is indeed one of their key advantages.

In this thesis we consider international portfolios comprised of stock and bond indices denominated in different currencies. The random variables of concern are the asset prices (or, equivalently, their returns) and the fluctuations of exchange rates. Statistical analysis of historical market data reveals that these random variables are correlated. Moreover, we observe that their historical values do not conform to normal distributions; they exhibit asymmetric distributions and heavy tails. These important features should be reflected in the postulated scenario sets that should capture the statistical characteristics of the random variables' empirical distributions. This becomes all the more necessary as we are concerned with controlling the downside risk in the tail of the portfolio's return distribution. Effectively capturing the observed skewness and excess kurtosis in the distributions of the random variables, as well as their correlations, becomes important. As a result, we do not employ any distributional assumptions in the scenario generation process. Instead, we apply a moment-matching procedure that generates scenarios so that specific statistical characteristics of the random variables match the corresponding values of their empirical distributions.

In the empirical tests we consider investments in four markets: United States (US), Great Britain (UK), Germany (GR) and Japan (JP), comprised of the following investment instruments in each market: a stock index, denoted as `Stk`, and bond indices with three different maturity bands: a short-term (1–3 years), an intermediate-term (3–7 years) and long-term (7–10 years), denoted `Bnd1`, `Bnd3` and `Bnd7`, respectively. Thus, a total of 16 assets are considered in each portfolio. The problem is viewed from the perspective of a US investor. Repositioning of investments between markets entails spot currency exchange transactions. Moreover, forward currency exchange contracts — with a term equal to a decision period, i.e., one month — are incorporated in the portfolio optimization model as means for (partly) hedging the currency risks of foreign investments. Hence, data for the spot and forward exchange rates of the foreign currencies to USD are also needed.

Market values of the stock indices were obtained from the Morgan Stanley Capital International, Inc. database (`www.mscidata.com`). The corresponding values for the bond indices and the currency spot and forward exchange rates were collected from `DataStream`. All time series have a monthly time-step and cover the period from April 1988 through December 2001. From the historical observations of index prices we computed their corresponding monthly returns (in domestic terms). Similarly, from the observed series of spot exchange rates we computed their corresponding monthly appreciation rates (proportional changes).

We analyze the statistical characteristics of the historical market data. As we can see from Table 4.1, both the domestic returns of the indices and the proportional changes of exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. Jacque-Bera tests on these data indicate that normality hypotheses can not be accepted for the majority of them[1]. The normal distribution cannot properly capture the observed joint behavior of the financial time series. Extreme returns are encountered far more often than predicted by the normal distribution. The corresponding correlations of the random variables — computed on the basis of their observed market data over the period 01/1990 — 12/2001 — are shown in Table 4.2.

[1]The Jarque-Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jarque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

Statistical Characteristics of Monthly Domestic Returns of Assets					
Asset Class	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
US.Stk	1.211%	4.097%	-0.413	3.649	6.11
UK.Stk	1.025%	4.254%	-0.018	3.371	0.606
GR.Stk	1.167%	5.807%	-0.561	4.301	13.92
JP.Stk	-0.170%	6.881%	0.017	3.836	2.45
US.Bnd1	0.567%	0.504%	-0.006	2.765	0.739
US.Bnd3	0.654%	1.118%	-0.112	2.626	1.028
US.Bnd7	0.710%	1.670%	-0.090	2.912	0.295
UK.Bnd1	0.680%	0.714%	0.995	7.461	150.815
UK.Bnd3	0.741%	1.349%	0.510	4.631	29.45
UK.Bnd7	0.793%	1.880%	0.134	3.390	1.533
GR.Bnd1	0.493%	0.480%	0.288	4.451	31.24
GR.Bnd3	0.552%	0.930%	-0.255	3.162	4.58
GR.Bnd7	0.583%	1.372%	-0.663	3.887	22.87
JP.Bnd1	0.283%	0.481%	0.663	4.708	12.98
JP.Bnd3	0.416%	1.121%	-0.099	4.401	26.37
JP.Bnd7	0.503%	1.678%	-0.519	5.434	47.62
Statistical Characteristics of Monthly Proportional Spot Exchange Rate Changes					
Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
UStoUK	-0.116%	2.894%	-0.755	5.672	102.39
UStoGR	-0.124%	3.091%	-0.215	3.399	5.69
UStoJP	0.030%	3.615%	0.942	6.213	105.27

Table 4.1: Statistical characteristics and Jacque-Bera statistic of historical monthly data for domestic returns of assets and proportional changes of spot exchange rates over the period 01/1990 — 12/2001.

	US.Stk	UK.Stk	GR.Stk	JP.Stk	US.Bnd1	US.Bnd3	US.Bnd7	UK.Bnd1	UK.Bnd3	UK.Bnd7	GR.Bnd1	GR.Bnd3	GR.Bnd7	JP.Bnd1	JP.Bnd3	JP.Bnd7	UStoUK	UStoGR	UStoJP
US.Stk	1																		
UK.Stk	0.69	1																	
GR.Stk	0.58	0.60	1																
JP.Stk	0.40	0.39	0.37	1															
US.Bnd1	0.18	0.07	-0.08	-0.02	1														
US.Bnd3	0.21	0.09	-0.05	0.01	0.96	1													
US.Bnd7	0.24	0.11	-0.02	0.01	0.89	0.97	1												
UK.Bnd1	0.13	0.40	0.03	0.00	0.38	0.37	0.35	1											
UK.Bnd3	0.21	0.48	0.11	0.03	0.41	0.44	0.44	0.94	1										
UK.Bnd7	0.26	0.50	0.16	0.05	0.42	0.47	0.49	0.84	0.97	1									
GR.Bnd1	0.05	0.20	0.03	-0.03	0.36	0.39	0.38	0.59	0.59	0.56	1								
GR.Bnd3	0.13	0.25	0.12	0.01	0.42	0.48	0.50	0.54	0.63	0.64	0.92	1							
GR.Bnd7	0.17	0.27	0.20	0.02	0.45	0.53	0.56	0.48	0.61	0.67	0.77	0.94	1						
JP.Bnd1	0.10	0.12	0.02	0.05	0.21	0.21	0.20	0.33	0.31	0.29	0.40	0.36	0.30	1					
JP.Bnd3	0.11	0.09	0.01	0.05	0.22	0.25	0.26	0.24	0.27	0.27	0.37	0.38	0.33	0.91	1				
JP.Bnd7	0.12	0.10	0.02	0.05	0.25	0.29	0.30	0.22	0.26	0.27	0.34	0.38	0.34	0.78	0.94	1			
UStoUK	-0.02	-0.20	-0.10	0.10	0.18	0.17	0.13	-0.17	-0.16	-0.10	-0.09	-0.08	-0.05	0.05	0.08	0.07	1		
UStoGR	-0.09	-0.22	-0.22	-0.07	0.25	0.25	0.19	-0.09	-0.10	-0.09	0.03	-0.02	0.00	0.10	0.12	0.13	0.71	1	
UStoJP	0.08	0.05	-0.15	0.04	0.11	0.13	0.08	0.12	0.11	0.10	0.14	0.08	0.04	0.07	0.06	0.07	0.36	0.46	1

Table 4.2: Correlation coefficients of monthly asset returns and proportional changes of exchange rates computed from historical market data over the period 01/1990 — 12/2001.

Given the empirically observed statistical characteristics of the random variables, we resort to a scenario generation procedure that does not assume any specific distributional form. Instead, we rely solely on the observed market data and generate scenarios so that key statistical properties of the random variables reproduce their corresponding historically observed values. We employ the moment-matching procedure that was proposed by Høyland and Wallace [95], as it was further specialized by Høyland et al. [93]. The method generates a set of scenarios so that selected statistical properties of the random variables match specified target values. Specifically, we match the following statistics: the first four marginal moments (mean, variance, skewness, and kurtosis), as well as the correlations of the monthly asset returns and currency exchange appreciation rates. We estimate the target values to be matched for these statistics on the basis of observed historical data.

The scenario generation method allows the user to specify a-priori the desired number of scenarios, thus controlling the size of the resulting portfolio optimization program. The model presented in the next section is fully dynamic and involves an arbitrary number of decision stages. We generate the scenarios incrementally, one stage at a time. From historical observations of market data we estimate the first four marginal moments and correlations of the random variables for a monthly time period — equal to the decision period of the model. We apply the moment-matching procedure to generate the joint realizations (a multivariate distribution) of the random variables for the first period ($t = 1$) — i.e., for the branches emanating from the root node of the scenario tree. Similarly, for each node at the first stage we generate a number of joint outcomes of the random variables for the next stage that match the same statistical properties. The same procedure is recursively applied to generate outcomes for all subsequent stages. The outcomes generated from any particular node of the scenario tree are equiprobable; this is not an absolute requirement as the moment-matching method can also generate outcomes with differing probabilities, at the expense of higher computational effort.

With this scenario generation procedure we do not account for possible inter-temporal dependencies of the random variables (e.g., mean reversion). That should be the subject of further research. However, we postulate dynamic evolutions of the random variables that are used as inputs in the portfolio optimization model to determine the respective sequences of portfolio rebalancing decisions and currency forwards exchange contracts.

At the root node we know with certainty the market prices of the assets, the spot exchange rates, and the forward rates for the term of the next period (i.e., one month). Using the projected outcomes of asset returns and currency appreciation rates for the first period we can easily compute the corresponding joint asset prices and spot currency exchange rates for every node of the scenario tree at the first stage ($t = 1$). Similarly, knowing the asset prices and spot exchange rates at a subsequent node of the scenario tree, as well as the projected asset returns and currency appreciation rates for the next stage for the branches emanating from that node, we can easily determine the asset prices and spot exchange rates at the end of the stage for each immediate successor node. For each tree node we also specify forward exchange rates for one-month forward currency transactions. These forward rates are set equal to the expected value of the respective spot exchange rates at the end of the next period — i.e., by taking the expectation of the spot exchange rates over the immediate successor nodes. This is done so as to eliminate arbitrage in currency exchanges.

4.3 The International Portfolio Management Model

The model specifies a sequence of investment decisions at discrete points in time (monthly intervals in this study). Decisions are made at the beginning of each time period. The portfolio manager starts with a given portfolio and with a set of postulated scenarios about future states of the economy, represented in terms of a scenario tree. This information is incorporated into a portfolio restructuring decision. At the beginning of the next period the manager has at hand a seasoned portfolio. The precise composition of the portfolio depends on the transactions at the previous decision point; its value depends on the outcomes of asset returns and exchange rates realized in the interim period. Thus, for every joint realization of the random variables we end up at a different state at the end of the period — associated with a descendant node on the scenario tree. Another portfolio restructuring decision is then made at that node based on the available portfolio and the subsequent possible outcomes of the random variables.

The problem of dynamic portfolio (re)structuring is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. Without loss of generality, no direct exchanges between foreign currencies are executed — either in the spot or in the forward market — in order to simplify the formulation of the model and to reduce its data needs, as well as the number of decision variables. All currency exchanges are executed with respect to the base currency. Thus, to reposition his investments from one market (currency) to another, the investor must first convert to base currency the proceeds of foreign asset sales in the market in which he reduces his presence and then purchase the foreign currency in which he wishes to increase his investments. The spot exchange rates of foreign currencies to USD applicable at the time are used for the currency exchange transactions. At the end of the holding period we compute the state-dependent value of each investment using its projected price at the respective node of the scenario tree. The USD-equivalent value is determined by applying the corresponding estimate of the appropriate spot exchange rate to USD at the same tree node.

The investor's portfolio is exposed to market risk in the domestic and foreign markets, as well as to currency exchange risk for foreign investments. To (partly) hedge currency risk at each decision node, the investor may enter into currency exchange contracts in the forward market. We allow forward currency exchanges with a term (maturity) of one period, i.e., one month. The critical decision is the amount of the contract. The actual exchange takes place at the end of the period using the forward rate that is specified at the time the decision is made (i.e., at the beginning of the period). These forward currency exchange contracts reflect hedging decisions to mitigate the respective exposure of foreign investments to currency risk.

Note that the value of asset holdings in a particular currency at the end of any period is not known with certainty at the beginning of the period, i.e., at the time that a forward contract is decided. This value depends on the realized asset returns during the interim period. Yet, an exact amount for the forward currency exchange contract must be specified beforehand, before the realized asset returns are observed. Clearly, forward currency exchanges can hedge only partially the currency risk exposure of foreign investments.

The notion of "full hedging" that is often referred to in the literature is unattainable with forward exchanges. This is because forward exchange contracts are decided beforehand, while the ending value of a foreign investment in uncertain as it depends on its return during the interim period. Only quanto options can fully hedge the risk exposure of a foreign asset holding. A quanto is an option written on a foreign underlying security but its strike price, its price and its payoff are expressed in a different reference currency; a prespecified exchange rate — usually the respective forward rate for the term of the option — is used for the value translations. Hence, a quanto can fully cover both the market and currency risk of a foreign asset holding. The use of quantos for risk hedging purposes is discussed in chapters 6 and 8 of this thesis. Here, we use only forward contracts as currency risk hedging instruments. Yet, the optimal selection of currency forward contracts is an integral part of the portfolio optimization model.

The stochastic programming model uses as critical input the representation of uncertainty as captured by the scenario tree. This tree postulates the dynamic evolution of the random variables over the planning horizon. The model minimizes the conditional value-at-risk (CVaR) of portfolio losses at the end of the planning horizon. That is, it minimizes the expected value in the lower tail (up to a specific percentile) of the portfolio value at the end of the planning horizon. The incorporation of the CVaR function in the stochastic programming model is done along the lines of Rockafellar and Uryasev [163]; its derivation is summarized in the appendix of this chapter.

We define the following additional notation:

User-specified parameters:

α	critical percentile for `VaR` and `CVaR`,
μ	target (minimum) expected return during the entire planning horizon.

Deterministic input data:

$\boldsymbol{C_0}$	set of markets (synonymously, countries, currencies),
$\ell \in \boldsymbol{C_0}$	index of investor's base (reference) currency (in our case USD),
$\boldsymbol{C}$	set of foreign markets; $\boldsymbol{C} = \boldsymbol{C_0} \setminus \{\ell\}$,
$\boldsymbol{I}_c$	set of available investments (asset classes) in market $c \in \boldsymbol{C_0}$,
b_{ic}	initial position (in number of units of face value) in asset $i \in \boldsymbol{I}_c$ of market $c \in \boldsymbol{C_0}$,
P_{ic}^0	current price of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$ (in units of domestic currency c)
c_c	initially available cash in currency $c \in \boldsymbol{C_0}$,
γ_{ic}	proportional transaction cost for sales or purchases of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$,
λ_c	transaction cost rate for currency $c \in \boldsymbol{C_0}$,
e_c^0	current spot exchange rate of currency $c \in \boldsymbol{C_0}$,
φ_c	currently quoted one-month forward exchange rate for foreign currency $c \in C$,
V_0	total value of initial portfolio (i.e., initial wealth, in units of reference currency):

$V_0 = \sum_{c \in \boldsymbol{C_0}} \left(c_c + \sum_{i \in \boldsymbol{I}_c} b_{ic} P_{ic}^0\right) e_c^0.$

Scenario dependent data:

p_n	probability of node $n \in \boldsymbol{N}$ — we generate equiprobable scenarios, thus $p_n = \frac{1}{\lvert \boldsymbol{N_t} \rvert}, \quad \forall n \in \boldsymbol{N_t}, \quad t = 0, 1, ..., T,$
P_{ic}^n	price of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$ on node $n \in \boldsymbol{N}$ (in units of domestic currency c),
e_c^n	spot exchange rate for currency $c \in \boldsymbol{C}$ at node $n \in \boldsymbol{N}$,
φ_c^n	one month forward exchange rate for foreign currency $c \in \boldsymbol{C}$ at node $n \in \boldsymbol{N}$,
V_n	the final value of the portfolio held at the end of the planning horizon under leaf node $n \in \boldsymbol{N_T}$ (in units of the base currency).

Decision variables (decisions are made at nodes of the scenario tree except for the leaves; thus separate variables are defined for each node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ to reflect the decisions made at the respective node):

x_{ic}^n	units of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$ purchased,
v_{ic}^n	units of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$ sold,
w_{ic}^n	units of asset $i \in \boldsymbol{I}_c,\ c \in \boldsymbol{C_0}$ held in the portfolio after revision,
g_c^n	expenditure of base currency for purchase of currency $c \in \boldsymbol{C}$ in the spot market,
q_c^n	revenue in base currency from sale of currency $c \in \boldsymbol{C}$ in the spot market,
f_c^n	amount of base currency collected from sale of currency $c \in \boldsymbol{C}$ in the forward market (i.e., amount of forward contract, in units of the base currency). A negative value indicates a purchase of the foreign currency in the forward market. These decisions are made at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, but the actual transaction is executed at the end of the respective period, i.e., at the successor nodes S_n.

Auxiliary variables:

y_n	loss shortfall beyond `VaR` at leaf node $n \in \boldsymbol{N_T}$,
z	variable in definition of `CVaR` — equals to `VaR` at the optimal solution.

All exchange rates are expressed as the equivalent amount of the base currency for one unit of the foreign currency. Obviously, $e_\ell^n = 1,\ \forall\, n \in \boldsymbol{N}$. Also $g_\ell^n = q_\ell^n = f_\ell^n = 0\,, \quad \forall n \in \boldsymbol{N}$; these variables are actually omitted in the formulation.

We formulate the stochastic programming model for the multi-period international portfolio management problem as follows:

$$\texttt{min.} \quad z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N_T}} p_n y_n \tag{4.1a}$$

$$\texttt{s.t.} \quad w_{ic}^0 = b_{ic} + x_{ic}^0 - v_{ic}^0, \qquad \forall c \in \boldsymbol{C_0}, \forall i \in \boldsymbol{I}_c \tag{4.1b}$$

$$w_{ic}^n = w_{ic}^{p(n)} + x_{ic}^n - v_{ic}^n, \qquad \forall c \in \boldsymbol{C_0}, \forall i \in \boldsymbol{I}_c, \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{4.1c}$$

$$c_\ell^0 + \sum_{i \in \boldsymbol{I}_\ell} v_{i\ell}^0 P_{i\ell}^0 (1-\gamma_{ic}) + \sum_{c \in \boldsymbol{C}} q_c^0 (1-\lambda_c) \tag{4.1d}$$

$$= \sum_{i \in \boldsymbol{I}_\ell} x_{i\ell}^0 P_{i\ell}^0 (1+\gamma_{ic}) + \sum_{c \in \boldsymbol{C}} g_c^0 (1-\lambda_c), \tag{4.1e}$$

$$c_c^0 + \sum_{i \in \boldsymbol{I}_c} v_{ic}^0 P_{ic}^0 (1-\gamma_{ic}) + \frac{g_c^0}{e_c^0} = \sum_{i \in \boldsymbol{I}_c} x_{ic}^0 P_{ic}^0 (1+\gamma_{ic}) + \frac{q_c^0}{e_c^0}, \quad \forall c \in \boldsymbol{C} \tag{4.1f}$$

$$c_\ell^n + \sum_{i \in \boldsymbol{I}_\ell} v_{i\ell}^n P_{i\ell}^n (1-\gamma_{ic}) + \sum_{c \in \boldsymbol{C}} q_c^n (1-\lambda_c) + \sum_{c \in \boldsymbol{C}} f_c^{p(n)}$$

$$= \sum_{i \in \boldsymbol{I}_\ell} x_{i\ell}^n P_{i\ell}^n (1+\gamma_{ic}) + \sum_{c \in \boldsymbol{C}} g_c^n (1-\lambda_c), \qquad \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{4.1g}$$

$$c_c^n + \sum_{i \in \boldsymbol{I}_c} v_{ic}^n P_{ic}^n (1-\gamma_{ic}) + \frac{g_c^n}{e_c^n}$$

$$= \sum_{i \in \boldsymbol{I}_c} x_{ic}^n P_{ic}^n (1+\gamma_{ic}) + \frac{q_c^n}{e_c^n} + \frac{f_c^{p(n)}}{\varphi_c^{p(n)}}, \quad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{4.1h}$$

$$L_n = -R_n, \qquad \forall n \in \boldsymbol{N_T} \tag{4.1i}$$

$$y_n \geq L_n - z, \qquad \forall n \in \boldsymbol{N_T} \tag{4.1j}$$

$$y_n \geq 0, \quad \forall n \in \boldsymbol{N_T} \tag{4.1k}$$

$$x_{ic} \geq 0, \quad w_{ic}^n \geq 0, \qquad \forall c \in \boldsymbol{C_0}, \forall i \in \boldsymbol{I}_c, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.1l}$$

$$0 \leq v_{ic}^0 \leq b_{ic}, \qquad \forall c \in \boldsymbol{C_0}, \forall i \in \boldsymbol{I}_c \tag{4.1m}$$

$$0 \leq v_{ic}^n \leq w_{ic}^{p(n)}, \qquad \forall c \in \boldsymbol{C_0}, \forall i \in \boldsymbol{I}_c, \forall n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{4.1n}$$

$$g_c^n \geq 0, \quad q_c^n \geq 0, \qquad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.1o}$$

$$f_c^n \leq \Phi_c^n, \qquad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.1p}$$

$$R_n = \frac{V_n}{V_0} - 1, \qquad \forall n \in \boldsymbol{N_T} \tag{4.1q}$$

$$\sum_{n \in \boldsymbol{N_T}} p_n R_n \geq \mu \tag{4.1r}$$

where

$$V_n = c_\ell^n + \sum_{i \in \boldsymbol{I}_c} w_{ic}^{p(n)} P_{ic}^n + \sum_{c \in \boldsymbol{C}} \left\{ f_c^{p(n)} + e_c^n \left(c_c^n + \sum_{i \in \boldsymbol{I}_c} w_{ic}^{p(n)} P_{ic}^n - \frac{f_c^{p(n)}}{\varphi_c^{p(n)}} \right) \right\} \quad \forall n \in \boldsymbol{N_T}. \tag{4.2}$$

The model is a linear program minimizing the Conditional Value at Risk of the portfolio losses at the end of the last period, while it parametrically sets a minimum target , μ, for the expected return at that time. Expectations are computed over the set of postulated scenarios (i.e., terminal nodes). The values of the CVaR and VaR of portfolio losses at the end of the planning horizon are obtained from the solution of the multistage program, which is structured in such a way that decisions at every time period influence subsequent decisions. Decisions explicitly depend on the sequence of postulated joint outcomes of the random variables. Clearly, this model falls in the general framework of multistage stochastic programs with recourse.

Equations (4.1b) and (4.1c) are the balance conditions for each asset, in each market, for the first and subsequent decision stages, respectively. Equations (4.1e) and (4.1f) impose the cash balance conditions at the first stage; the former for the base currency ℓ and the latter for the foreign currencies $c \in \boldsymbol{C}$. In each case, total availability of funds stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of funds include the total expenditure for the purchase of assets and outgoing currency exchanges in the spot market. Note that no holdings in cash are allowed after portfolio restructuring. Hence, we do not need to model the interest rates in each market. Similarly, equations (4.1g) and (4.1h) impose the state-dependent cash balance conditions in every currency at subsequent decision stages. These equations additionally account for forward currency exchange contracts that are decided at the predecessor node.

The constraints in (4.1l), (4.1m) and (4.1n) disallow short positions in the assets; they ensure that asset sales cannot exceed the quantities in the portfolio at hand. Constraints (4.1o) ensure that the amounts of currencies purchased or sold in the spot market are nonnegative. The constraints in (4.1p) impose limits on the currency forward exchange contracts at every decision state. We consider three alternative hedging policies, by appropriately setting the allowable limits in these constraints:

$$(i) \quad \Phi_c^n = \sum_{i \in \boldsymbol{I}_c} e_c^n (w_{ic}^n P_{ic}^n), \quad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.3}$$

$$(ii) \quad \Phi_c^n = \sum_{m \in S_n} p_m e_c^m (\sum_{i \in \boldsymbol{I}_c} w_{ic}^n P_{ic}^m), \quad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.4}$$

$$(iii) \quad \Phi_c^n = \infty, \quad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.5}$$

$$(i\nu) \quad \Phi_c^n = 0, \quad \forall c \in \boldsymbol{C}, \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{4.6}$$

The first case permits a forward contract to cover an amount equal to the current value of total asset holdings in the respective currency, with the foreign holdings valued at the time that the forward contract is decided. This alternative does not account for the fact that the value of the asset holdings will change due to the uncertain returns and exchange rates during the period. In the second case, the level of a forward contract is bounded by the expected value (in units of the base currency) of the respective foreign asset holdings in the portfolio at the end of the decision period — thus, the expectation is taken over the outcomes at the successor nodes, so as to reflect the expected exposure of the foreign asset positions. In the third case, no restriction is explicitly imposed on the level of forward contracts, which are then treated simply as alternative investment opportunities. In this case, forward positions are allowed regardless of the value of asset holdings in the respective currency in the portfolio at hand. The last case sets the currency forward contracts identically equal to zero (in fact, we eliminate form the model the variables f_c^n in this case). This case corresponds to totally unhedged international investments.

Constraints (4.1j), and (4.1k) are the definitional constraints for determining CVaR, as they are introduced in the general formulation (4.14) (see the Appendix), while equation (4.1i) is the definition of the loss function. Equation (4.1q) defines the portfolio return at leaf node $n \in \boldsymbol{N_T}$. Finally, constraint (4.1r) sets a minimum target μ on the expected return over the planning horizon. By parametrically changing this target level we can generate solutions that trade off expected return for a reduction in total risk exposure (as captured in the objective function). The final value of the portfolio at leaf node $n \in \boldsymbol{N_T}$ is computed in (4.2). This equation takes into consideration exogenously available cash and revenues from the liquidation of asset holdings in each market. The contribution of foreign investments to the total value of the portfolio takes into account the settlement of any outstanding forward contracts. The residual amount is valued in terms of the base currency by using the projected spot exchange rates.

The use of proportional transaction costs γ_{ic} for the assets in a sense creates a proportional bid-ask spread of $2\gamma_{ic}P^n_{ic}, \quad \forall c \in \boldsymbol{C_0}, \quad \forall i \in I_c, \quad \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. Note that the effective purchase price for an asset in a cash balance equation for node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ is $P^n_{ic}(1+\gamma_{ic})$ while the effective sale price for the same asset is $P^n_{ic}(1-\gamma_{ic})$. In the computational tests we use a uniform proportional transaction cost $\gamma_{ic} = 0.05\%$ for all assets (i.e., $\forall c \in \boldsymbol{C_0}, \quad \forall i \in I_c$). As defined in the formulation, the model permits a different transaction cost for each asset. Similarly, transaction costs may be applied to only one side of a transaction (say, purchases only). A transaction cost is similar by applied to the spot currency exchange transactions with the parameters λ_c. We use a transaction cost $\lambda_c = 0.01\%$ for all foreign currencies in the numerical implementation of the model. No transaction cost is charged to forward exchange contracts.

Starting with an initial portfolio, and using a representation of uncertainty for the asset prices and exchange rates by means of a scenario tree, the multistage portfolio optimization model determines optimal decisions under the contingencies of the scenario tree. The portfolio rebalancing decisions at each node of the tree specify not only the allocation of funds across markets but also the position in each asset. Moreover, the respective currency forward contracts are determined so as to (partly) hedge the currency risk exposure of the foreign investments during the holding period (i.e., until the next portfolio rebalancing decision). Simple variants of the model can reflect alternative policies for hedging currency risks with the use of currency forward contracts, as we indicate in equations (4.3) – (4.6). We employ the model so as to analyze empirically its effectiveness in managing international portfolios of stock and bond indices, and to compare the performance of risk hedging alternatives.

4.4 Empirical Results

We have implemented the multistage stochastic programming model presented in the previous section in the General Algebraic Modeling System (GAMS) [40]. We solved single-stage and two-stage instances of the model. The aims of the numerical experiments are: (i) to investigate the efficacy of the stochastic programming model as a practical decision support tool for managing international investment portfolios, (ii) to compare the performance of alternative risk hedging strategies so as to assess the impact of currency forward contracts in controlling currency risk, and (iii) to contrast the performance of the two-stage stochastic programming model with that of its single-stage counterpart.

We examine the performance of various decision strategies in static as well as in dynamic tests to identify the most promising tactics. In static tests we compare the risk-return profiles (efficient frontiers) generated with appropriate variants of the model at a certain point in time (specifically, August 2001). The observed monthly asset returns and currency appreciation rates during the previous ten years were used to calibrate the scenario generation procedure. From the historical data we computed estimates of the first four marginal moments and correlations of the random variables. Using these estimates as target values we invoked the moment-matching procedure to generate the scenarios for the portfolio optimization models as we described in section 4.2.2. The scenarios represent joint realizations of asset prices and exchange rates at the decision stages of the models.

The resulting optimization models were run with alternative bounds on currency forward contracts as specified in equations (4.3) – (4.6). The static tests considered portfolio selection problems. The initial portfolio involved only a cash endowment in the base currency — which the model apportioned optimally to the available assets — and no holdings in any other asset. The models were repeatedly run by varying parametrically the level of minimum expected return, μ, at the end of the planning horizon. The solutions trace the corresponding efficient frontiers of expected portfolio return vs the CVaR risk metric of portfolio losses (at the $\alpha = 95\%$ percentile) over the planning horizon. Clearly, the efficient frontiers are determined in-sample; that is, with respect to the postulated scenarios that are used in the optimization phase.

The static tests yield useful insights as they reveal the potential of the various decision strategies with respect to the postulated distributions (scenarios) at a specific point in time. However, this potential does not necessarily materialize in practice. We additionally ran dynamic tests to assess the actual performance of the models in backtesting simulations. The models were run, on a rolling horizon basis, at each successive month in the period 04/1998-11/2001 (i.e., for a total of 43 months) as follows. Starting with an initial cash endowment in April 1998 each model was set up and executed to decide the initial portfolio composition. The statistics of the random variables, computed from their observed market values during the previous 10 years, were matched in generating the scenarios. The appropriate model was solved and the first-stage decisions were recorded. The clock then advanced one month. The realized return of the optimal portfolio was determined on the basis of the revealed market prices of the assets and the exchange rates. Any outstanding currency forward contracts were settled and the resulting cash positions in each currency were updated accordingly. A new set of scenarios was then generated by matching the statistics of the random variables to their estimates from the monthly observations of their market values during the previous ten years. With these scenarios as input, and using the portfolio composition and cash positions resulting from the previous decisions as a starting point, the new model was solved. The process was repeated for each successive month and the ex post realized returns were recorded and compounded. Thus, the backtesting simulations demonstrate the actual returns that would have been realized had the decisions of the models been implemented during the simulation period 04/1998-11/2001.

4.4.1 Assessment of Hedging Strategies

Figure 4.2 presents the efficient frontiers resulting from alternative permissible levels of currency forward positions — according to their respective bounds indicated in (4.3) – (4.6). This figure depicts the tradeoffs between the expected return and the CVaR risk measure of losses during a two-month planning horizon for optimal portfolios of two-stage stochastic programming models. The two-stage models in these tests used 15,000 scenarios composed of 150 joint realizations of the random variables in the first decision stage, each followed by a set of 100 further outcomes in the second period. The postulated outcomes for each decision period were generated so that the statistical properties of the random variables in each stage matched their empirical values for the 10 years prior to August 2001. Thus, these static tests considered portfolio selection problems at August 2001. The results obtained at this time are typical of the models' observed behavior at other periods.

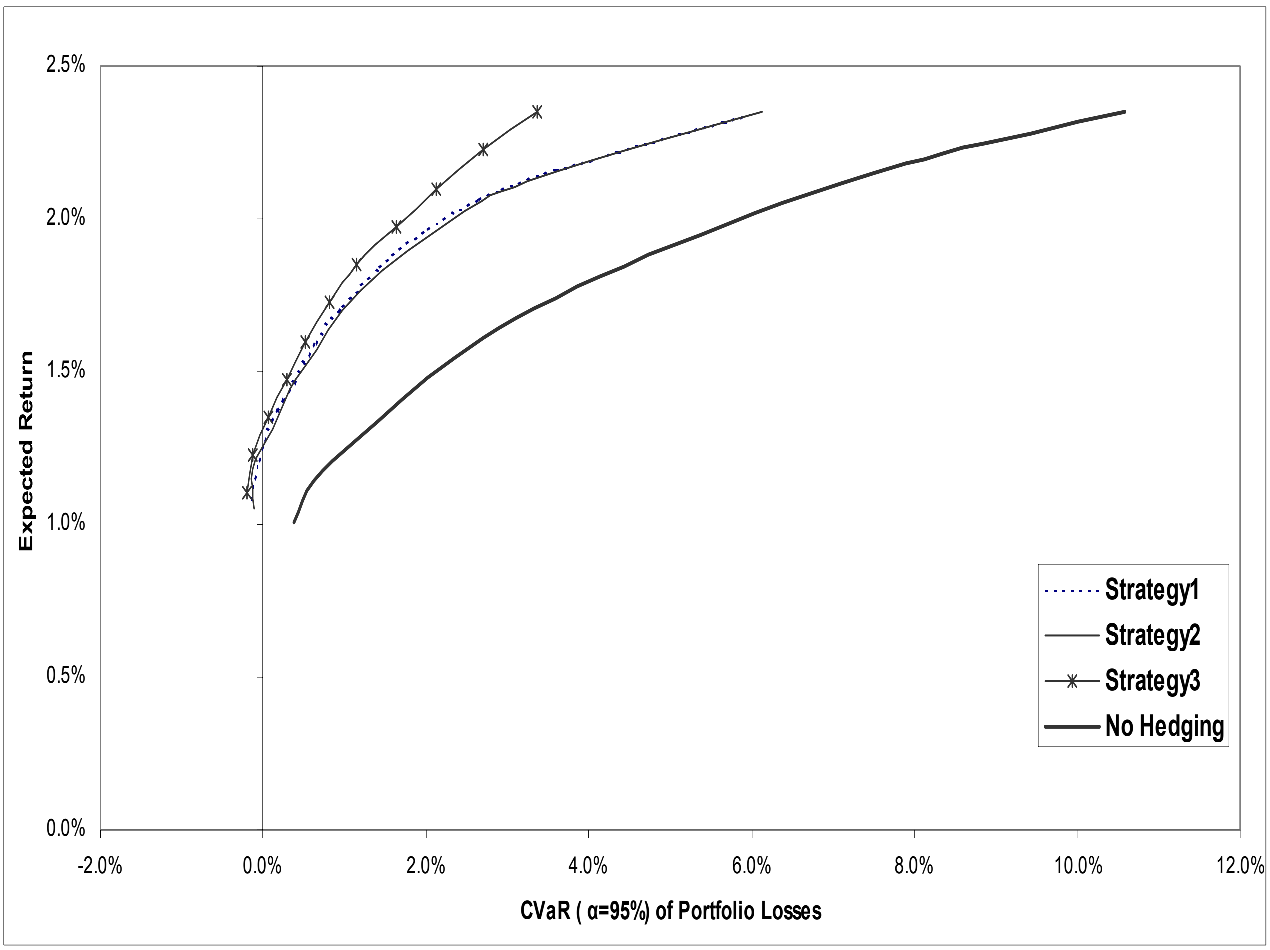


Figure 4.2: Comparison of risk-return efficient frontiers with two-stage models for alternative hedging strategies (August 2001).

From these results we observe the following. The efficient frontier of the optimal unhedged portfolios — that do not involve currency forward contracts — is clearly dominated by the efficient frontiers of the optimal, selectively hedged portfolios. Incorporating decisions for currency forward positions within the portfolio management model improves the risk-return profile of the resulting optimal portfolios as indicated by the shift of the efficient frontier to the left. That is, for any value of target expected return, the optimal hedged portfolios exhibit a lower level of risk. This potential gain from risk reduction is increasing for more aggressive targets of expected portfolio return.

The three strategies that permit currency forward contracts reflect alternative views of the selective hedging approach, as they allow the hedge ratios to be different — and indeed they come out to be different — across currencies. The third strategy — that does not place any restriction on the currency forward positions — exhibits the dominating risk-return profile in the static tests. This results from the fact that this strategy corresponds to the least-constrained use of currency forward positions. In this case, the currency forward positions are not tied to the foreign asset holdings; they are regarded as unconstrained choices in the investment opportunity set. Thus, they can be selected not necessarily to hedge currency risk but also for speculative purposes. We should expect that the adoption of this strategy in practice may, at least on occasion, lead to the selection of riskier portfolios. The other two strategies restrict the use of currency forward contracts so as to (partially) cover the exposure in foreign currency investments. The first strategy (see eq. (4.3)) bounds the levels of currency forward contracts to the respective current values of foreign investments in the revised portfolio, while the second strategy (see eq. (4.4)) bounds the levels of such contracts to the expected value of the respective foreign asset holdings at the end of the decision period. We see that these two strategies exhibit practically indistinguishable risk-return profiles.

We applied the same hedging tactics in backtesting experiments in order to investigate whether their potential in the static tests actually materializes in a practical setting. Figure 4.3 contrasts the ex post realized returns of the different hedging policies over backtesting simulations covering the period 04/1998-11/2001. The results were generated by successively applying the respective two-stage models in each month of the simulation period and implementing each time the first-stage optimal decisions of the model. Each instance of the model again used 15,000 scenarios (150 outcomes for the first stage, each associated with 100 further outcomes for the second period). The scenarios were generated so that the statistics of the random variables in the postulated outcomes for each stage matched their corresponding empirical values from the 120 prior monthly observations.

The first graph in Figure4.3 presents the results for the minimum risk case — i.e., when the models simply minimize the CVaR risk measure at the end of the planning horizon without imposing a target on expected portfolio return. The results in the second graph were generated when a target $\mu = 2\%$ for expected return over the two-month horizon was imposed at each instance of the two-stage portfolio optimization model (i.e., for an aggressive investor).

In the minimum risk case, all three selective hedging strategies resulted in essentially the same performance. Only the most liberal (third) strategy fell a little behind in the fall of 1998, but traced closely the performance of the other two strategies in all other periods. The second strategy was a very slight winner in these simulations. The performance of optimal unhedged portfolios was not very different either in this case. We can detect only a very slight advantage of the optimal hedged portfolios in comparison to the optimal unhedged portfolios in the minimum risk case, as they demonstrate just slightly more stable return paths. The optimal portfolios were positioned almost exclusively in short-term government bond indices throughout these simulation exercises. We note that in the minimum risk case the models selected diversified portfolios of short-term international bond indices that weathered the storm of the September 11, 2001 crisis unscathed, and actually generated profits during that period. That crisis affected primarily the stock markets for a short period and had no material impact on the bond markets, especially the international bond markets.

The differences in the performance of the alternative hedging strategies are more pronounced when we use a more aggressive target for expected return, as shown in the second graph of Figure 4.3. In this case all three selective hedging strategies demonstrate material benefits from the reduction of currency risk through the use of currency forward positions for hedging purposes. Their realized return paths over the simulation period are discernibly more stable than the corresponding path of the optimal unhedged portfolios. We note particularly that the international hedged portfolios were affected much less than the undhedged portfolios during market downturns (e.g., 08/1998, 01–02/2000, 04/2001, 08–11/2001). Again, the first two strategies demonstrated very similar ex post performance; with the second strategy being a slight favorite. The third strategy — that can take unrestricted positions in currency forwards — lagged a bit behind, particularly in periods of down markets. In these simulations the models selected portfolios that varied more substantially over time, in comparison to the runs for the minimum risk case, in their attempt to meet the high expected return target. The optimal portfolios also involved sizable positions in the US stock index for most of the simulation period, and thus did not avoid the effects of the crisis in September 2001 — even if more mildly than the unhedged portfolios.

The results of the static and the dynamic tests show that benefits can be gained, in terms of risk reduction, by internalizing decisions for currency forward positions within the portfolio management models. Regardless of fairly minor details on how permissible currency forward contracts may be controlled in the portfolio optimization models, we observe that the use of these instruments has a positive impact on reducing risk. The stochastic programming models prove to be useful and practical decision support tools for international portfolio management. They provide a flexible framework for incorporating alternative risk hedging strategies in a dynamic decision setting.

We adopted the second strategy for the tests that are presented below as it exhibited slightly more effective performance than the other two hedging alternatives. Recall that in this strategy the allowable positions in one-month currency forwards are bounded by the expected value of the portfolio holdings in a respective currency during the same term.

4.4.2 Comparison of Single- and Two-Stage Models

We now turn to a comparative assessment of single- and two-stage variants of the stochastic programming models for international portfolio management. We first examine the performance of the models in static tests on August 2001. Again, the 15,000 scenarios generated for the previous static tests were used as inputs to the models in these tests as well. We set up and solved two-stage and single-stage instances of the portfolio selection model. The single-stage model had exactly the same horizon (two months) and used exactly the same scenarios for the random variables as the two-stage model. Thus, the two models contained exactly the same information content in terms of the outcomes of the random variables (scenarios), and optimized the same risk measure (CVaR of portfolio losses at the end of the two-month horizon), starting with the same initial portfolio — a cash endowment in the base currency only — and parametrically imposed target levels on expected portfolio return over their two-month horizon. The only fundamental difference was that, unlike the two-stage model, the single-stage model did not allow portfolio rebalancing decisions during the interim month.

The resulting risk-return efficient frontiers of the two models are shown in Figure 4.4. Clearly, the two-stage model exhibits a dominating risk-return profile over the common two-month horizon of the two models. These results confirm the intuition that when the two models use the same scenario sets and have the same horizon, then the two-stage model should produce superior results owing to its additional flexibility to incorporate rebalancing decisions at an intermediate stage. The introduction of rebalancing decisions in the two-stage model can essentially be viewed as a relaxation of constraints in comparison to the corresponding single-stage model. The intermediate decisions permit the reconfiguration of the portfolio during the planning horizon for each descendant node of the root in the scenario tree, i.e., in response to each specific outcome of the first stage. The same principle applies to extensions of the model with additional decision stages as long, of course, as the postulated outcomes (scenarios) remain the same over the planning horizon.

For a specific representation of uncertainty (scenario tree) regarding future outcomes of the random variables, and a specific horizon, it is preferable to allow portfolio rebalancing in as many stages as captured in the scenario tree (i.e., branching nodes), rather than to aggregate decision stages. Of course, this modeling choice has implications on the size and computational complexity of the resulting stochastic programs.

Figure 4.4 shows that, for any target of expected return, the two-stage model yields optimal solutions with lower risk at the end of the horizon than the optimal solutions of the respective single-stage model. The reduction in risk for the two-stage model can be attributed to the allowable portfolio adjustments at an intermediate stage during the horizon. The benefits seem to be marginal at the minimum risk case. This is because at the minimum risk case both the single- and two-stage models exhibit a “flight to safety”, i.e., they select very similar portfolios composed of the most secure assets. However, the potential benefits for risk reduction by adopting the two-stage model in comparison to its single-stage counterpart increase for increasingly aggressive (higher) targets of expected return that dictate the selection of riskier portfolios. In these cases, the flexibility of an interim readjustment of the portfolio during the planning horizon in response to changing economic conditions carries a higher additional value.

Finally, we contrast the performance of the single- and two-stage models in dynamic, backtesting experiments with real market data. Again, the models are set up and executed repeatedly at successive time periods according to the procedure we had explained earlier. From each instance of the models we keep and implement only the optimal decisions for the first stage. The single-stage model has a horizon of one month, while the two-stage model has a horizon of two months, partitioned into two monthly decision stages. The necessary scenarios are generated on a rolling horizon basis; at each month the scenarios are generated on the basis of the historical data during the prior ten years.

In order to assess the effect of adding information (i.e., additional joint outcomes of the random variables in the scenario set) we experimented with the following variants of the models:

- A two-stage model that used 15,000 scenarios (150×100) generated as described before,
- A single-stage model that used the 150 first-stage outcomes of the two-stage model as its scenario set,
- A single-stage model with a finer representation of uncertainty for its monthly horizon, comprised of 15,000 scenarios.

Hence, the first two models share the same information regarding potential outcomes in the month ahead, but the two-stage model incorporates additional information for potential outcomes in the following month. The third model involves a much finer representation for the distribution of the random variables in the month ahead.

The ex post realized returns of these models during backtesting simulations over the period 04/1998-11/2001 are shown in Figure 4.5. The first graph of this figure corresponds to the experiments with minimum risk models, e.g., models that minimize the risk measure without any constraint on target expected return. The second graph represents the use of more aggressive return targets in the models (i.e., a target expected return $\mu = 1\%$ over the monthly horizon of the single-stage models, and a target expected return $\mu = 2\%$ over the two-month horizon of the two-stage model). We observe that, in all cases, the two-stage model produces the superior performance, while the single-stage model with the limited set of 150 scenarios yields the worst performance. Clearly, the addition of information in the representation of uncertainty — either with more scenarios for the single-stage model, or with the extension of the horizon to consider further outcomes in a second decision period in a two-stage model — improves the performance of the models.

Let us compare the performance of single- and two-stage models that both use 15,000 scenarios. In the minimum risk case, the performance of the models is quite similar. As we explained earlier, this is a consequence of the selection of very similar portfolios (composed of the most secure assets) by both models. But, as we expected, the differences in realized performance are more evident when higher target returns are imposed on the model, forcing the selection of riskier portfolios. In this case the two-stage model yields a clearly superior performance.

Finally, Figures 4.6 and 4.7 compare the compositions of the optimal portfolios for the single- and two-stage models throughout the simulation period for the minimum risk case, and under more aggressive expected return targets, respectively. We see that in the minimum risk case both models select very similar portfolios. These consist primarily of positions in the short-term US bond index and hedged positions in the short-term government bond indices of the other three countries; these instruments exhibited the most stable performance over the backtesting period. The portfolios followed by the two-stage model are somewhat more stable across time than those selected by the single-stage model, thus indicating less active portfolio turnovers.

For the more aggressive targets of expected return, we observe in Figure 4.7 that the models resort to more diversified portfolios that include also holdings of stock indices (particularly in the US stock index). The optimal portfolios of the two-stage model are more diversified and, again, more stable over time compared to those of the single-stage model. This can be attributed to the look-ahead feature of this model that has a longer horizon and considers longer-term effects in comparison to the myopic model when deciding the portfolio composition.

In closing, we list in Table 4.3 the size and computational effort required to solve the stochastic programming models. The reported solution times reflect an average for typical instances of the respective problems. The problems were solved with IBM's Optimization Subroutine Library (OSL) on an IBM RS/6000 44P worskstation (Model 170 with a 400 MHz Power 3 Risk processor, 1Gb of RAM, running AIX 4.3). We note the almost proportional increase in solution time with the increasing number of scenarios for the single-stage model. We note also the larger size and required solution time for the two-stage model in comparison to a single-stage model with the same number of scenarios. Clearly, these solution times are by no means prohibitive for realistic instances of the models with today's available computing technologies. The problems can be solved much more efficiently by employing specialized algorithms that exploit the structure of stochastic programming models. However, issues of computational efficiency are not of primary concern in this study.

Model	Number of constraints	Number of variables	Number of nonzeros	Approx. solution time (sec)
Single-Stage 150 scenarios	325	360	3860	0.1
Single-Stage 15000 scenarios	30026	30060	375111	55
Two-Stage 15000 scenarios	36782	39969	444499	88

Table 4.3: Size and solution times of models

4.5 Conclusions and Further Research

In this chapter we extended the work of the previous chapter in significant ways. We developed and implemented an integrated simulation and optimization framework for managing international portfolios of financial assets, and we demonstrated its viability through extensive computational experiments using real market data. The multistage stochastic programming model can address portfolio management problems in a dynamic setting. The model provides a useful and flexible decision support framework as it captures many features of practical problems. It considers a multi-period horizon capturing decision dynamics, it accommodates a multistage representation of the stochastic evolution of the random variables by means of a scenario tree, it incorporates practical considerations (e.g., transaction costs) and it can accommodate alternative objective functions to reflect the decision maker's risk taking attitudes. Our implementation of the model controls the risk exposure of the portfolios by minimizing the conditional value-at-risk measure of portfolio losses at the end of the planning horizon. It parametrically imposes a target on expected return during the planning horizon via a constraint. However, alternative objective functions can be easily accommodated.

We have employed a flexible and realistic scenario generation method to represent the uncertainty of random variables. This method does not rely on any distributional assumptions, but rather relies on principles of moment-matching. It generates scenarios so that key statistical properties of the random variables are matched to their empirical values. Hence, the scenario generation method captures the correlations of the random variables as well as asymmetries and excess kurtosis that are empirically observed in historical data of international index returns and exchange rates. The moment-matching method had been criticized that it does not safeguard that the fundamental no-arbitrage condition is met by the scenarios (see, Klaasen [122]). We have thoroughly checked the scenario sets we have employed in all our empirical tests and we verified that the no-arbitrage condition was satisfied in all cases. We have not observed a case in which the scenario generation method failed to meet the no-arbitrage condition.

The extension of the international portfolio management model to a dynamic setting is an important contribution of this study. Another novel contribution concerns the internalization of decisions for currency forward positions in the portfolio management model, as means of mitigating currency risk. This enables the consideration of alternative risk hedging tactics. The inclusion of explicit decisions for currency forward contracts in the model enables the determination of optimal selective hedging decisions to control currency risk exposure. The model determines jointly the optimal portfolio composition (i.e., specifies the appropriate holdings in the available assets in each currency) as well as the appropriate positions in currency forwards so as to minimize the portfolio's total risk. Thus, the portfolio selection and currency risk hedging decisions, that are traditionally considered separately, are here cast in a unified decision framework.

Using the stochastic programming model as a basis, we investigated the performance of several hedging strategies in extensive empirical tests. We examined the performance of alternative decision strategies both in static as well as in dynamic tests. We demonstrated that appropriate use of currency forward contracts materially contributes to the reduction of risk of international portfolios. Selective hedging strategies proved effective in controlling risk and generating stable return paths in backtesting simulations with real market data. Our results also demonstrate the incremental benefits that can be gained from the adoption of dynamic portfolio optimization models instead of myopic, single-period models. Two-stage variants of the model exhibited superior performance in comparison to single-stage models, both in static as well as in dynamic tests. They, generally, lead to more diversified portfolios and lower turnover compared to the decisions of myopic models.

The multistage stochastic programming model formed the fundamental basis for the implementation and empirical investigation of alternative decision strategies. This model provides the foundation for further developments in subsequent chapters. Specifically, we introduce options as means of controlling risk exposure. For this purpose, we adapt procedures for pricing options consistently with the scenario sets that are considered in scenario-based portfolio optimization models; this is the subject of chapter 5. We then apply the option pricing methods to incorporate various types of options in the stochastic programming model. We examine the effectiveness of option-based strategies to control the risk exposures of international portfolios, either separately or jointly. Chapter 6 focuses on the use of options on international stock indices as means of mitigating market risk, while chapter 7 incorporates currency options in the decision process and contrasts their risk hedging performance with that of currency forward positions. Finally, in chapter 8 currency options, options on stock indices and currency forwards are all incorporated in the multistage stochastic programming model so as to address the issue of total risk management of international portfolios in an integrated manner.

4.6 Appendix: CVaR in terms of Portfolio Losses

The portfolio optimization problem is based on a multistage model of investment. In the general T-stage stochastic program the stochastic process of the random variables is :

$$\boldsymbol{\omega} = \{\boldsymbol{\omega_t} : t = 0, 1, ..., T\} \tag{4.7}$$

In scenario-based multistage stochastic programs the probability distribution of $\boldsymbol{\omega}$ is discrete, and concentrated on a finite number of points. A path between states from $t = 0$ to T is a sequence $\{\omega_0, \omega_1, ..., \omega_T\} \in \{\boldsymbol{\omega_0}, \boldsymbol{\omega_1}, ..., \boldsymbol{\omega_T}\}$. Such a sequence of states is a scenario. Thus, each complete scenario distinguishes a sequence of joint realizations of the random variables during all periods of the planning horizon. It has a one-to-one correspondence with a terminal (leaf) node of the scenario tree. The constituent joint realizations of the random variables during each period are identified by the nodes on the unique path from the root node to the corresponding leaf node associated with the scenario (such as the highlighted path in Figure 4.1). We denote N_t the number of distinct nodes of the tree at time period $t = 0, 1, ..., T$.

The portfolio decision process is a measurable function of $\boldsymbol{\omega}$:

$$\boldsymbol{x}(\boldsymbol{\omega}) = \{\boldsymbol{x_t}(\boldsymbol{\omega_t}) : \ t = 0, 1, ..., T\} \tag{4.8}$$

At each stage t, the portfolio decision $\boldsymbol{x_t}$ depends on the sequence of decisions $\{\boldsymbol{x_1}, \boldsymbol{x_2}, ..., \boldsymbol{x_{t-1}}\}$ and on the sequence of the realizations in $\boldsymbol{\omega_t}\}$. The decision process is nonanticipative in the sense that decisions taken at any stage of the process do not depend on future realizations of the random parameters or on future decisions.

For a specific set of decisions $\boldsymbol{x}$ over the entire horizon, let the loss function associated with each leaf node $n \in \boldsymbol{N_T}$, be $f(\boldsymbol{x}, \boldsymbol{\omega})$. This loss function takes the values

$$f(\boldsymbol{x}, \omega^N) \ = \ L_n, \quad \forall n \in \boldsymbol{N_T} \tag{4.9}$$

The probability that the loss function does not exceed some threshold value z, in this discrete scenario setting, is given by the probability function

$$\Psi(\boldsymbol{x}, z) = \sum_{\{n \in \boldsymbol{N_T} \mid z \geq L_n\}} p_n \tag{4.10}$$

where p_n is the probability of occurrence of a certain leaf node $n \in \boldsymbol{N_T}$. It is determined by multiplying the conditional probabilities of the outcomes on the path from the root to the specific leaf node; thus, by compounding the conditional probabilities of the constituent outcomes that lead to the specific leaf node $n \in \boldsymbol{N_T}$.

The *Value-at-risk* (`VaR`) is defined as the maximum allowable loss with a certain confidence level $\alpha * 100\%$. Thus,

$$\texttt{VaR}(\boldsymbol{x}, \alpha) = \min\{z \in \mathbb{R} \,:\, \Psi(\boldsymbol{x}, z) \geq \alpha\} \tag{4.11}$$

$\texttt{VaR}(\boldsymbol{x}, \alpha)$ is the $\alpha * 100\%$ percentile of the distribution of portfolio losses. The percentile α is the left endpoint of the nonempty interval consisting of the values z such that $\Psi(\boldsymbol{x}, z) = \alpha$ (see, Uryasev [162]).

The *Conditional value-at-risk* (`CVaR`) is defined as the conditional expectation of losses exceeding `VaR` at a given confidence level (`VaR` is also defined as a percentile of the loss function in this case). As introduced by Rockafellar and Uryasev [162], for continuous distributions, `CVaR` is defined as

$$\texttt{CVaR}(\boldsymbol{x}, \alpha) = \mathcal{E}\left[L_n \mid L_n \geq \mathrm{VaR}(\boldsymbol{x}, \alpha)\right], \quad n \in \boldsymbol{N_T} \tag{4.12}$$

This definition of `CVaR` for continuous distributions measures the expected value of the $(1-\alpha) * 100\%$ greater losses for portfolio $\boldsymbol{x}$ (i.e., the conditional expectation of portfolio losses above $\texttt{VaR}(\boldsymbol{x}, \alpha)$).

The definition of `CVaR` in equation (4.12) applies to the case of a continuous distribution $\Psi(\boldsymbol{x}, z)$ of portfolio losses. When the random portfolio returns $\tilde{\mathbf{r}}$ are modeled in terms of a discrete distribution (i.e., scenarios) the function $\Psi(\boldsymbol{x}, z)$ is discontinuous in z and $\boldsymbol{x}$. A problem particularly arises when $\Psi(\boldsymbol{x}, z)$ exhibits a jump at the critical percentile $z = \texttt{VaR}(\boldsymbol{x}, \alpha)$. An alternative definition of `CVaR` for general distributions (including discrete distributions) is introduced in Rockafellar and Uryasev [163]:

$$\texttt{CVaR}(\boldsymbol{x}, \alpha) = \frac{1}{1-\alpha}\left(\sum_{\{n \in \boldsymbol{N_T} \mid z \geq L_n\}} p_n - \alpha\right) z \; + \; \frac{1}{1-\alpha} \sum_{\{n \in \boldsymbol{N_T} \mid z < L_n\}} p_n L_n \,, \tag{4.13}$$

where $z = \texttt{VaR}(\boldsymbol{x}, \alpha)$. As we use scenarios to model the distribution of random variables in this thesis, we will adopt this alternative definition of `CVaR`. This definition of `CVaR` for discrete distributions may not be exactly equal to the conditional expectation of portfolio losses above $\texttt{VaR}(\boldsymbol{x}, \alpha)$ when $\Psi(\boldsymbol{x}, z))$ has a jump at $\texttt{VaR}(\boldsymbol{x}, \alpha)$. Rockafellar and Uryasev [163] discuss the subtle differences in this case.

Lets define for every leaf node $n \in \boldsymbol{N_T}$ an auxiliary variable

$$y_n^+ \;=\; \max\left[0, L_n - z\right],$$

which measures the excess losses with respect to $\texttt{VaR}(\boldsymbol{x}, \alpha)$ under the respective leaf node. These piecewise linear functions can be expressed in terms of the following set of linear inequalities

$$\left\{\, y_n^+ \;:\; y_n^+ \geq 0\,,\; y_n^+ \geq L_n - z\,,\; n \in \boldsymbol{N_T} \,\right\}.$$

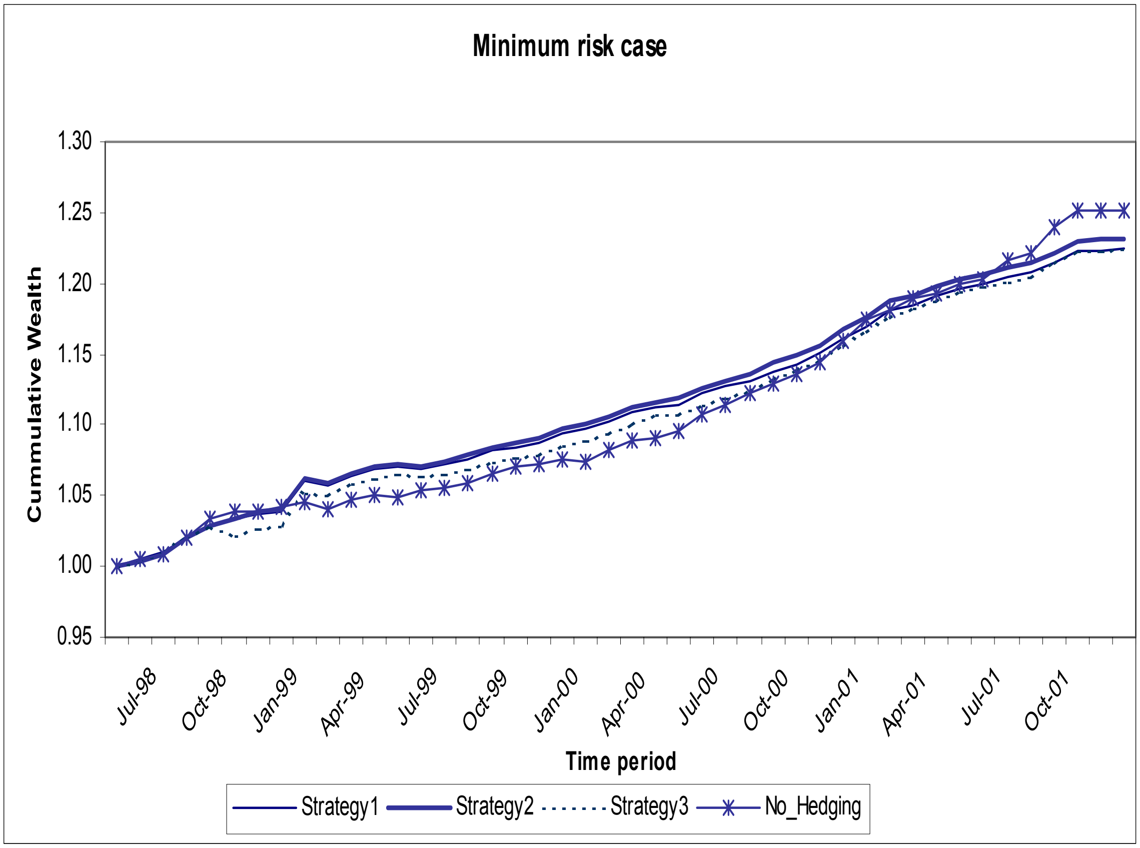
Using these auxiliary variables we have

$$\begin{aligned}
\sum_{n\in \boldsymbol{N_T}} p_n y_n^+ &= \sum_{\{n\in \boldsymbol{N_T} | z\geq L_n\}} p_n y_n^+ \;+ \sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n y_n^+ \\
&= \sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n (L_n - z) \\
&= \sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n L_n \;- z \sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n \\
&= -z(1-\alpha) \;-\; \left(\left(\alpha - \sum_{\{n\in \boldsymbol{N_T} | z\geq L_n\}} p_n\right) z \;+ \sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n L_n\right).
\end{aligned}$$

Dividing both sides of the equation by $(1-\alpha)$ and rearranging terms we get

$$z + \frac{1}{1-\alpha}\sum_{n\in \boldsymbol{N_T}} p_n y_n^+ = \frac{1}{1-\alpha}\left(\sum_{\{n\in \boldsymbol{N_T} | z\geq L_n\}} p_n - \alpha\right) z \;+\; \frac{1}{1-\alpha}\sum_{\{n\in \boldsymbol{N_T} | z< L_n\}} p_n L_n \qquad (4.14)$$

From equations (4.13) and (4.14) we observe that the right hand side term of (4.14) is $\texttt{CVaR}(\boldsymbol{x}, \alpha)$. Therefore, the conditional value-at-risk of portfolio losses can be optimized using a linear program with the left hand side expression of (4.14) as the objective function.



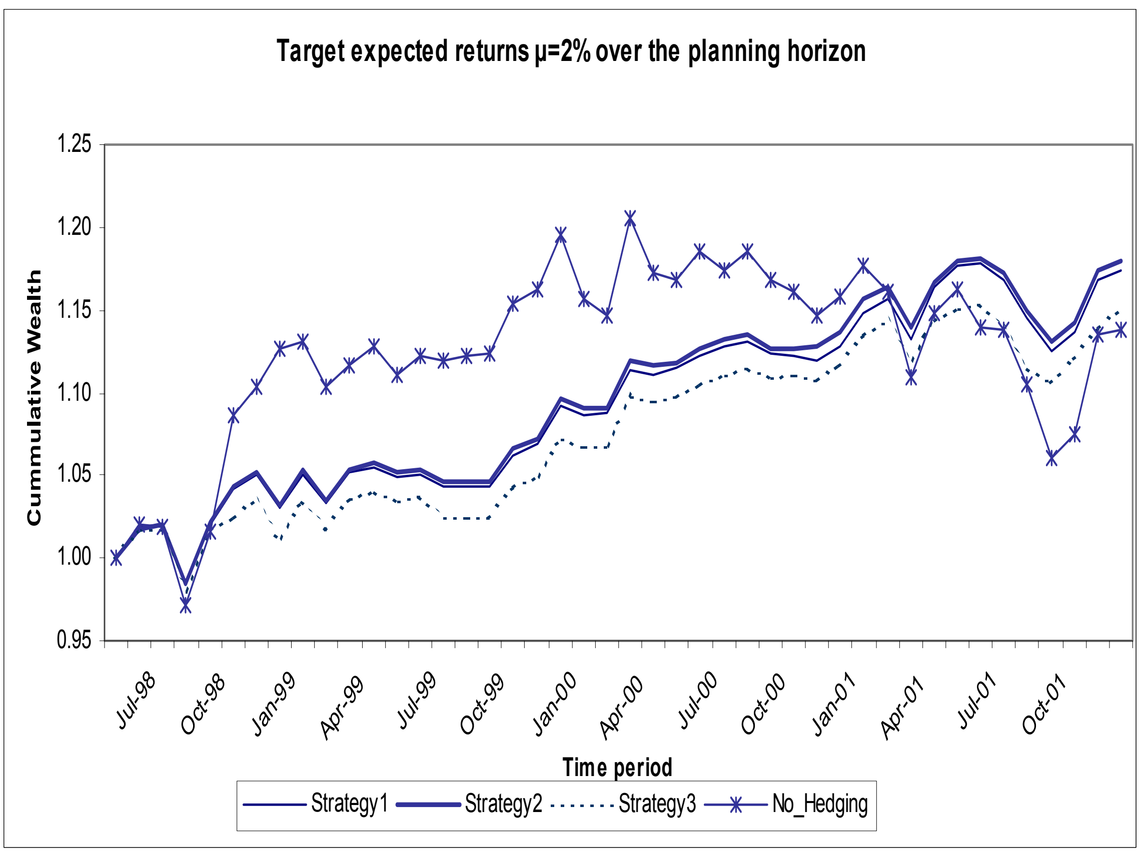


Figure 4.3: Ex-Post realized performance of alternative hedging strategies in two-stage models. The first graph corresponds to the minimum risk case, while for the second graph, the target expected return at horizon is $\mu = 2\%$.

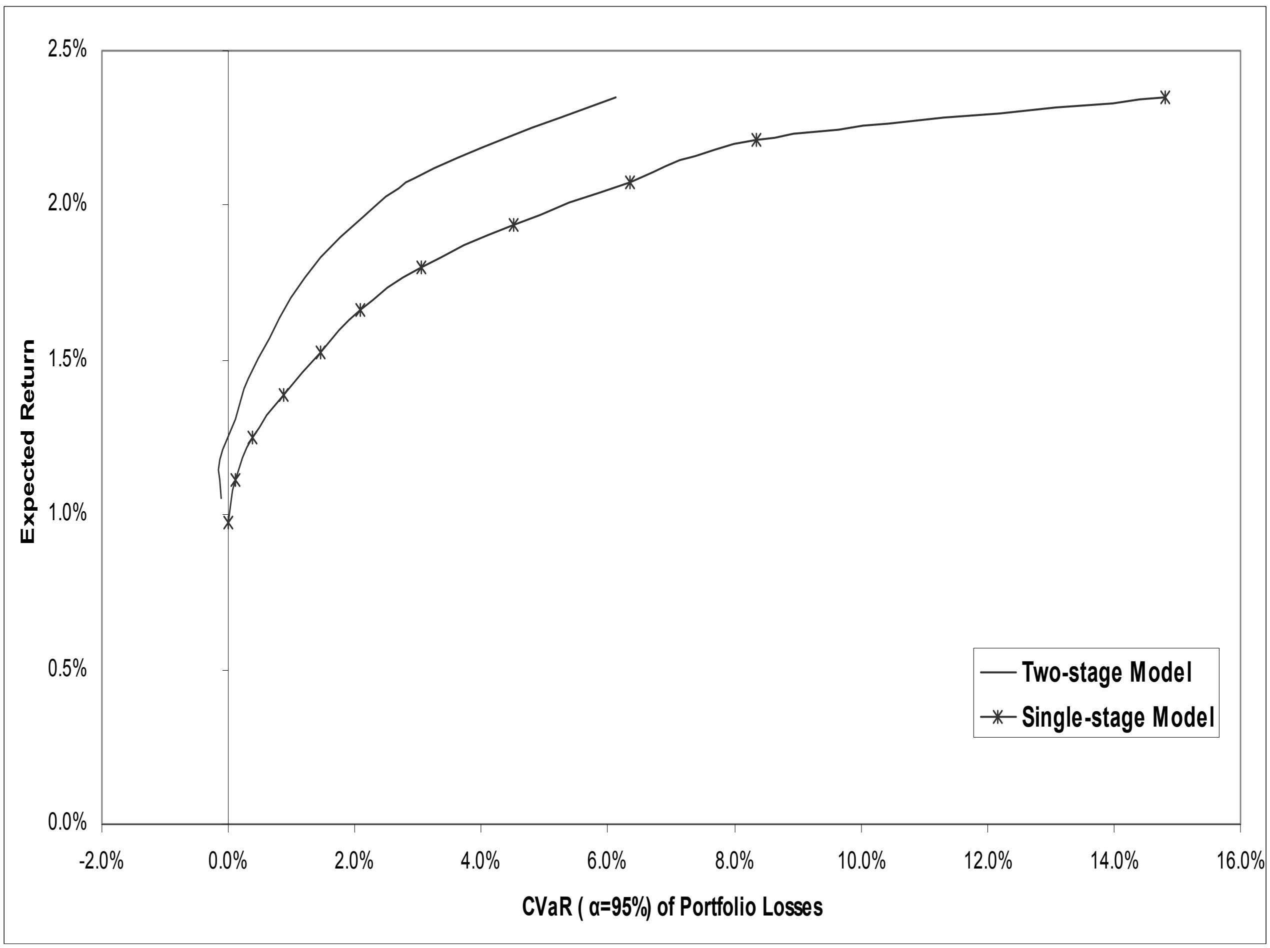
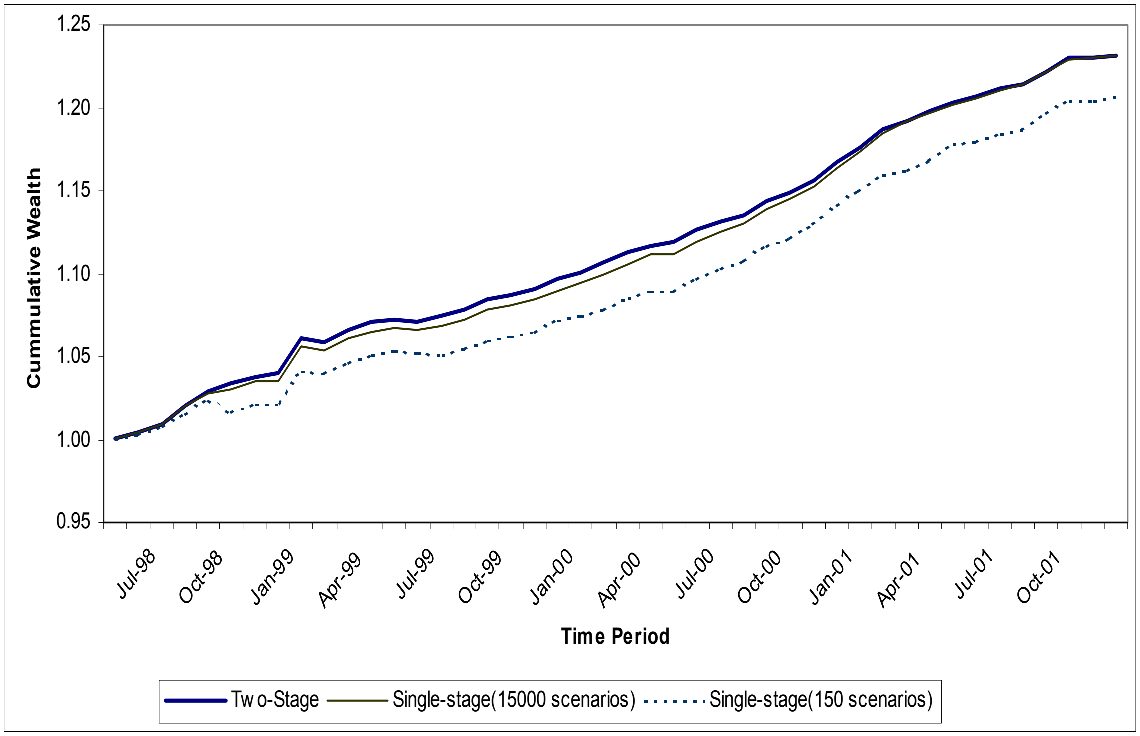


Figure 4.4: Comparison of efficient frontiers for single- and two-stage models (August 2001).



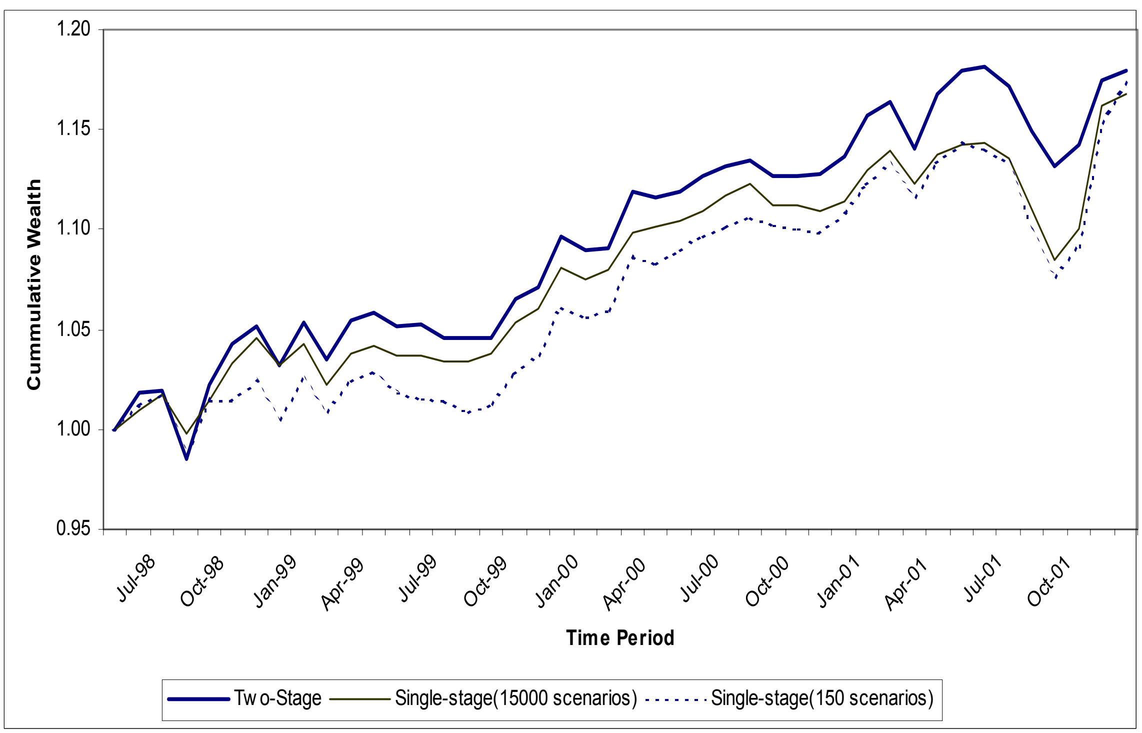
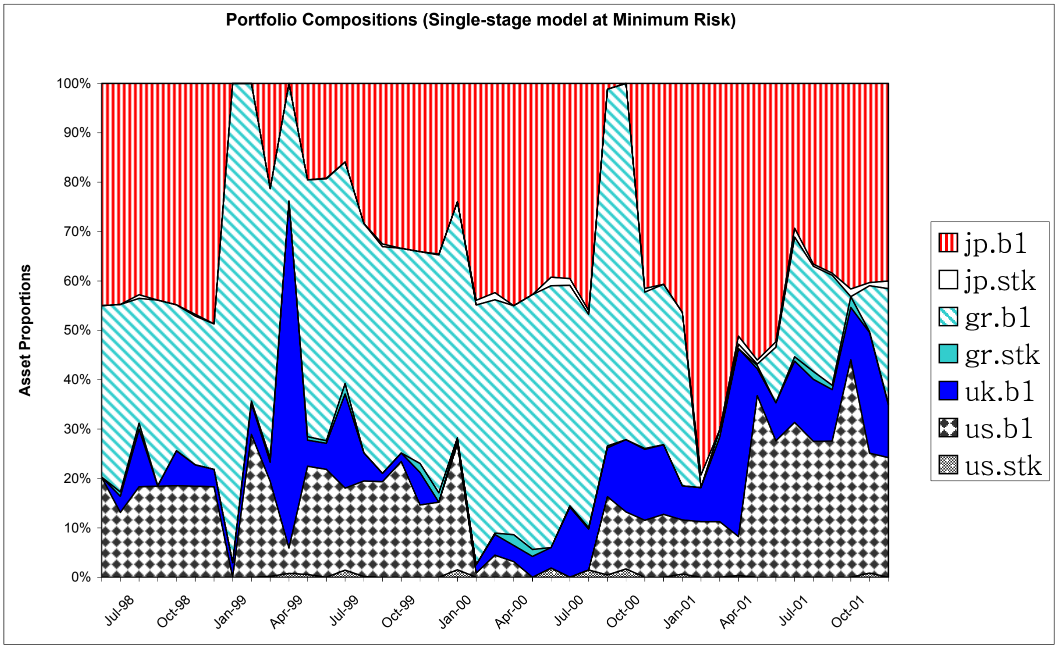


Figure 4.5: Ex-post realized performance of single- and two-stage models. The first graph corresponds to the minimum risk case, while for the second graph the target expected returns during the planning horizon are $\mu = 1\%$ for the single-stage, and $\mu = 2\%$ for the two-stage models).



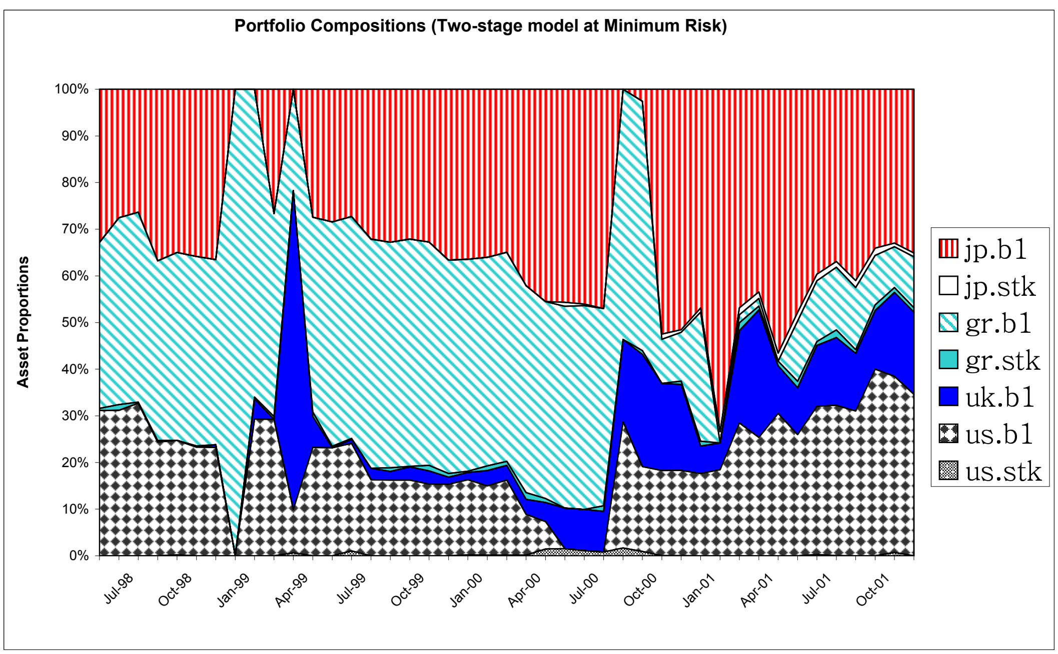
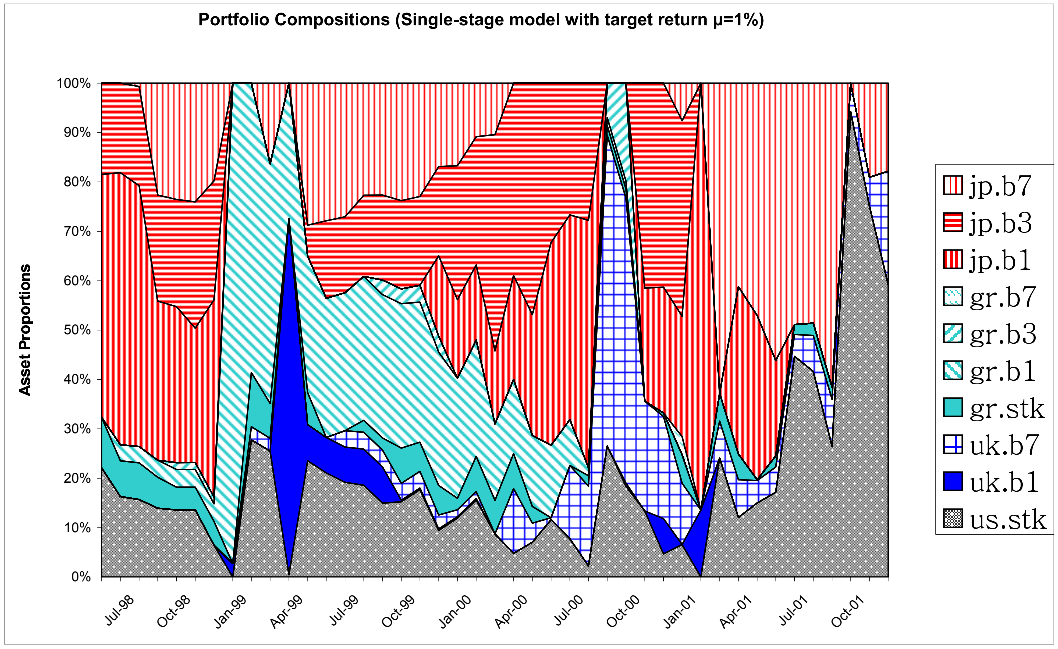


Figure 4.6: Compositions of selectively-hedged international portfolios (minimum risk case) during backtesting simulations. The first graph shows the portfolios decided by the single-stage model, while the second graph shows the portfolios decided by the two-stage model.



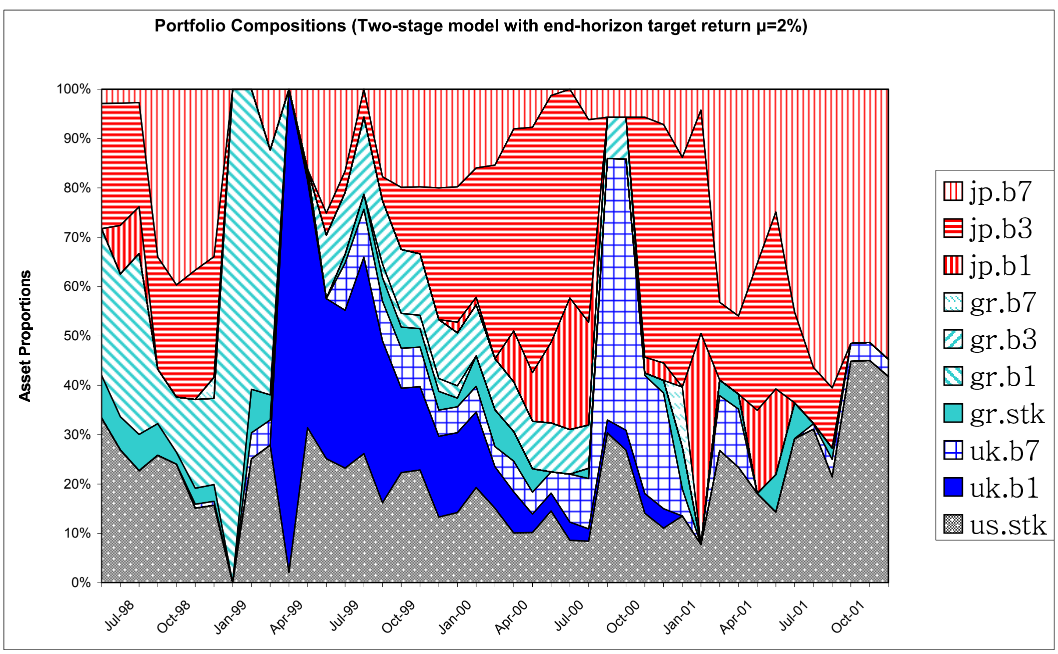


Figure 4.7: Compositions of selectively-hedged international portfolios (aggressive portfolios) during backtesting simulations. The first graph shows the portfolios decided by the single-stage model, while the second graph shows the portfolios decided by the two-stage model.

Chapter 5

Incorporating Options in Stochastic Programming Models for Portfolio Management

The ability to solve large-scale, scenario-based optimization programs allows the consideration of financial portfolios with many securities and the control of multiple elements of uncertainty (risk factors) in a unified framework. The deployment of stochastic programming models in practice is now possible due to flexible modeling systems, significant algorithmic developments for solving large-scale stochastic programs by exploiting their special structures, and the advancements of modern computing technology. As a result, stochastic programs are gaining increasing acceptance as viable tools for addressing diverse financial planning problems under uncertainty. A number of stochastic programming applications in practical financial planning problems have been reviewed in previous chapters, and particularly in chapter 2.

Stochastic programming models are gaining popularity because of their flexibility and several advantageous attributes. Multistage stochastic programs provide an effective basis to model problems that involve decision dynamics over multi-period planning horizons. They can incorporate practical considerations such as transaction costs, turnover constraints, limits of collective holdings in asset groups, no short sales, as well as additional managerial and regulatory requirements that can be expressed as linear constraints on the decision variables. Moreover, stochastic programs can accommodate different objective functions to express the decision maker's risk taking attitude and the primary decision goals. For example, utility functions, penalties on deviations from benchmarks, and alternative risk measures can be considered in the stochastic programming framework. These features provide substantial leeway in modeling effectively practical financial planning problems.

One of the key advantages of dynamic stochastic programs with recourse is their flexibility to cope with diverse representations of uncertainty, as long as they are expressed in terms of discrete distributions. In stochastic programs, uncertainty in input parameters is represented by means of discrete distributions (scenarios) that reflect the joint co-variation of the random variables. The discrete outcomes take the form of a scenario tree in the case of multistage problems so as to depict the progressive evolution of the random variables over multiple time periods. The scenarios can be generated with diverse methods so as to reflect the desired perceptions about future conditions in accordance with economic principles. See, Dupačová et al. [65] for a review of alternative scenario generation methods. Asymmetric and heavy-tailed distributions of financial random variables can be considered in scenario-based portfolio optimization models. Thus, features that are typically observed in financial time series can be flexibly reflected in discrete scenario sets.

Risk management is a particularly fertile application domain for stochastic programs. Risk management of internationally diversified portfolios is the main theme of this thesis. So, it is obviously natural to consider, in the context of portfolio management models, appropriate financial instruments and other means that can serve the risk management function. Derivative securities are particularly well-suited for this purpose. By construction, options have highly asymmetric payoffs that can effectively cover against adverse price movements of the underlying security. For the cost (premium) of an option, the holder locks in a specific exercise (strike) price for the underlying. For example, with the purchase of a European put option the holder has the right, but not the obligation, to sell the underlying security at the exercise price at the time of maturity (term) of the option. Obviously, the option will yield no payoff if the market price of the underlying security exceeds the exercise price at the time of the option's expiration; the option will simply be left to expire in this case and the holder will lose the cost paid for the purchase of the option. However, the holder will realize a profit if the security's market price falls below the exercise price. The gain grows linearly with the decline of the security's market price at the option's expiration time. Hence, a put option can provide coverage against potential declines in the price of the underlying security.

Options can also be used for speculative purposes. For example, with the purchase of a European call option the holder acquires the right, but not the obligation, to buy the underlying security at a predetermined (exercise) price at the maturity of the option. The cost (option price) for this right is paid upfront. Clearly, the option will not yield a payoff, and will be left to expire, if the market price of the security remains below the exercise price at maturity. But, in the event that the market price of the security exceeds the strike price of the option at maturity the holder reaps an immediate profit. By exercising the option, he will purchase the predetermined amount of the security at the exercise price, which he can then solve in the market at the higher price and reap the profit from the difference of the market price to the strike price. Hence, a long position in a call option accentuates the upside potential in the event of market upturns.

By employing specific combinations of options, it is possible to shape a desired payoff profile over the range of possible movements in the market price of the underlying. Obviously, options can provide effective means for risk hedging in various circumstances. Consequently, the incorporation of options in portfolio management models should lead to improved risk management tools. This goal is the primary motivation for the developments pursued in the remaining chapters of this thesis.

The following key issue needs to be resolved. On the one hand, traditional option pricing methods typically rely on specific distributional assumptions regarding the stochastic price of the underlying security. On the other hand, we want to make full use of the flexibility afforded by stochastic programming models to cope with alternative representations of uncertainty in asset prices by means of discrete scenario sets. It is often desirable, and even necessary, to capture in the scenario sets skewness and excess kurtosis features that are empirically observed in financial time series. As we demonstrated in the previous chapters, this is an important consideration for the international portfolio management problem. Hence, the postulated scenarios for the asset prices do not necessarily conform to the distributional assumptions on which popular option pricing methods are based. As a result, these option pricing methods cannot be employed if the options are to be incorporated in stochastic programming models that use a different representation of uncertainty for the prices of the underlying assets. Otherwise, the models can give rise to arbitrage opportunities and imply the attainment of spurious profits. The results of such models would have questionable validity.

Clearly, the stochastic programming framework must be internally consistent. We need to reconcile the pricing of options with the scenarios of asset prices that are employed in stochastic programs for portfolio management, without confining the two to restrictive distributional assumptions. For this purpose, it is necessary to devise suitable procedures for pricing the options consistently with the postulated distributions of the asset prices, as expressed by the set of scenarios, while also satisfying fundamental economic principles (e.g., no-arbitrage condition).

In this study we apply a moment matching method (see, Høyland and Wallace [95] and Høyland et al. [93]) to generate the price scenarios of the securities; however, this is not a restrictive requirement for the developments that follow. With this method we generate scenarios so that key statistical properties of the asset returns match prespecified target values. Specifically, the generated scenarios are such that the first four marginal moments (mean, variance, skewness and kurtosis) of the asset returns — as well as their correlations, when multiple assets are considered — are matched to specific targets. The target values are estimated from historical time series of these financial variables. Hence, empirically observed key statistical properties of the random variables are reflected in the scenario sets.

The main purpose of this study is the adaptation, implementation and empirical validation of appropriate methods to price options in accordance with discrete scenario sets that depict the distribution (and dynamic evolution) of asset prices for the underlying securities. We confine our attention to European-type options that can be exercised only at the expiration time (maturity). The adopted option pricing methods take into consideration the statistical characteristics of the random asset prices (e.g., skewness and excess kurtosis) as reflected in their discrete distributions, which in turn are generated in compliance with historical observations. In this manner, we can build internally consistent stochastic programming models for portfolio management, encompassing options that are priced in accordance with the models' discrete distributions (scenarios) for the prices of the underlying securities. The portfolio management models determine optimal positions in the assets and the options so as to meet specific objectives (e.g., minimization of total risk, subject to a minimum target of expected portfolio return). A key concern is to ensure that the scenarios of asset prices, in conjunction with the resulting option prices, satisfy the fundamental no-arbitrage condition. The satisfaction of this requirement is verified through numerical tests.

We propose two different methods to price the options in accordance with the discrete distributions of a scenario tree. We validate the methods using market prices for the options and their underlying security. We study the impact of higher moments in the distribution of the underlying on the prices of options, as well as implied volatility smile effects that are typically observed in practice. The proposed option valuation methods are motivated from the need to achieve internal consistency in stochastic programming models when options are incorporated, but their use is not confined to these models only. The proposed methods hold their own as promising option pricing methods. We demonstrate through empirical validation tests using market prices for the S&P500 stock index and options on this security that our proposed methods yield prices that are closer to the market prices of the options than those obtained by the Black-Scholes method, especially for deep out-of-the-money options. They also allow us to investigate the effects of higher moments (skewness, kurtosis) of the underlying asset on the resulting option prices. We analyze whether asymmetric distributions lead to option prices considerably different than the Black-Scholes prices, and to what extent nonzero skewness and excess kurtosis cause significant deviations. Moreover, the methods allow us to estimate directly the risk-neutral distribution of the underlying asset and to study its characteristics, thus explaining skewness premia effects.

In subsequent chapters we apply the option pricing procedures developed in this chapter, so as to incorporate stock index options and/or currency options in stochastic programming models for international portfolio management. We find that such derivatives provide effective means for controlling market and/or currency risk exposures of international portfolios.

5.1 Introduction

Stochastic programs provide an effective framework for modeling diverse financial management problems. They have been attracting a growing interest from researchers and practitioners in recent years. Stochastic programs posses several attractive features that make them applicable in diverse practical settings. Uncertainty in input parameters of stochastic programs with recourse is represented by means of discrete distributions (scenarios) that depict the joint co-variation of the random variables. In multistage problems the progressive evolution of the random variables is expressed in terms of a scenario tree, as we showed in the previous chapter. The scenarios are not restricted to follow any particular distribution or stochastic process; alternative joint distributions of the random variables can be flexibly accommodated.

Scenario generation is a critical step in the modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution of the underlying financial variables and is consistent with market observations and financial theory. The collection of scenarios should satisfy the no-arbitrage properties. This means that the scenarios of future asset prices should not allow arbitrage opportunities. Thus, the portfolio optimization programs should be bounded, and should not imply the existence of spurious profits.

Various scenario generation methods have been proposed for financial management problems under uncertainty. These include bootstrapping from historical observations, sampling from assumed distributions that are fitted on the basis of market data econometric models that account for varying volatility and other dynamic effects, moment-matching and clustering methods, etc. Often these methods are augmented with subjective expert opinions on future economic conditions. A review of scenario generation methods for dynamic stochastic programs is provided by Dupačová et al. [65]. Clearly, these scenario generation procedures can yield alternative representations of uncertainty in financial random variables. Empirically observed features, such as asymmetric and heavy-tailed distributions for the random variables can be captured by a discrete scenario tree.

The ability to solve large-scale stochastic programs with a large number of scenarios allows us to consider in a common framework multiple elements of risk and to deal simultaneously with many securities. This, together with the flexibility of stochastic programs to consider alternative discrete empirical distributions for the underlying random variables, without being confined to restrictive assumptions about the functional form of these distributions, makes them essential tools for risk management applications.

Derivative securities are playing an increasingly important role as risk hedging instruments. Essential for hedging various risks are the options that are traded in many exchanges. The asymmetric payoffs of options (individual options or appropriate combinations), provide the means to protect the value of holdings in an asset in the event of substantial variations in the market price of the underlying security. A long position in a put option provides coverage against potential declines in the market value of the underlying security. Conversely, a long position in a call option generates its payoffs in the event of upside changes in the market value of the underlying. Combinations of options may be used to shape a payoff profile according to the preferences of an investor so as to yield a desired tradeoff between payoff risk and expected return. The introduction of options in portfolio management models broadens the investment opportunity set and provides instruments geared towards risk control due to the asymmetric form of option payoffs.

For the cost (premium) of an option, the holder locks in a specific exercise (strike) price for the underlying. For example, with the purchase of a European put option the holder has the right, but not the obligation, to sell the underlying security at the exercise price, at the time of maturity (term) of the option. Obviously, the option will yield no payoff if the market price of the underlying security exceeds the exercise price at the time of the option's expiration; the option will simply be left to expire in this case and the holder will lose the cost paid for the purchase of the option. However, the holder will realize a profit if the security's market price falls below the exercise price. The gain grows linearly with the decline of the security's market price at the option's expiration time. Hence, a put option can provide coverage against potential declines in the price of the underlying security.

Clearly, the option will not yield a payoff, and will be left to expire, if the market price of the security remains below the exercise price at maturity. But, in the event that the market price of the security exceeds the strike price of the option at maturity, then the holder reaps an immediate profit. By exercising the option, he will purchase the predetermined amount of the security at the exercise price, which he can then solve in the market at the higher price and reap the profit from the difference of the market price to the strike price. Hence, a long position in a call option accentuates the upside potential in the event of market upturns.

By employing specific combinations of options, it is possible to shape a desired payoff profile over the range of possible movements in the market price of the underlying. Obviously, options can provide effective means for risk hedging in various circumstances. Consequently, the incorporation of options in portfolio management models should lead to improved risk management tools. This goal is the primary motivation for the developments pursued in the remaining chapters of this thesis.

There is a vast literature on option pricing methodologies. It is not our intention to provide a complete review of the option pricing literature; we only cite some representative works. Option valuation methods can be classified in two general approaches: Analytical solutions and derivation of the risk neutral distribution, and numerical methods or simulations (lattices and Monte Carlo).

The basis for the first approach is the work of Osborne [154] who suggested that stock prices follow a geometric Brownian motion. Using this assumption, Black and Scholes [29] derived explicit formulas for pricing both put and call options. Merton [142] extended the B-S formulas, to account for the effect of dividends. Several empirical studies (e.g., Rubinstein [165, 166]) showed that the B-S model misprices deep out-of-the money options. The volatility estimates in the Black-Scholes formula, implied by observed market data for options and their underlying securities, differ across exercise prices and maturities, and form “smile” patterns. A volatility smile indicates implied volatilities that are convex and monotonically decreasing functions of exercise prices. In contrast, the assumption of a geometric Brownian motion implies constant volatility relative to exercise prices and maturities. Moreover, if extreme events are more frequent than it is assumed in the normal case, then out-of-the money options will be more expensive than the values derived from the Black-Scholes formula, as is often the case. Finally, empirical estimates of the implied risk-neutral probability density of asset returns reveal negatively skewed and leptokurtic distributions, in contrast to the lognormal distribution assumed in the Black-Scholes model.

Several extensions have been proposed to the seminal work of Black-Scholes. Merton [143] proposed that the underlying stock returns are generated by a mixture of both continuous and jump processes. Hull and White [97] proposed a stochastic volatility model. They showed that when volatility is stochastic, but uncorrelated with the underlying asset price, the price of a European option is the BS price obtained on the basis of the average volatility during the life of the option. Bates [16, 17] extended the jump-diffusion process for the underlying stock returns to incorporate stochastic volatilities. Heston [89] showed that a closed-form solution for a European call can be derived as an integral of the future security price density which can be calculated by an inverse Fourier transform. This method may also be applied when the correlation between the increments of the driving Brownian motion of the underlying asset and the volatility is nonzero. Bakshi et al. [13], and Das and Sundaram [57] analyzed the extent to which models under either stochastic volatility or random jumps are capable of resolving the well-known limitations of the BS model. They showed that neither class of models can provide an adequate explanation of the biases of the B-S model that are reported in empirical studies. In both papers, stochastic volatility models seemed to behave slightly better than models with jumps.

Another approach is to infer the risk-neutral probability measure for the price of the underlying security. The observed financial time series and risk premia, imply an associated “risk-neutral” probability measure that can be used to price any derivative as the expected discounted value of its future payoff. While the objective and risk-neutral probability measures are related, and may share some common parameters, they are identical only in the case of zero risk premia on all relevant risks. Current models attribute to risk premia any discrepancies between objective properties of stock index returns, and risk-neutral probability measures inferred from the volatility smirk. The fundamental theorem of asset pricing states that in the absence of arbitrage opportunities in option prices, there exists some pricing kernel that can reconcile the two measures. This result has prompted the studies by Ait-Sahalia and Lo [7], Jackwerth [100], and Rosenberg and Engle [164] regarding general characteristics of the pricing kernel.

An alternative approach for dealing with nonconstant volatility was suggested by Rubinstein [166], Jackwerth and Rubinstein [101], and in a series of related papers by Derman and Kani [61], Derman et al. [62] and Dupire [67]. Instead of imposing a parametric functional form for volatility, they developed binomial or trinomial lattices in order to approximate the whole structure of market prices. In this way perfect fit with observed option prices is achieved. This procedure captures, by construction, the most salient characteristics of the data. In particular, the implied tree employed in the numerical estimation correctly reproduces the volatility smile. The most popular models within this family use recombining binomial trees implied by the smile from observed prices of European options. Moreover, Rubinstein [166] showed how to compute the implied probability distribution using quadratic programming. Jackwerth and Rubinstein [101] generalized this approach using nonlinear programming to minimize four different objective functions.

Finally, the last option pricing approach is based on simulations. Cox et al. [54] proposed a numerical option valuation procedure with a binomial tree to model the evolution of stock prices. This model assumes a perfect market for a contingent claim. A hedging portfolio containing stocks and bonds is rebalanced to have the same payoff as that of the target stock option at each time period and for each outcome on the lattice. Boyle [33] applied Monte Carlo simulations as an alternative to the binomial model for pricing options. Monte Carlo simulation has the advantage that its convergence rate is independent of the number of state variables, while that of the binomial model is exponential in the number of state variables. Analysts use Monte Carlo simulations to generate paths for the price of the underlying asset until maturity. Then, they discount the expected payoff on each path by the riskless rate and compute the expectation under the risk-neutral measure (Boyle et al. [34]).

The analytical solution method is unsuitable when options are to be introduced in stochastic programming models. As we already stressed, stochastic programs can flexibly accommodate alternative representations of uncertainty by means of arbitrary discrete distributions (scenarios). Such distributions can capture asymmetries and fat tails that are often observed in historical data of financial time series, as is the case in the international portfolio management problems examined in this thesis. The distributions of the underlying random variables do not usually conform to the functional forms for which analytic option pricing formulae exist. We cannot use one of the popular pricing formulas to price these options (e.g., the Black-Scholes formula) because the discrete distribution of our postulated scenarios for the underlying assets does not usually satisfy the distributional assumptions of the existing formulas (lognormal, in the BS framework).

Hence, the incorporation of options in scenario-based portfolio optimization models requires that these options should be priced consistently with the scenarios of the underlying instruments, while also conforming to fundamental financial principles (e.g., exclusion of arbitrage), in order to provide an internally consistent model. Pricing using analytical methods would lead to arbitrage opportunities, because of the inconsistency of the pricing method and the discrete distribution (in terms of scenarios) of the underlying asset. Thus, the problem we address in this chapter is to price European options when the distribution of the underlying asset is depicted by a set of discrete scenarios, which is generated on the basis of empirical observations, without explicit distributional assumptions. The scenario generation method we use to produce the scenario tree, is the moment matching scenario generation method discussed in the previous chapter. The scenarios are generated so that the first four marginal moments of the underlying random variable match their corresponding historical values. Thus, the scenario tree is based on the empirical distribution of the underlying asset, capturing any asymmetries and fat tails.

The last approach is again inappropriate for this framework. Simulation techniques aim to converge with a known (e.g., normal) functional form for the distribution of the underlying asset, which is not the case in the flexible stochastic programming framework which is free of explicit distributional assumptions.

In order to price the options and incorporate them in stochastic programming models, we adapt two existing pricing methods; the first is based on the methodology developed by Bakshi et al. [15], while the second is based on the methodology developed by Corrado and Su [52]. According to the first method, in order to price the options we derive the equivalent risk-neutral probability measure from the physical probability measure that is associated with the postulated outcomes on the scenario tree, using the change of measure pricing kernel in power utility economies. Once the risk-neutral probabilities for the nodes of the scenario tree are derived, pricing the options is straightforward. The price of a European call option is the discounted expected payoff conditional upon finishing "in the money", times the probability finishing in the money. The expectation is taken over the appropriate nodes of the scenario tree under the risk-neutral probability measure.

According to the second method, we start with a known distribution and we add correction terms which depend on nonzero skewness and excess kurtosis of the empirical distribution that is depicted by the discrete scenarios. The resulting option pricing formula is expressed as the sum of three parts: a Black-Scholes option price, plus two separate adjustments for skewness and excess kurtosis.

The novel contribution of this work is the adaptation of suitable methods for pricing and incorporating options in scenario-based stochastic programming models. The resulting option prices are consistent with the postulated scenario set. We verify numerically that the pricing procedures satisfy the no-arbitrage conditions. Using these methods, we can price the options at any node of the scenario tree, thus allowing transactions with options at any decision stage. The option pricing methods are not restricted to the specific scenario generation method that is employed in this study. They are not dependent on any explicit assumption regarding the discrete distribution that represents the stochastic prices of the underlying security. In principle, they can be applied to any arbitrary discrete distribution (scenario tree) for the prices of the underlying. Of course the appropriateness (i.e., satisfaction of no-arbitrage conditions) and the effectiveness of these methods to properly price options needs to be empirically verified in each case. We apply extensive empirical tests to validate the proposed option pricing methods when the scenario sets for the prices of the underlying security are generated by the moment-matching method.

We compare prices derived from the proposed methods and from the Black-Scholes formula with observed market prices of European options on the $S\&P500$ stock index. The results indicate that the proposed methods produce prices that are much closer to the observed market prices than the corresponding Black-Scholes prices, especially for deep out-of-the-money options. This can be explained by the fact that the risk-neutral distribution of the $S\&P500$ index does not satisfy the assumptions underlying the Black-Scholes formulas. Particularly, the deeper "out-of-the-money" the options are the greater the deviation of the Black-Scholes prices from the observed market prices, while our procedures still produce prices fairly close to the observed prices. Keeping in mind that for risk management purposes "out-of-the-money" put options are appropriate instruments to eliminate losses from unfavorable movements in the price of the underlying asset, it is clear that the Black-Scholes pricing formulas are ineffective in this framework since they underprice these options and lead to arbitrage opportunities when the options are incorporated in the portfolio management models.

We demonstrate through empirical tests that the pricing of European options based on flexible scenarios that consider empirical dynamics, asymmetries and heavy tails of the underlying random variables, is an effective valuation methodology. The resulting option prices are very close to observed market prices, the pricing procedure is free from explicit distributional assumptions, while no arbitrage conditions are met.

The rest of the chapter is organized as follows. In section 5.2 we summarize the general form of the scenario tree for that depicts uncertain prices of the underlying assets. Moreover, we present two different methods for pricing options based on this scenario tree for the underlying asset. In section 5.3 we describe the numerical tests to verify that the no-arbitrage condition is satisfied. In section 5.4 we investigate the effects of changing moments of the underlying asset's distribution on the resulting option prices. In section 5.5 we discuss the volatility smile that implies negatively skewed risk-neutral distributions. In section 5.6 we compare option prices on the $S\&P500$ index computed with the proposed methods to observed market prices. Finally, section 5.7 concludes this chapter.

5.2 Pricing Options on Discrete Scenario Sets for the Underlying Asset

In order to solve multistage stochastic programming models for portfolio management, the distribution of the asset prices has to be represented in terms of a relatively small number of nodes in an event, or scenario, tree of the form shown in Figure 5.1. The generation of the scenario tree was described in detail in section 4.2. The root node of the tree corresponds to the current moment ($t = 0$), from which a number of branches extend, representing possible outcomes that the random variables may take at the second decision time ($t = 1$). Each of these branches ends to a successor node at time $t = 1$, which has further branches and this procedure is repeated until every decision time in the tree is represented by a collection of nodes. Decisions (i.e., potential portfolio revisions) are made at the nodes (states) of each discrete time $t = 0, 1, ..., T$.

As in the previous chapter, we define the following notation to describe the scenario tree and the associated outcomes:

$\boldsymbol{N}$	is the set of nodes of the scenario tree,
$n \in \boldsymbol{N}$	is a typical node of the scenario tree ($n = 0$ denotes the root node at $t = 0$),
$\boldsymbol{N_t} \subset \boldsymbol{N}$	is the set of distinct nodes of the tree at time period $t = 0, 1, ..., T$,
$\boldsymbol{N_T} \subset \boldsymbol{N}$	is the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios,
$p(n) \in \boldsymbol{N}$	is the unique predecessor node of node $n \in \boldsymbol{N}$,
$M_n \subset \boldsymbol{N}$	is the set of immediate successor nodes of node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n.
p_t^n	is the conditional probability for the outcome associated with the transition from the predecessor node $p(n)$ to node $n \in \boldsymbol{N}$,
$\hat{p}_n$	is the objective probability of the state associated with node $n \in \boldsymbol{N}$, as resulting from the scenario generation process.

We intend to incorporate options in the stochastic programming model for portfolio management. This entails decision variables corresponding to option purchases at each decision stage. Thus, we need a procedure to price options at each node of the scenario tree, taking into consideration the postulated outcomes of the random variables on the subtree emanating from the node of interest. Although in our applications we typically consider options with maturity equivalent to a single decision period (i.e., one month in our case), we present the pricing methods for options with arbitrary term — but with expiration dates coinciding with the decision periods of the scenario tree, i.e., multiples of months.

A critical condition that must be satisfied by the scenarios is the no-arbitrage requirement. The presence of arbitrage in the outcomes represented by the scenario tree will lead to solutions of the

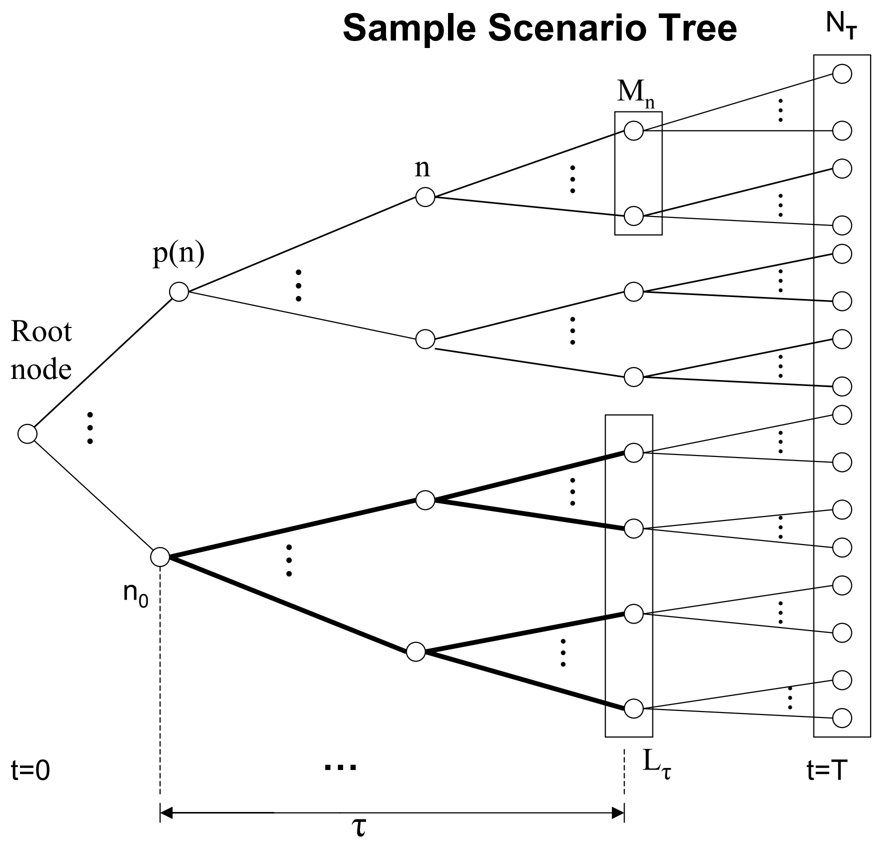


Figure 5.1: General form of a scenario tree

portfolio optimization program that imply spurious profits that are not attainable in practice. Such solutions are biased and of no practical usefulness. So, we must ensure that the postulated scenarios for the asset prices are arbitrage free. The absence of arbitrage is a fundamental condition for option pricing; we ensure that this condition is satisfied in the option pricing procedures. If the prices of European call and put options do not satisfy the put-call parity relationship then arbitrage opportunities exist.

In the next subsections we present two different methods to price options based on the postulated scenario tree of Figure 5.1. Both methods can be used in order to price the options at the root, as well as at any intermediate node of the scenario tree.

Lets say we want to price a European option with term (maturity) τ periods (months) at an arbitrary node $n_0 \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ of the scenario tree. Of course the expiration date of the option must not extend beyond the depth of the tree. We consider the subtree rooted at node n_0 and extending forward in time. At the maturity date of the option the subtree has a number of terminal nodes $\boldsymbol{L_\tau}$ that represent the possible states of the economy (i.e., respective asset prices) at that time, conditional on starting from node n_0. This construct is highlighted on Figure 5.1.

We stress that we do not assume that the prices of the underlying security evolve according to a discrete process (either in time or in outcomes). We assume that the prices of the securities evolve according to an unknown (not explicitly specified) continuous stochastic process which is approximated by the moment-matching scenario generation procedure. The outcomes corresponding to the nodes of the scenario tree simply represent an appropriate discretization of this process. The time discretization is taken at the decision periods (equidistant every one month), which also correspond to the expiration dates of the options. The nodes of the scenario tree at each stage represent a discretization of the continuous distribution of the random asset prices at the respective stage.

The structure of the scenario tree reflects the conditional transitions of the discretized price process. The discretization can be made finer by increasing the number of outcomes (nodes) at each stage, i.e., the branching factor from each node. In the limit, the discrete scenario tree converges to the continuous process for the underlying securities. Of course, the addition of the nodes has a direct impact on the size of the resulting stochastic program and the required solution effort. A balance is needed between a finer discretization — for increased accuracy in the representation of the stochastic price process — and the computational complexity of the resulting stochastic program.

So, conditional on starting from node n_0, the price process of the underlying security can make the transitions shown on the highlighted subtree on Figure 5.1 during the lifetime of the option. Of course the tree is not binomial as shown in Figure 5.1 simply for illustration purposes; it typically has a much larger number of branches per node. The final states of the price precess at the expiration date of the option are represented by the node set $\boldsymbol{L_\tau}$ (always conditional on starting from the state of node n_0 at the issue date of the option).

The fact that portfolio rebalancing decisions are allowed during the interim decision periods (i.e., at the states of the intermediate nodes of the subtree) is of no consequence for the option pricing procedure. Recall that we consider European options that can be exercised only at their expiration date. To price the European options we need only the discrete approximation of the probability distribution of the underlying security's price at the expiration date (as represented by the nodes $\boldsymbol{L_\tau}$), and the discount factor (risk-free interest rate) for the term of the option.

Let us define the following notation:

τ	is the term of the option issued at state n_0,
S_0	the price of the underlying asset at node n_0 where the option is to be priced (i.e., at the root node of the subtree),
$\hat{S}_\tau$	is the price (random variable) of the underlying asset at the expiration date of the option, conditional on the price S_0 at the option's issue date,
S^n	the price of the underlying asset at leaf node $n \in \boldsymbol{L_\tau}$ of the subtree. These prices and their associated probabilities over all terminal nodes of the subtree $n \in \boldsymbol{L_\tau}$ reflect the discrete distribution for the asset prices at the option's expiration date, conditional on the price starting from the value S_0 at the issue date (i.e., from state n_0),
r_f	the riskless rate applicable during the lifetime of the option, (i.e., from issue date to expiration date),
P	the objective probability measure for the discrete representation of the price process on the scenario tree. The objective probabilities of the tree nodes of the scenario tree are obtained from the scenario generation procedure. Our implementation of the moment-matching procedure generates equiprobable outcomes at each decision stage. Thus, $\hat{p}_n = \frac{1}{\lvert \boldsymbol{N_t} \rvert}, \quad \forall n \in \boldsymbol{N_t}, \quad t = 0, 1, ..., T$,
$\bar{P}$	is the risk-neutral probability measure for the outcomes of the scenario tree, derived from equilibrium principles,
c_0	is the price of the European call option on the underlying asset, at node n_0,
p_0	is the price of the European put option on the underlying asset, at node n_0,
K	is the exercise price of the option.

Recall that the options we look to price are European options, issued at state n_0 with an initial price S_0 for the underlying asset. They have a term τ, and the distribution of the underlying asset's price – and associated payoffs – at the expiration date is approximated by the states corresponding to the nodes in set $\boldsymbol{L_\tau}$. First, we rescale the objective probabilities of these nodes to reflect their conditional value for a starting asset price S_0 at the issue date:

$$p_n = \frac{\hat{p}_n}{\hat{p}_{n_0}}, \quad \forall n \in \boldsymbol{L_\tau}$$

We now consider two approaches for pricing options under the discrete representation of the underlying asset's stochastic price that was described above. In the first method we determine an appropriate risk-neutral probability measure for the discrete outcomes ($S^n, \quad \forall n \in \boldsymbol{L_\tau}$) of the asset prices. The risk-neutral probabilities are obtained through a transformation of the objective probabilities of these outcomes that applies the change of measure pricing kernel in power utility economies. Once the risk-neutral probabilities for the discrete outcomes of the asset price are determined, pricing the options is straightforward. The price of a European option is simply the expectation — using the risk-neutral measure — of its discounted payoffs over the discrete outcomes at the option's expiration date. This procedure is presented in section 5.2.1.

The second method prices an option over an empirical distribution for the stochastic price of the underlying that is expressed (approximated) by its Gram-Charlier series expansion. The series is truncated after the fourth term, as the first four moments of the underlying distribution of asset returns should, essentially capture all material effects on option prices. The moments of asset's return distribution are estimated from historical observations of market data. Recall that the moment matching method generates the discrete outcomes of asset prices on the scenario tree so as to match these empirical moment estimates. Although this pricing procedure does not explicitly depend on the specific outcomes of the scenario tree, it conforms to the discrete distribution they represent as it uses the exact same values for the first four moments of the asset returns. If the series expansion is truncated after two terms then we end up with a normal distribution; this corresponds to the familiar Black-Scholes setting. The additional consideration of the third and fourth empirical moments accounts for the effects of skewness and kurtosis in the distribution of asset returns on the option price. The procedure computes the price of the option as the sum of three components: the Black-Scholes price (accounting for the first two moments), plus two additional terms that adjust for the contribution of skewness and excess kurtosis in the distribution of the asset returns. As we show in section 5.2.2 the procedure is simple to implement.

The ability of the two proposed option pricing procedures to capture the impact of higher moments on option prices is numerically demonstrated in section 5.4. Section 5.5 shows how these methods adjust to the smile effects exhibited by market prices of options. The validity of the option pricing procedures is investigated in section 5.6 using real market data. The numerical results show that both procedures produce consistent option prices that are closer to observed market prices of options than those generated by the Black-Scholes formula, especially for deep-out-of-the money options.

5.2.1 Method 1: Deriving Risk-neutral Probabilities

The absence of arbitrage is a fundamental condition in option pricing. Arbitrage exists if some options are cheap or expensive in comparison to a fair price, and thus transactions with these options can yield an immediate profit without involving any risk. In the no-arbitrage framework, the price of a European option is computed by taking the expectation of its payoff with respect to a risk-neutral density, and then discounting this expectation at the riskless rate for the term of the option. Harisson and Kreps [86] applied this valuation framework and introduced the notion of a pricing functional which operates on the payoff of a contingent claim. We need to find this equivalent risk-neutral probability measure $\bar{P}$.

We need to determine an appropriate risk-neutral probability distribution $\bar{P}$ for the postulated outcomes of asset prices. Options can then be priced in a straightforward manner on the basis of the risk-neutral distribution. There are two basic theorems in risk-neutral valuation (see Jacob and Shiryaev [102] for these theorems in discrete time). The first states that a model of asset prices is arbitrage free iff there exists a probability measure $\bar{P}$ under which the discounted process of the underlying asset prices is a martingale. That is, under this measure the expected return of each asset is equal to the riskless rate. In order to satisfy the martingale condition we should determine the risk-neutral probability mass $\bar{p}_n$ for each discrete outcome of asset prices $(S^n,\ n \in \boldsymbol{L_\tau})$ so that the implied expected return of the underlying asset is equal to the riskless rate. A discrete probability measure $\bar{P}$ that satisfies the martingale property can be obtained from the solution of the following system of linear equations (and inequalities):

$$\sum_{n\in \boldsymbol{L_\tau}} \bar{p}_n S^n = S_0 (e^{r_f})^\tau \tag{5.2}$$

$$\sum_{n\in \boldsymbol{L_\tau}} \bar{p}_n = 1 \tag{5.3}$$

$$\bar{p}_n \geq 0, \quad \forall n \in \boldsymbol{L_\tau} \tag{5.4}$$

Given that we use a large number of outcomes to represent the distribution of asset prices, the system in (5.2) — (5.4) is underdetermined (it has only two equations but many $|\boldsymbol{L_\tau}|$ unknowns, $\bar{p}_n$); hence, it admits multiple solutions.

According to the second basic theorem, this risk-neutral probability measure $\bar{P}$ must be unique (for the market to be complete). In order to have a unique solution to the system in (5.2) – (5.4), the number of underlying assets must be equal to the number of postulated outcomes $|\boldsymbol{L_\tau}|$-1. This is hardly ever the case as we use many outcomes to effectively approximate the stochastic asset prices. The problem then is how to uniquely select an appropriate discrete probability measure over those that satisfy the martingale condition to apply for option pricing purposes.

As the martingale principle by itself is not sufficient in this case to provide a unique risk-neutral probability measure, we impose the additional requirement that the measure sought should be the closest to the risk-neutral measure implied by equilibrium principles. In equilibrium, the objective and the risk-neutral probability measures are related through a pricing kernel. The basic idea is to determine a solution to (5.2) – (5.4) — i.e., that satisfies the necessary martingale requirement — while being the closest (in the Euclidean sense) to the risk-neutral measure implied by equilibrium principles. This idea has also been used by de Lange et al. [58]. They also require prices to be arbitrage-free, and in addition they require that prices are as close as possible to reasonable equilibrium prices. The objective is to minimize a metric between prices that satisfy equilibrium and prices that satisfy the martingale requirement. Rubinstein [166] used an analogous idea. First, he established the risk-neutral probabilities in a standard binomial tree. Then he derived the implied posterior risk-neutral probabilities, using a quadratic program. These are, in the least square sense, closest to lognormal that cause the present value of the underlying asset and all the options calculated with these probabilities to fall between their respective bid and ask prices.

In equilibrium, the current price of an asset whose random value at time τ is $\hat{S_\tau}$ is given by:

$$S_0 = (e^{-r_f})^\tau E_P[\xi \hat{S}_\tau] \tag{5.5}$$

where ξ is the pricing kernel (stochastic discount factor) and $E_P[.]$ denotes the expectation operator with respect to the objective probability measure P. Explicit dynamic modelling of the joint stochastic process of asset returns and the pricing kernel can be found in the consumption-based equilibrium asset pricing literature (see, Ait-Sahalia and Lo [7], Jackwerth [100], Rosenberg and Engle [164] for applications to option pricing).

Bakshi et al. [15] developed a method for relating the objective and risk-neutral probability measures through a pricing kernel. We follow their methodology, which we adapt to the case of a discrete distribution of asset returns. The equilibrium condition (5.5) relates to the risk-neutral measure $\bar{P}$ as follows:

$$S_0 = (e^{-r_f})^{\tau} E_P[\xi \hat{S}_{\tau}] = (e^{-r_f})^{\tau} \int_{\Omega} \hat{S}_{\tau}(\omega)\xi(\omega)dP(\omega) = (e^{-r_f})^{\tau} \int_{\Omega} \hat{S}_{\tau}(\omega)d\bar{P}(\omega) \tag{5.6}$$

implying

$$\xi(\omega)dP(\omega) = d\bar{P}(\omega). \tag{5.7}$$

Let the objective and the risk-neutral probability measures P and $\bar{P}$, respectively, be defined on the measurable discrete space (Ω, F). These probability measures are said to be equivalent, if

$$P(Z) = 0 \Leftrightarrow \bar{P}(Z) = 0 \tag{5.8}$$

for all $Z \subseteq \Omega$. According to the Radon-Nikodym theorem, there exists a strictly positive random variable $\xi \in L^1(\Omega, F, P)$ which is unique, and is such that

$$\frac{d\bar{P}}{dP} = \xi \quad a.s. \tag{5.9}$$

This result takes on a very concrete form when Ω is a discrete set, $(\omega^1, \omega^2, ..., \omega^k)$, as is the case with the discrete outcomes $S^n, \;\; n \in \boldsymbol{L_{\tau}}$, we use to represent the distribution of the stochastic asset price $\hat{S}_{\tau}$. In this case, F comprises all subsets of Ω. From equation (5.9), the random variable ξ now takes the values

$$\xi(\omega^n) = \frac{\bar{P}(\omega^n)}{P(\omega^n)} \;\Rightarrow\; \bar{P}(\omega^n) = \xi(\omega^n)P(\omega^n), \quad \forall \omega^n \in \Omega.$$

Bakshi et al. [15] derive the following relationship between the objective and the risk-neutral probabilities:

$$\bar{p}_n = E_P[\xi|S^n]p_n, \quad n \in \boldsymbol{L_{\tau}} \tag{5.11}$$

where $E_P[\xi|S^n]$ is the filtration generated by the asset price. They conjecture a form for the Radon-Nikodym derivative,

$$\bar{p}_n = \frac{E_P[\xi|S^n]p_n}{\sum_{n \in \boldsymbol{L_{\tau}}} E_P[\xi|S^n]p_n}, \quad n \in \boldsymbol{L_{\tau}} \tag{5.12}$$

where ξ can be interpreted as a general unnormalized change-of-measure pricing kernel. The normalization factor in the denominator ensures that $\bar{P}$ is a probability measure that sums to one. Under the common hypothesis of a power utility function of wealth, the pricing kernel can be specialized to

$$E_P[\xi|S^n] = (S^n)^{-\gamma} = e^{-\gamma ln S^n}, \quad n \in \boldsymbol{L_{\tau}} \tag{5.13}$$

where γ is the coefficient of the relative risk aversion. Substituting (5.13) in (5.12) and dividing both the denominator and the numerator by $S_0^{-\gamma}$ we obtain

$$\bar{p}_n = \frac{e^{-\gamma ln(\frac{S_n}{S_0})} p_n}{\sum_{n \in \boldsymbol{L_\tau}} e^{-\gamma ln(\frac{S_n}{S_0})} p_n} = \frac{e^{-\gamma R^n} p_n}{\sum_{n \in L_\tau} e^{-\gamma R^n} p_n}, \qquad n \in \boldsymbol{L_\tau} \tag{5.14}$$

where $R^n = ln(\frac{S_n}{S_0})$ is the return of the underlying asset at leaf node $n \in \boldsymbol{L_\tau}$, conditional on the initial price S_0 (recall the subtree depicted in Figure 5.1).

The risk-neutral density is obtained by exponentially tilting the objective density. The normalization factor in the denominator of the above equation ensures that $\bar{P}$ is a proper density function that sums to unity.

In the probability measure $\bar{P}$, derived from equilibrium principles, the individual outcomes $S^n, \ n \in \boldsymbol{L_\tau}$ of the discrete distribution of asset prices at the option's expiration date have corresponding probability mass $\bar{p}_n$, as specified by equation (5.14). We seek risk neutral probabilities $\bar{p}'_n, \ \ n \in \boldsymbol{L_\tau}$ over the same discrete outcomes that are close (in the sense of the Euclidiean distance) to $\bar{p}_n$, while additionally satisfying the martingale property. These probabilities are obtained from the solution of the following quadratic program:

$$\begin{aligned} \texttt{minimize} \quad & \sum_{n \in \boldsymbol{L_\tau}} (\bar{p}'_n - \bar{p}^n)^2 \\ \texttt{s.t.} \quad & \sum_{n \in \boldsymbol{L_\tau}} \bar{p}'_n S^n = S_0 (e^{r_f})^\tau \\ & \sum_{n \in \boldsymbol{L_\tau}} \bar{p}'_n = 1 \\ & \bar{p}'_n \geq 0, \quad \forall n \in \boldsymbol{L_\tau} \end{aligned} \tag{5.15a}$$

The risk neutral probability measure represented by $\bar{p}'_n, \quad n \in \boldsymbol{L_\tau}$, satisfies the required no-arbitrage conditions for option pricing. As they meet the constraints of the quadratic program (5.15) they satisfy the martingale condition. They are also uniquely determined, as they are closest, in the least square sense, to the corresponding probabilities implied by equilibrium principles.

Once we determine the risk-neutral probabilities for the discrete outcomes of the underlying asset's price, according to the procedure described above, pricing the options is straightforward. The "fair" price of a European call option at node n_0, with strike price K, is computed as the expected value of its payoffs at the expiration date over the risk-neutral measure for the discrete price outcomes, discounted by the riskless rate:

$$c_0(S_0, K) = (e^{-r_f})^\tau \sum_{n \in \boldsymbol{L_\tau}} \bar{p}'_n [max(S^n - K, 0)], \tag{5.16}$$

The "fair" price of a European put option with strike price K is similarly computed by:

$$p_0(S_0, K) = (e^{-r_f})^\tau \sum_{n \in \boldsymbol{L_\tau}} \bar{p}'_n [max(K - S^n, 0)]. \tag{5.17}$$

Note that in order to use this option pricing method, in the context of a multistage portfolio management problem, we need to determine a risk-neutral probability distribution for the asset prices at each node of the scenario tree. For this purpose, a quadratic program of the form (5.15) needs to be set up and solved at each non-terminal node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ of the scenario tree. Obviously, the computational effort is considerable if the scenario tree includes many decision stages and a large number of outcomes (nodes).

The advantage of the method lies in its generality. It is applicable for any discrete representation (scenario tree) of the random asset prices, and prices the options by explicitly considering these discrete distributions. The numerical behavior and practical viability of this option pricing method are examined in subsequent sections of this chapter.

5.2.2 Method 2: Accounting for Skewness and Kurtosis of Asset Returns

We explore another option pricing method based on an idea introduced by Jarrow and Rudd [106] and further developed by Corrado and Su [52] (Brown and Robinson [41] provided a correction note on their approach). The method augments the Black-Scholes approach by further accounting for nonzero skewness and excess kurtosis in the distribution of asset returns. Corrado and Su apply a Gram-Charlier series expansion to represent an empirical probability density of asset log returns, in order to derive a semiparametric option pricing formula. A truncation of the series to only two terms results in a lognormal distribution of asset returns and produces the familiar Black-Scholes pricing formula. Corrado and Su derive the pricing formula based on a four-term Gram-Charlier series expansion of the probability density function, thus including the effects of skewness and kurtosis of asset returns. The moments of asset returns are estimated from historical data.

Let

τ is the term of the European option,

S_0 the price of the underlying asset at the time of the option's issue,

$\hat{S}_\tau$ the price (random variable) of the underlying asset at the expiration date,

r_f the riskless rate,

K is the exercise price of the option.

The derivation of the pricing formula can be summarized as follows:

Consider the log return of the asset during the holding period:

$$\hat{r}_\tau = ln\hat{S}_\tau - lnS_0 = ln(\frac{\hat{S}_\tau}{S_0}) \tag{5.18}$$

Then

$$\hat{S}_\tau = S_0 e^{\hat{r}_\tau} \tag{5.19}$$

and the conditional distribution of $\hat{S}_\tau$ depends on that of the log return $\hat{r}_\tau$.

In the "risk-neutral" setting, the price of the call option depends on the conditional distribution of $\hat{r}_\tau$:

$$c_0 = (e^{-r_f})^\tau E[(\hat{S}_\tau - K)^+] = (e^{-r_f})^\tau \int_{ln(K/S_0)}^{\infty} (S_0 e^x - K) f(x)) dx \tag{5.20}$$

where $f(.)$ is the conditional density of $\hat{r}_\mathcal{T}$. Corrado and Su use a Gram-Charlier four-term series expansion for the conditional density $f(.)$ and solve (5.20) to generate the following expression for the price of the call option:

$$c_0 = C_{BS} + \gamma_1 Q_3 + (\gamma_2 - 3) Q_4 \tag{5.21}$$

where C_{BS} is the Black-Scholes price, and Q_3 and Q_4 represent the marginal effects of nonnormal skewness and kurtosis respectively, on the option price c_0. They show that:

$$\begin{aligned} C_{BS} &= S_0 N(d) - K e^{-r_f \mathcal{T}} N(d - \sigma_\mathcal{T}) && (5.22)\\ Q_3 &= \frac{1}{3!} S_0 \sigma_\mathcal{T} [(2\sigma_\mathcal{T} - d)\varphi(d) + \sigma_\mathcal{T}^2 N(d)] && (5.23)\\ Q_4 &= \frac{1}{4!} S_0 \sigma_\mathcal{T} [(d^2 - 3d\sigma_\mathcal{T} + 3\sigma_\mathcal{T}^2 - 1)\varphi(d) + \sigma_\mathcal{T}^3 N(d)] && (5.24)\\ d &= \frac{ln(S_0/K) + r_f \mathcal{T} + \sigma_\mathcal{T}^2/2}{\sigma_\mathcal{T}} && (5.25) \end{aligned}$$

where φ is the standard normal density, N is the cumulative normal density, γ_1 and γ_2 are the Fisher parameters for skewness and kurtosis, μ_i is the i^{th} central moment.

$$\begin{aligned} \gamma_1 &= \frac{\mu_3}{\mu_2^{3/2}}, \quad \gamma_2 = \frac{\mu_4}{\mu_2^2} \\ \varphi(d) &= (2\pi)^{-1/2} exp(-d^2/2) \end{aligned}$$

The price p_0 of a corresponding put option with the same strike price K is given by the put-call parity:

$$p_0 = c_0 + K e^{-r_f \mathcal{T}} - S_0 \tag{5.26}$$

This option pricing method is simple to implement and is computationally efficient. Unlike the previous method, it does not require the solution of an optimization program or other involved computations. It involves only simple algebraic calculations and can be readily applied to price options at any node of the scenario tree. Its deterministic inputs include the riskless rate, r_f, the initial price of the asset, S_0, and the strike price of the option, K. It additionally requires estimates of the moments of the asset returns during the holding period. These are determined on the basis of the discrete distributions of the asset prices at the option's expiration date, as specified at the leaf nodes $n \in \boldsymbol{L_\mathcal{T}}$ of the subtree shown in Figure 5.1. Recall that with the moment-matching scenario generation procedure the discrete outcomes of asset prices on the scenario tree are generated so that the moments of asset returns are matched to their historical values.

This procedure forms the foundation of our developments to price and incorporate options in multistage stochastic programs for international portfolio management. It is applied to currency options (chapter 7) and to stock options (chapter 8). In chapter 8 we extend this procedure so as to price quanto options. Quantos are options on a foreign asset (stock index) but with their payoffs and price expressed in a different reference currency using a predetermined exchange rate for value translations between the currencies involved. Hence, they can be used to hedge the market risk and the currency risk of investments in the underlying foreign asset.

5.3 Checking for Arbitrage-free Conditions

We already stressed the need for the discrete distribution of asset prices on the scenario tree, and the associated option prices, to satisfy the fundamental no-arbitrage requirements in order for the resulting portfolio management model to yield meaningful results. We explicitly verify that the no-arbitrage condition is satisfied in our modelling framework.

For simplicity of notation and presentation, we discuss the relevant tests only for the single-stage case. That is, for a portfolio problem that concerns only a single portfolio structuring decision with a one-period horizon. The same principles can be readily extended to multistage decision models, but the required multistage stochastic programs are much more involved.

We set up and execute the tests specified in Klaassen [123]. Arbitrage opportunities of two types can be detected as follows. An arbitrage opportunity of the first type exists if there is an allocation (including short sales) to the available investment instruments (options and the underlying assets) with a zero initial cost, that results in non-negative payoffs in all future economic states (e.g., scenarios of subsequent outcomes), and produces a strictly positive payoff in at least one scenario. This implies the existence of a riskless self-financed strategy that results in positive expected return.

Let

- I the set of all available investment instruments (assets and option),
- $\boldsymbol{N}$ the set of scenarios (outcomes) for asset prices at the end of the holding period,
- R_i^n the holding period return of instrument $i \in I$ under scenario $n \in \boldsymbol{N}$,
- x_i the allocation (position) in instrument $i \in I$.

The existence of an arbitrage opportunity of the first type implies:

$$\begin{aligned} \sum_{i \in I} x_i &= 0 \\ \sum_{i \in I} x_i R_i^n &\geq 0 \quad \forall n \in \boldsymbol{N} \\ \sum_{i \in I} x_i R_i^n &> 0 \quad \text{for at least one } \; n \in \boldsymbol{N} \end{aligned}$$

In order to test whether an arbitrage opportunity of the first type exists, we determine the maximum total return of a self-financed portfolio by solving the following linear program:

$$\begin{aligned} \texttt{maximize} \quad & \sum_{n \in \boldsymbol{N}} \sum_{i \in I} x_i R_i^n \\ \texttt{s.t.} \quad & \sum_{i \in I} x_i = 0 \\ & \sum_{i \in I} x_i R_i^n \geq 0 \quad \forall n \in \boldsymbol{N} \end{aligned}$$

If the optimal value of this linear program is positive, then an arbitrage opportunity of the first type exists; indeed the problem is unbounded in such case. In the absence of an arbitrage opportunity the optimal value is identically equal to zero. This indicates that there is no self-financed strategy that can produce positive profit.

An arbitrage opportunity of the second type exists if there is a portfolio allocation x (including short sales), that has non-negative payoffs in all scenarios and also has negative initial cost (i.e., generates an immediate cash surplus without any potential subsequent costs). This would be implied by a feasible solution to the following system of inequalities:

$$\begin{aligned} \sum_{i \in I} x_i &< 0 \\ \sum_{i \in I} x_i R_i^n &\geq 0 \quad \forall n \in \boldsymbol{N} \end{aligned}$$

To detect the existence of such an arbitrage opportunity we solve the linear program

$$\begin{aligned} \texttt{minimize} \quad & \sum_{i \in I} x_i \\ \texttt{s.t.} \quad & \sum_{i \in I} x_i R_i^n \geq 0 \quad \forall n \in \boldsymbol{N} \end{aligned}$$

A negative optimal value indicates the existence of an arbitrage opportunity. The problem would again be unbounded in such a case. In the absence of arbitrage the obvious solution is $x_i = 0, \ \forall i \in I$.

We use these models to detect the existence of arbitrage opportunities in the discrete sets of asset prices that we generate with the moment-matching scenario generation procedure and the associated option prices. We observe that when we apply these tests to portfolios that contain options priced with the Black-Scholes formula (see, e.g., (5.22)) we detect arbitrage opportunities; both linear programs are unbounded. This is a consequence of the fact that the Black-Scholes formula underprices out-of-the-money put options and implies higher potential returns for these options than can be actually obtained. This was an immediate motivation to seek alternative schemes to price options in accordance with the postulated discrete distributions of asset returns in the context of stochastic programming models.

In contrast, we did not found arbitrage opportunities when we applied the same tests to portfolios comprised of options priced with either of the methods presented in the previous subsections, as well as the underlying assets. Both tests were satisfied. This indicates that the moment-matching method for generating scenarios of asset returns (capturing their empirical higher order moments), in conjunction with the two proposed option pricing methods, satisfy the fundamental requirement for a no-arbitrage price framework.

5.4 Effects of Changes in Higher Moments

We investigate the effects that changes in higher moments (skewness, kurtosis) of the distribution of the underlying asset's returns have on the resulting option prices. The purpose is to examine whether asymmetric and heavy-tailed distributions lead to considerably different option prices from the Black-Scholes prices, and to quantify the impact that nonzero skewness and excess kurtosis have on option prices.

We generate scenarios for the $S\&P500$ stock index using the moment matching scenario generation method. The scenarios are generated so that the first four marginal moments of the index return match the corresponding historical moments. Keeping the mean, the variance, and the kurtosis constant (to their historical values), we generate various scenario sets that differ only in the skewness of the discrete distribution for the $S\&P500$ index returns (from -25% of the historical skewness to $+25\%$, varied by 5% each time, thus having 7 different sets of 15000 scenarios each). We repeat the exercise by keeping the skewness, as well as the first two moments, constant (to their historical values), while varying the kurtosis in a similar fashion. Using as inputs the different scenario sets, we compute the option prices resulting from the two methods described above.

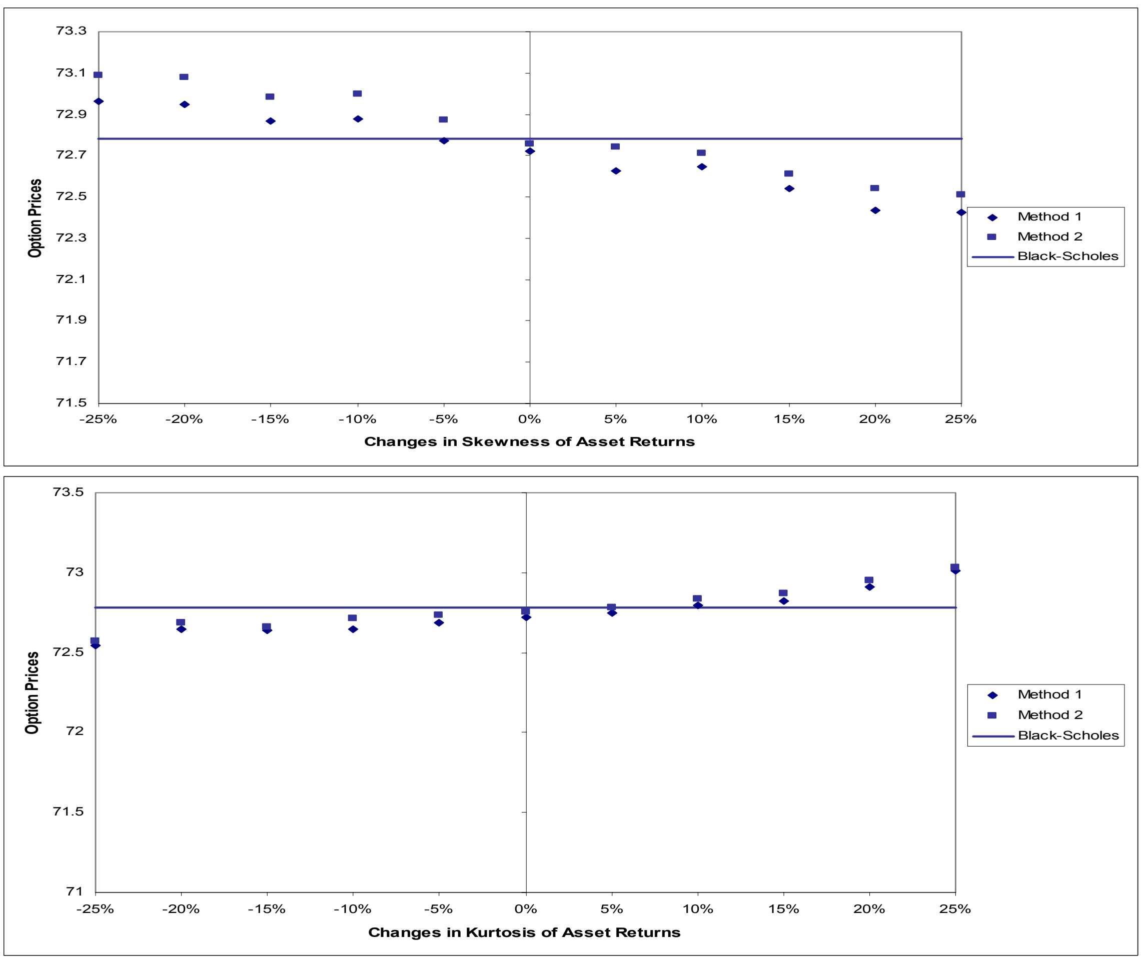


Figure 5.2: The effect of changing skewness and kurtosis of asset returns on ITM call option prices.

The tests are performed on the basis of market data on November 20th, 1999. At that time the price of the index was $S_0 = 1424.26$. The moments of the monthly log returns were estimated on the basis of monthly observations of the index over the period between November 1989 and October 1999. The empirical values of these moments were: mean=0.007356, variance=0.018762, skewness=0.011508, excess kurtosis=0.029589. These moments are initially matched in the generated scenario set of index prices, and then additional scenario sets are generated by varying either the skewness or the kurtosis value according to the procedure described above.

We consider two different call options. One with a strike price $K = 0.95 * S_0$ (in-the-money, ITM) and one with a strike price $K = 1.05 * S_0$ (out-of-the-money, OTM). The results of these tests are shown in Figures 5.2 and 5.3, for the in-the-money and out-of-the-money call options, respectively. From the results we make the following immediate observations:

a) The two methods produce very similar option prices in all cases.

b) For the ITM options, both methods result in prices that are very close to the Black-Scholes price (when the historical moments of the index return are used). However, the Black-Scholes formula overprices the OTM call options.

c) Both models exhibit consistent behavior in terms of the variation of option prices with respect to changes in the higher moments (skewness or kurtosis) of the index' returns; the observed effects on their computed option prices follow the same patterns. Of course the Black-Scholes price is unaffected by changes in the values of higher moments, as it depends only on the first two moments.

d) Clearly, the ITM call option is much more expensive that the OTM, as is to be expected.

Analyzing more closely the results we observe the following. For the ITM call option (Figure 5.2), a decrease in skewness of the underlying asset's return — especially negative skewness — causes an increase in the price of the option, while an increase in skewness results in lower option prices. The opposite trend is true for changes in the kurtosis of the underlying index' returns. Changes in the higher moments do not have substantial (proportionally) impact on ITM call prices. Of course this sensitivity of ITM call option prices to changes in the values of higher moments depends on the level of the induced changes and on the degree of the option's "moneyness".

More interesting are OTM options, whether call or put, which are cheaper than ITM options. It is these options that are more frequently of interest in practice. Long positions in OTM put options provide downside risk coverage, while long positions in OTM call options enhance upside potential.

For the OTM call option (Figure 5.3), we observe that the resulting option price is an increasing function of both the skewness and the kurtosis of the underlying index' returns. Except for very high values of kurtosis or/and skewness, the Black-Scholes approach overprices the OTM call option in comparison to the proposed methods. This is an empirically observed bias of the Black-Scholes approach (i.e., it underprices deep OTM put options and overprices deep OTM call options). The results illustrate that the proposed methods can price options more accurately than the B-S method; this is indeed verified empirically in section 5.6 where we contrast computed option prices against market data. Finally, we observe a higher (proportional) sensitivity of OTM call option prices to changes in the higher moments of asset returns, in comparison to ITM call options.

These results are consistent with those reported by Corrado and Su [52] and Theodosiou and Trigeorgis [179] regarding the effect that higher order moments of asset returns have on option prices. Of course, method 2 is that proposed by Corrado and Su; but Theodossiou and Trigeorgis employ a range of different distributions in their analysis. The results also demonstrate that the proposed pricing procedures are more flexible, and more reliable in approximating market data of options, in comparison to the Black-Scholes approach. They can capture and react to the values of higher moments of the underlying asset's returns. In the following sections we demonstrate that the proposed methods produce prices that reflect empirically observed characteristics of option prices, and are closer to observed market data of options than Black-Scholes prices (especially for OTM options).

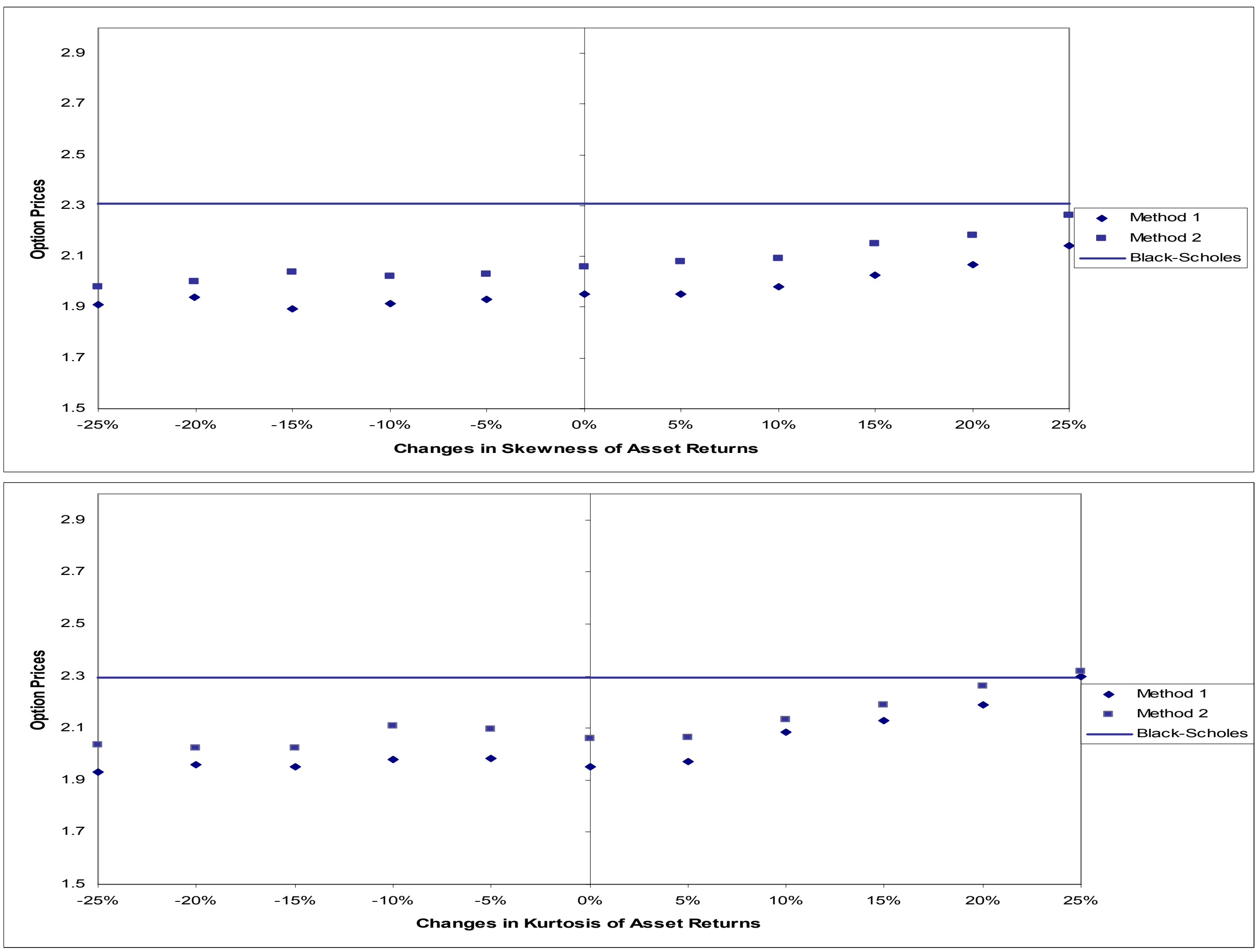


Figure 5.3: The effect of changing skewness and kurtosis of asset returns on OTM call option prices.

5.5 Empirical Stylistic Features

The insight from equations (5.16) and (5.17) is that OTM calls reflect conditions in the upper tail of the risk-neutral distribution, while OTM puts reflect conditions in the lower tail. As Bates [16] notes, the symmetry or asymmetry of the risk-neutral distribution is reflected in the relative prices of these OTM calls and puts. This is because OTM calls yield positive cashflows only if the asset price rises above the exercise price of the option, while OTM puts yield positive payoffs only when the asset price falls below the exercise price of the option. Symmetric risk-neutral distributions imply equal prices for OTM calls and puts, when their exercise prices are spaced symmetrically around the mean asset price ($S_0 e^{r_f \tau}$) at the expiration date.

A skewness measure of the risk-neutral distribution is the "skewness premium". The skewness premium is defined by Bates [16] as the percentage deviation of a% OTM call prices from a% OTM put prices. Mathematically is:

$$SP(a) = \frac{c_0(S_0, K_c)}{p_0(S_0, K_p)} - 1 \tag{5.29}$$

where $K_c = (1+a)S_0 e^{r_f \tau}$ is the strike price price of the OTM call option, $K_p = (1-a)S_0 e^{r_f \tau}$ is the strike price of the OTM put option, S_0 is the price of the underlying asset at the issue date, r_f is the riskless rate for the options' term τ, c_0 and p_0 are the prices of the call and put options, respectively.

Method 1 allows us to estimate directly the risk-neutral distribution of the asset returns and to study its characteristics. Using this method, we determine the risk-neutral distribution of monthly returns for the $S\&P500$ index at each month during the period of January 1999 to November 2002. Based on these distributions, we calculate the skewness premiums for 5% and 2% OTM call and put options, according to equation (5.29). The resulting skewness premiums are shown in Figure 5.4.

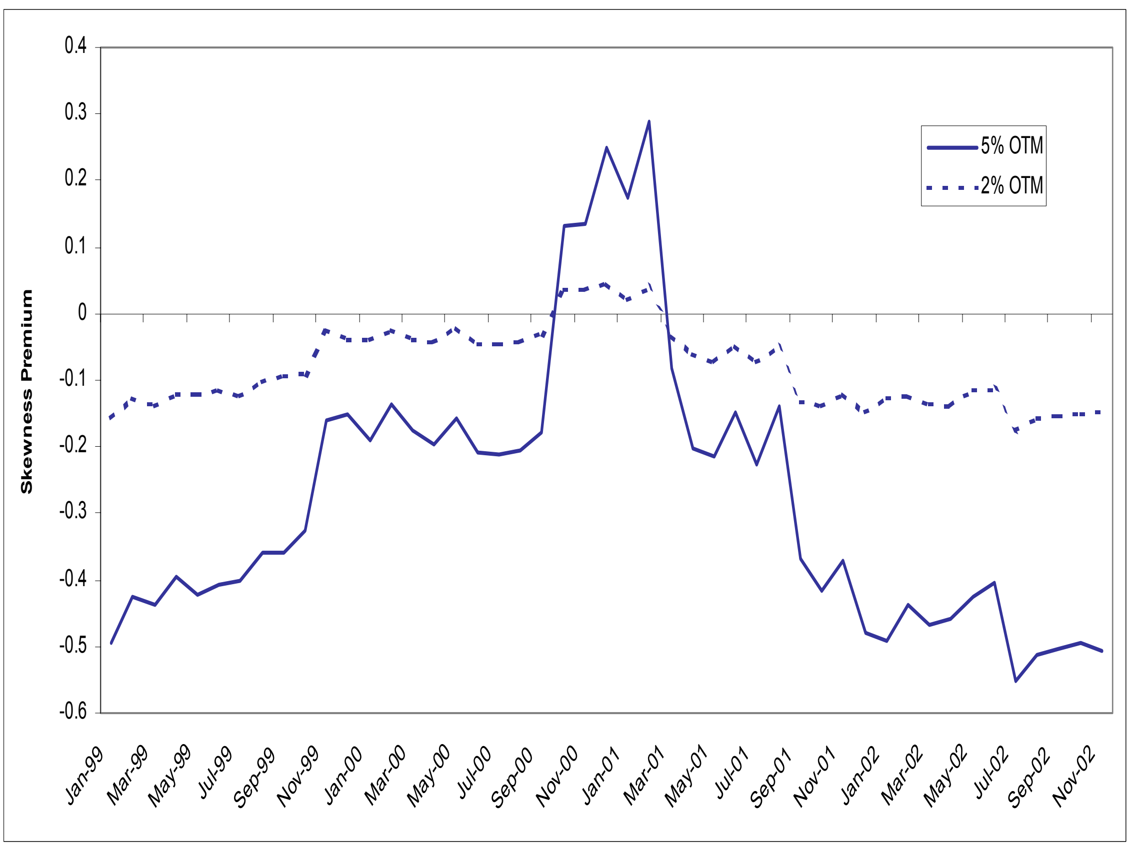


Figure 5.4: Empirical estimates of skewness premiums for the $S\&P500$ index.

Figure 5.4 confirms that the risk-neutral distributions of the $S\&P500$ index during this period are not symmetric around their means. The period can be divided to three subperiods. Over Jan-99 –

Sep-00, OTM puts are more expensive than OTM call options, yielding negative skewness premiums. Positive skewness premiums are observed during the short period between Oct-00 and Feb-00, while from March-00 until Nov-02, the skewness premiums are again negative. Figure 5.4 also confirms that the deeper OTM the options are, the higher, in absolute value terms, the respective skewness premiums.

The fact that OTM puts are usually more expensive than the symmetric OTM calls can be explained by the fact that the demand for put options for hedging and insurance purposes is larger than the demand for OTM call options. As portfolio managers try to hedge their downside risk, they usually buy put options that provide coverage against market downturns, while maintaining the upside potential. Bollen and Rasiel [30] note that portfolio insurers prefer OTM puts rather than ITM puts. As the demand for ITM puts is less than for OTM puts, ITM puts are relatively less expensive. In this case, put-call parity implies that OTM call options would be relatively less expensive.

Skewness premiums provide an indication of market expectations. If skewness premiums are negative, the market considers a decrease in the value of the underlying index more likely than an increase. If such a market expectation is correct, the probability that an OTM put option will be exercised at maturity is greater than the corresponding probability for an OTM call option. Thus the OTM put options would be more expensive than the respective OTM call options. Figure 5.5 plots the $S\&P500$ index between October 1988 and December 2002.

Comparing Figures 5.4 and 5.5 we observe that skewness premiums vary in a similar pattern to the underlying asset — the $S\&P500$ index in this case — which they seem to follow. They exhibit an upward trend during periods of market advancements, while they take increasingly negative values during periods of market declines. The skewness premiums become positive at, or right after, the period during which the $S\&P500$ had peaked (Oct. 2000).

We estimate the risk-neutral distribution of the $S\&P500$ stock index at two points in time; the first when the skewness premium was positive (December 2000) and the second when it was negative (June 2002). Figure 5.6 plots these distributions. The value of the index, S_0, at the corresponding date and the strike prices K_c and K_p for $a = 5\%$ OTM call and put options, respectively, are also marked on these graphs.

As we can see, the risk neutral distribution of the index returns on December 2000 (first graph) is positively skewed. The probability that the index will exceed the strike price, K_c, of the OTM call option is higher than the probability that it will fall below the strike price, K_p, of the symmetric OTM put option. This explains why the OTM call is more expensive than the respective OTM put in such a case, giving rise to a positive skewness premium.

In the Black-Scholes framework, asset prices are assumed to follow a geometric Brownian motion. This implies that the risk-neutral distribution of the asset prices is lognormal (i.e., positively skewed). When the empirically determined risk-neutral distribution is indeed positively skewed — as is the case for the $S\&P500$ index returns on 12/2000 (first graph of Figure 5.6) — then the two proposed pricing methods will produce prices of OTM call and put options that are close to the Black-Scholes prices.

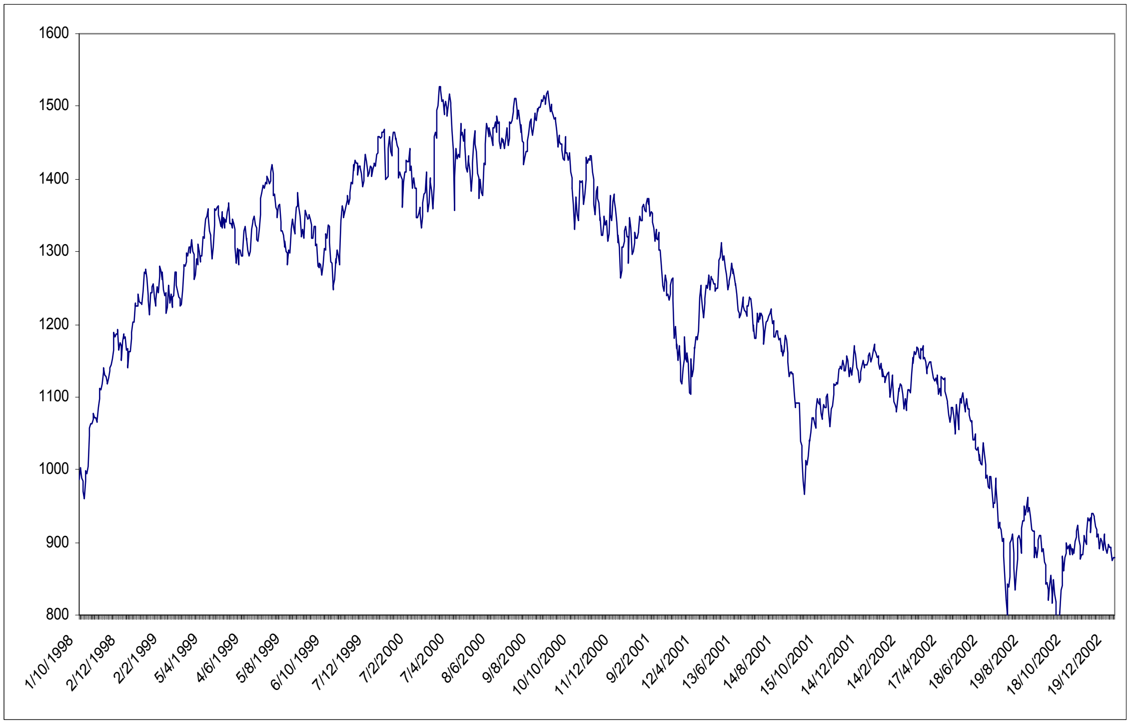


Figure 5.5: The $S\&P500$ index through time

Their differences will depend only on the degree that the risk-neutral distribution is approximated by the lognormal. In this case, the differences in OTM option prices resulting from the alternative methods should be small, and should also approximate well the market prices of these options.

This does not hold when the estimated risk-neutral probability distribution for the asset prices comes out to be negatively skewed. This is the case for the $S\&P500$ index returns on June 2002 (second graph in Figure 5.6). We now observe that the probability that the index will fall below the strike price, K_p, of the OTM put option is higher than the probability that it will surpass the strike price, K_c, of the symmetric OTM call option. Hence, the OTM put is relatively more expensive that the respective OTM call, which is reflected in the negative skewness premium. In this case, which is more frequent in practice, the Black-Scholes formula underprices OTM puts, while it overprices OTM calls. The proposed pricing methods should approximate more closely than the Black-Scholes formula the market prices of options in such case.

The conjectures stated above regarding the relative performance of the two proposed option pricing procedures in comparison to the Black-Scholes formula are empirically verified in the next section, where option prices computed with the alternative methods are compared to market prices of $S\&P500$ index options at the two periods, December 2000 and June 2002.

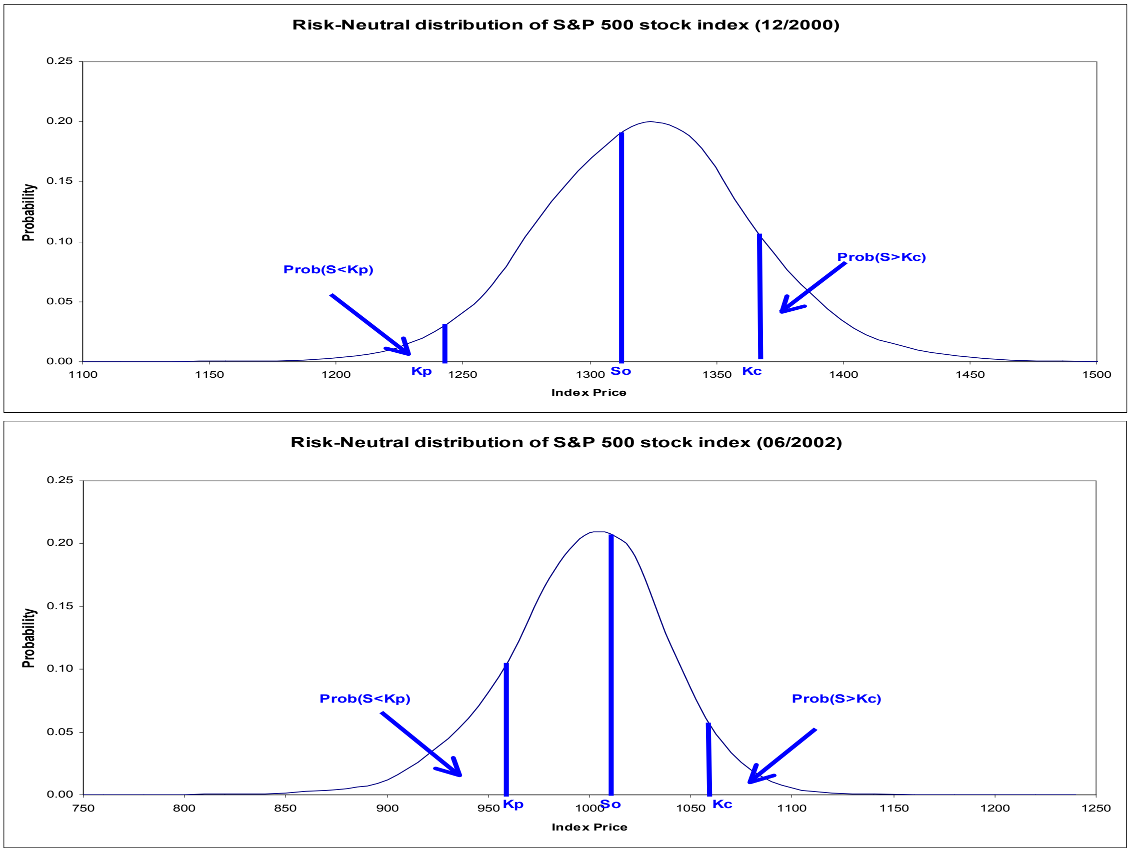


Figure 5.6: The estimated risk-neutral distribution of the $S\&P500$ index returns at two different points in time. The distribution is positively skewed for December 2000 and negatively skewed for June 2002.

5.6 Validity of Methodologies using Market Option Prices

We carry out numerical tests to empirically validate the performance and practical reliability of the two proposed option-pricing procedures. To this end, we compare option prices obtained with the proposed valuation procedures, and with the Black-Scholes approach, to market prices of European options on the S&P500 stock index.

Bates [19] notes several practical difficulties faced in comparative studies of option pricing methods against actual market prices of options. These difficulties include methodological as well as data related issues. For example, abundance of data from disparate sources — that can be inconsistent and even conflicting at times, — incomplete market data for symmetric call and put options to enable the verification of estimated skewness premiums with actual option prices, lack of contemporaneous records of option prices and corresponding prices of the underlying asset to enable accurate calibration of valuation models and accurate comparisons, etc. The diversity of comparative tests that may be applied also poses a problem.

Jarrow and Madan [105] point out that the absence of arbitrage does not guarantee that theoretical option prices computed as the expected discounted cahsflows under a martingale measure will necessarily match the market prices of the corresponding options. They attribute the differences between theoretical prices determined by option pricing models and the market prices of the options to market valuation of asymptotic cash flow positions in the time and state direction, that they term "rational bubbles." A related view of the relationship between market prices and the valuations obtained from option pricing models is adopted in Jacquier and Jarrow [103]. They identify, in addition to observation or measurement errors, errors induced by parameter uncertainty. Another possibility is that the observed option prices are systematically distorted, and that one can make a profit in the options market by exploiting such mispricings.

So, several factors have been suggested as possible justifications for observed differences between model prices and market prices of options. Notable among these are, measurement and parameter estimation errors, model errors — due to approximations adopted in modeling, — possible presence of rational bubbles, market mispricing due to possible delays in market reaction to fundamentals (i.e., delays of market adjustments to information flows), etc. Admittedly, no option valuation approach reproduces exactly the option prices observed in the markets. Nevertheless, empirical evidence of a valuation model's ability to effectively approximate market prices of options, without systematic biases, is the proper way to assess its appropriateness and practical reliability. This is exactly the purpose of the tests reported in this section.

From equations 5.22 and 5.26 we note that a critical input of the Black-Scholes option pricing formulae is the instantaneous volatility (σ) of the underlying asset. There are two possible ways to apply the Black-Scholes formulae for call and put options. We can either use an estimate of volatility based on historical observations of asset prices, or we can compute an estimate of volatility that is implied by quoted option prices at a particular time of interest. Using the market price of an option, one can infer the value of volatility that, when substituted in the appropriate Black-Scholes formula, would exactly reproduce the quoted price of the option. The implied volatility (σ_{ISD}) is obtained so as to minimize the pricing error of the Black-Scholes model over a set of options with the same expiration date. Thus, the implied volatility is obtained as the solution of the following quadratic problem:

$$\sigma_{\text{ISD}} \;=\; \text{argmin}_\sigma \sum_{i=1}^{m} \left(C_{i,M} - C_{i,BS}(\sigma)\right)^2 ,$$

where

$C_{i,M}$ is the observed market price of option $i = 1, \ldots, m$,

$C_{i,BS}(\sigma)$ is the Black-Scholes price for option $i = 1, \ldots, m$. This price results from the corresponding Black-Scholes formula — depending on whether the option is a call or put option — using a volatility value σ.

The implied volatility σ_{ISD} can then be used in the Black-Scholes formulae to price options with the same term, but different strike prices. We apply both alternatives in the numerical tests.

The options considered in the tests are European call options on the S&P500 stock index, with maturity (τ) of one month. All data for the analysis are obtained from Datastream. We value European call options of one-month maturity on the S&P500 stock index at two different dates:

December 20, 2000 and June 20, 2002. Our estimates of the skewness premiums at these two dates have opposite sign, implying different market regimes during those times. Basic data for the analysis are summarized in Table 5.1.

	Dec. 20, 2000	June 20, 2002
Value of S&P500 stock index (S_0)	1305.6	1006.29
Monthly riskless interest rate (r_f)	0.06578	0.018762
Skewness Premiums from Estimated Risk Neutral Distributions		
2% OTM	0.044	-0.114
5% OTM	0.247	-0.403
Skewness Premiums Implied by Market Prices of Options		
∼3% OTM	0.082	-0.1853
Estimated Statistics of Monthly Returns of the S&P500		
Mean (μ_1)	0.01208	0.0073
Standard Deviation (μ_2)	0.1826	0.1926
Skewness (μ_3)	-0.00344	-0.115
Kurtosis (μ_4)	0.025	0.029
Implied Volatility (σ_{ISD})	0.1909	0.1969

Table 5.1: Basic data for option valuation assessments on 20/12/2000 and 20/06/2002.

We see that the skewness premiums implied by market prices of options confirm the corresponding premiums obtained from estimates of the risk neutral distribution of the S&P500 index for the respective dates.[1] A negative skewness premium reflects a negatively skewed risk-neutral distribution for the underlying. In such a case, the Black-Scholes approach systematically overprices OTM call options and underprices OTM put options; the pricing error is higher for deep OTM options. We show that this is indeed observed with quoted market prices of options. As we see in the empirical results, the proposed option valuation procedures approximate more closely than the Black-Scholes approach the market prices of OTM options.

Tables 5.2 and 5.3 compare the performance of alternative option valuation methods, and contrast their estimated option prices against market prices of the options on 20/12/2000 and 20/06/2002, respectively. A number of one-month European call options on the S&P500 stock index with different strike prices are considered in each case. Call options with strike prices lower, respectively higher, than the price of the S&P500 index (S_0) at the time represent ITM, respectively OTM, call options; these are separated by the dividing line in each Table.

The valuation methods include the two procedures described in section 5.2, and the two variants

[1]The skewness premiums from observed market prices of options are approximate, as there were no price quotations for exactly symmetric call and put options (i.e., same OTM level). We used the available price quotations for options whose strike prices were closest to 2% OTM. The results confirm the different sign of the skewness premiums at the two dates.

of the Black-Scholes approach: B-S_h refers to the B-S method using an estimate of volatility σ from historical observations of the index, while B-S_I refers to the B-S method using the volatility estimate σ_{ISD} implied by the entire set of option quotations on the respective date.

		Estimated Option Prices							
Strike	Market	Method 1		Method2		B-S_h		B-S_I	
Price	Price		(error)		(error)		(error)		(error)
950	354.5	360.2	1.61%	360.4	1.66%	360.8	1.78%	360.8	1.78%
975	329.9	335.3	1.64%	335.1	1.58%	335.9	1.84%	335.9	1.84%
995	310.1	314.7	1.48%	315.0	1.57%	316.0	1.91%	316.0	1.91%
1025	291.8	286.4	-1.83%	285.8	-2.04%	286.2	-1.90%	286.2	-1.90%
1050	267.0	261.3	-2.13%	261.1	-2.21%	261.3	-2.12%	261.3	-2.12%
1075	242.4	236.4	-2.47%	236.5	-2.42%	236.5	-2.43%	236.5	-2.43%
1100	216.3	211.2	-2.35%	210.8	-2.53%	211.6	-2.15%	211.6	-2.15%
1125	192.9	186.4	-3.36%	185.9	-3.62%	186.8	-3.16%	186.8	-3.15%
1150	168.1	161.1	-4.18%	161.0	-4.24%	162.0	-3.64%	162.1	-3.60%
1200	117.9	111.4	-5.49%	111.2	-5.66%	113.4	-3.83%	113.7	-3.58%
1250	58.5	63.1	7.86%	63.0	7.69%	68.8	17.65%	69.6	19.03%
1270	45.4	48.0	5.79%	48.3	6.45%	53.5	17.80%	54.5	20.03%
1275	42.4	45.8	8.08%	46.1	8.79%	49.9	17.76%	51.0	20.26%
1285	36.0	38.1	5.83%	38.4	6.67%	43.2	19.98%	44.3	23.14%
1300	28.0	30.2	7.86%	30.4	8.57%	34.2	21.98%	35.4	26.33%
1320	21.0	22.2	5.71%	22.5	7.14%	24.1	14.76%	25.3	20.67%
1325	18.8	19.6	4.53%	19.9	6.13%	21.9	17.02%	23.2	23.59%
1345	13.1	14.3	8.95%	14.4	9.71%	14.7	11.76%	15.8	20.43%
1350	11.4	12.4	8.92%	12.5	9.89%	13.17	15.79%	14.3	25.48%

Table 5.2: Observed vs estimated prices of European call options on the S&P500 index (December 20,2000).

We observe that in all cases the two proposed procedures yield estimates of the option prices that are quite close. Although the manner in which the two methods account for the effect of higher order moments of the underlying is different, yet they consistently produce very similar estimates of option prices. These option price estimates approximate more closely than the Black-Scholes model the quoted market prices for deep ITM, and especially for OTM, options. The Black-Scholes prices have smaller errors only for some ITM call options. The most important observation is that the Black-Scholes approach substantially overprices deep OTM call options — and would correspondingly underprice deep OTM put options; the proposed procedures are more effective in these circumstances. OTM puts are the more suitable instruments for controlling downside risk and are also inexpensive. The systematic mispricing (bias) of OTM options by the Black-Scholes method is particularly evident when the skewness premium is negative, i.e., in June 2002 (see, Table 5.3). The mispricing is more severe when an implied volatility estimate is used in the Black-Scholes formula.

Strike	Market	Estimated Option Prices							
		Method 1		Method2		B-S_h		B-S_l	
Price	Price		(error)		(error)		(error)		(error)
800	207.7	207.9	0.10%	207.6	-0.02%	207.6	-0.04%	207.5	-0.09%
825	183.2	182.8	-0.25%	182.6	-0.33%	182.6	-0.30%	182.5	-0.36%
850	159	157.6	-0.87%	157.6	-0.90%	157.7	-0.82%	157.6	-0.88%
875	135.2	132.7	-1.83%	132.6	-1.94%	132.8	-1.79%	132.7	-1.83%
900	112.9	107.9	-4.42%	107.5	-4.82%	108.0	-4.30%	108.1	-4.23%
925	89.4	83.4	-6.74%	82.3	-7.98%	83.9	-6.15%	84.3	-5.74%
930	72.5	77.5	6.96%	77.3	6.60%	79.2	9.26%	79.7	9.88%
935	67.6	72.7	7.61%	72.4	7.06%	73.1	8.18%	75.1	11.12%
950	54.2	57.7	6.40%	58.2	7.36%	61.2	12.95%	62.0	14.44%
960	48.6	48.6	0.07%	49.4	1.71%	52.8	8.70%	53.8	10.77%
965	43.2	45.3	4.78%	45.3	4.90%	48.8	13.02%	49.9	15.57%
975	37	37.7	1.86%	37.9	2.34%	41.3	11.51%	42.5	14.97%
990	27.2	26.7	-2.00%	27.4	0.87%	31.1	14.46%	32.6	19.94%
995	24.3	23.7	-2.60%	24.4	0.49%	28.1	15.70%	29.7	22.03%
1005	20.3	19.9	-1.77%	20.4	0.72%	22.6	11.49%	24.2	19.35%
1010	17.5	17.8	1.74%	17.9	2.55%	20.1	14.99%	21.8	24.44%
1020	14.2	15.0	5.51%	15.3	7.52%	15.8	11.34%	17.4	22.48%
1025	12.1	13.1	8.52%	13.3	9.64%	15.0	24.08%	15.5	27.70%
1030	10.6	11.6	9.77%	11.7	10.79%	13.2	24.94%	13.7	28.99%
1040	7.6	8.6	13.25%	8.7	14.90%	10.2	33.90%	10.6	39.17%
1050	6.3	6.7	6.83%	6.8	8.62%	7.7	22.00%	8.0	27.77%
1060	4.2	4.7	12.09%	4.8	13.76%	5.7	35.83%	6.0	43.47%

Table 5.3: Observed vs estimated prices of European call options on the S&P500 index (June 20, 2002).

5.7 Conclusions

This chapter considered appropriate ways to price options in a scenario-based setting so as to incorporate them in stochastic programming models for portfolio management. This is motivated by the fact that options constitute particularly suitable instruments for risk management purposes due to the asymmetric form of their payoffs. Put options provide coverage against downside risk while preserving upside potential. Moreover, trading strategies that involve specific combinations of options can be suitably designed so as to generate desirable payoff patterns. Thus, such trading strategies can provide coverage against downside risk and also enhance upside potential. Hence, the incorporation of options as hedging instruments in portfolio management models should lead to improved risk management tools.

The problem addressed in this chapter concerns the development of an internally consistent modeling framework, whereby the options are priced in accordance with the scenarios (discrete distributions) of asset returns that are used to represent uncertainty in input parameters of stochastic programs. The stochastic programming framework affords the flexibility to generate scenarios through alternative procedures, so as to represent the decision maker's view regarding future market conditions. Thus, these scenario sets do not usually conform to the assumed distributions of asset prices on which traditional option valuation methods are based, making these popular pricing methods inapplicable for the stochastic programming framework. In this study, the scenarios of asset returns are generated by means of a moment-matching procedure so that the first four marginal moments of the asset returns — and their correlations in multi-asset cases — match their corresponding historical values. Hence, the postulated scenarios reflect the empirical distributions of asset returns, including asymmetries and heavy tails that are often observed in practice.

We devise two alternative procedures to price options in accordance with such discrete distributions of asset returns. The objective of this work is not the pursuit of novel theoretical developments in option pricing methods. Instead, our contribution lies in the adaptation of existing valuation methods in accordance with discrete distributions of asset returns, as depicted by means of scenario trees.

The proposed option valuation procedures take into account higher order moments of the empirical distribution of the underlying asset's returns. In section 5.4 we showed that the higher order moments do clearly affect the estimated prices of options; their effects can explain, at least partly, the difference between Black-Scholes prices and the market prices of options. Moreover, we demonstrated empirically that the proposed valuation procedures price options in a consistently effective manner. They result in estimates of option prices that are consistently similar, and approximate more closely than the Black-Scholes approach market quotations of option prices. The results of the numerical tests show that the proposed valuation procedures are notably more effective in those cases that the Black-Scholes approach is known to exhibit systematic biases (i.e., overpricing OTM call options and underpricing OTM put options, especially when the skewness premium is negative). Such conditions are common in practice (e.g., see Figure 5.3). Moreover, OTM puts are of special interest — either individually or as constituents of more complex trading strategies — for risk management purposes. Therefore, the use of the Black-Scholes approach becomes ineffective in these cases to price options in the context of portfolio optimization models.

As the Black-Scholes approach underprices (deep) OTM put options, arbitrage opportunities are typically detected in the portfolio optimization model if the B-S option prices are used as inputs. We verified through numerical tests that, on the contrary, our proposed framework meets the arbitrage-free requirement. We explicitly tested that the discrete outcomes of asset returns generated by the moment-matching scenario generation method are free from arbitrage. The corresponding option valuation procedures that are based on the discrete distributions reflected in these scenario sets are constructed so as to satisfy the fundamental no-arbitrage condition. The conformity of the entire datasets (scenarios of asset prices and associated option prices) to the no-arbitrage principle was exhaustively tested numerically, using the approach outlined in section 5.3.

The numerical tests establish the effectiveness and practical reliability of the proposed valuation procedures. As we indicated in section 5.2, these procedures can be applied in a fairly straightforward manner to price European options at any decision node of the scenario tree, provided that the option's expiration date coincides with a subsequent decision stage on the tree. As a result, these procedures provide flexible tools to incorporate options in stochastic programming models for portfolio management.

The developments of this chapter provide the foundation for the studies in subsequent chapters. The first method is applied in chapter 6 to introduce options on stock indices as means of controlling market risk in international portfolios. The second method forms the basis for the studies in the remaining chapters. It is used in chapter 7 to price currency options which are then employed for currency risk hedging purposes in the context of multistage stochastic programs for international portfolio management. This method is extended in chapter 8 to price quanto options (integrated options on foreign stock indices but having payoffs and price quotations in another reference currency). This provides the capability to analyze in chapter 8 the effectiveness of alternative strategies that use options (on currencies, on stock indices, and quantos) for controlling total risk in international portfolios. Thus, proposed option valuation procedures of this chapter provide the means to introduce various types of options in stochastic programming models, and to empirically explore the relative effectiveness of different decision strategies for managing risk in international portfolios.

Chapter 6

Managing Market Risk with Options on Stock Indices

In this chapter we build on and extend the developments of the previous chapters. We introduce to the investment opportunity set of the portfolio management models options on stock indices as additional means to control risk exposures. Specifically, we consider two different types of European options on stock indices: unhedged (simple) and fully protected options (quantos). The first type of option has its strike price and payoffs expressed in the same currency as the stock index that constitutes the underlying; hence, it is also priced in the same currency units. This type of option is intended to hedge the market risk associated with the underlying stock index and does not account for changes in currency exchange rates. We use the term "simple option" to refer to stock index options of this type. On the contrary, quantos are options written on a foreign stock index that also protect against currency movements as they apply a prespecified exchange rate to convert the payoffs of the option to a different currency (i.e., the reference currency of the investor). The strike price and payoffs of this type of option are expressed in units of the base currency by using a prespecified exchange rate (usually the forward rate for the term of the option) as the conversion factor. Hence, the underlying of a quanto is the product of the value of the foreign stock index augmented by a fixed exchange rate. A quanto is an integrative instrument as it jointly protects against both the market risk of the stock index as well as the exchange risk between the index and the base currency; it is priced in units of the base currency.

The introduction of these options broadens the investment opportunity set and provides instruments geared towards risk control due to the asymmetric and nonlinear form of option payoffs. We suitably extend the portfolio optimization models so as to incorporate the options and we empirically investigate the ex ante and the ex post impact that these options have on the performance of international portfolios of financial assets. The residual currency risk from other foreign holdings (e.g., in bond indices) can be covered with forward currency exchange contracts that are also included in the models. The goal is to control the portfolio's total risk exposure and to attain an effective balance between portfolio risk and expected return. The results indicate that the inclusion of these types of options provides an efficient and effective way to control risk and to improve the performance of international portfolios. In fact, we find that the performance of the international portfolios is improved as we take a progressively integrated view towards total risk management; that is, as we progressively control more risk factors with appropriate hedging instruments.

In this chapter we confine our attention to single-period models in order to simplify the notation and the presentation of the novel concepts. Generalizations that incorporate options in multistage instances of the scenario-based optimization models so as to address the dynamic aspects of the problem are demonstrated in subsequent chapters. The problem addressed here concerns a single portfolio restructuring decision with a single-period planning horizon. The models take as primary inputs the composition of an initial multi-currency portfolio, along with the current prices of the assets, spot currency exchange rates, forward exchange rates for a term equal to the planning horizon, and the option prices. The stochastic elements concern the returns of the assets over the planning horizon - and consequently the asset prices at the horizon - the spot currency exchange rates and the option payoffs at the end of the horizon. The uncertainty in these parameters is depicted by means of discrete distributions (scenario sets). The scenarios are generated using a moment matching procedure that was described in chapter 4 and is also briefly explained at a later section of this chapter 6.2.2. Specifically, the scenarios of the random variables - the asset returns (in domestic terms) and the proportional changes of spot exchange rates over the planning period - are generated so that their first four marginal moments and correlations match their corresponding historical values over the previous 10 years. The decision variables of the optimization models determine the required transactions for each asset that result in a revised portfolio. They also determine the forward currency exchange contracts as well as the positions in the options. Thus, a number of interrelated decisions that have traditionally been considered separately are cast in the same framework with consequent improvements in decision effectiveness.

We had noted earlier the flexibility of stochastic programming models to accommodate alternative representations of uncertainty by means of arbitrary discrete distributions (scenarios). Such distributions can capture asymmetries and fat tails that are often observed in historical data of financial time series, as is the case in the international portfolio management problems examined in this thesis. However, these distributions do not usually conform to the functional forms for which analytic option pricing formulae exist. Hence, as we pointed out in the previous chapter, the incorporation of options in scenario-based portfolio optimization models requires that these options should be consistently priced with the scenarios of the underlying instruments, while also conforming to fundamental financial principles (e.g., exclusion of arbitrage), in order to provide an internally consistent model. To this end, we employ the procedures described in section 5.2 to price the simple options on stock indices. Quanto options are similarly priced with a suitably adapted procedure as illustrated in Appendix 6.6.

6.1 Introduction

Derivative securities are being increasingly used for hedging purposes. Options, due to their asymmetric and nonlinear payoffs, provide the means to protect the value of holdings in an asset in the event of substantial variations in the market price of the underlying security. They can also be used for speculative purposes so as to generate profits in the event of large changes in the value of the underlying. A long position in a put option provides coverage against declines in the market value of the underlying security. Conversely, a long position in a call option generates its payoffs in the event of upside changes in the market value of the underlying. Combinations of options may be used to shape a payoff profile according to the preferences of an investor so as to yield a desired tradeoff between payoff risk and expected return.

An important aspect of risk in the context of our international portfolio management problem is the exposure to market risks in the various countries from holdings of stock indices in these countries. Options on these stock indices provide the means to mitigate these risks. In this chapter we consider two types of European options on stock indices. The first type yields its payoffs in the currency of the underlying stock index; the strike price is also expressed in the same currency, as is also the price of the option. Hence, the sole concern of this option - which we call a "simple option" - is to hedge the market risk of the underlying stock index, without any consideration for the currency risk. The second type of option is designed so as to additionally cover the currency risk. Options of this type are known as "quantos"; they are written on a stock index but their strike price and payoff are expressed in units of a different currency (i.e., the base currency of the investor). This is accomplished by applying a predetermined exchange rate (usually the forward currency exchange rate for the term of the option) as the conversion factor. These options are priced and marketed in the base currency and jointly cover against the market risk and currency risk of an investment in a foreign stock index.

The inclusion of the options in the international portfolios is intended to cover the relevant risk exposures from positions in the stock indices. As the portfolio may also involve holdings in foreign bond indices our models additionally allow forward currency contracts in order to hedge the residual currency risk from these investments. Our aim is to control the total portfolio risk and to achieve a desirable balance between portfolio risk and expected return. Obviously, it is more effective to consider all the decisions in an integral manner rather than to separate the investment selection decisions from the risk hedging decisions. Our optimization models serve this purpose as they take a holistic view of the risk management problem for the international portfolios. The models incorporate in a unified framework the decisions for investments in various assets across countries and the positions in the appropriate hedging instruments (i.e., options on stock indices and forward currency exchange contracts). These decisions have typically been considered separately in practice.

A primary contribution of this work is that it unifies in a common optimization framework all the relevant decisions in the international portfolio management problem. The resulting benefits in the performance of international portfolios are demonstrated through empirical tests with real market data.

This study extends the work of Topaloglou et al. [182] - which also appears as chapter 3 of this thesis. Similar to that paper, the current chapter considers a single-period problem. However, the optimization model is extended in several ways. First, we consider a portfolio restructuring model that accounts for the initial composition of the portfolio as well as for transaction costs. Second, the decisions regarding forward currency exchanges are operationalized as the model includes separate decisions to explicitly specify the optimal amounts of forward currency contracts instead of approximating the level of currency hedging through forward exchanges. Most importantly, the novel features of the current models concern the additional inclusion of options on stock indices (domestic and foreign) for risk hedging purposes in the context of international asset allocation. The simple options provide the means to mitigate market risks, while quantos integrally control market and currency risk of holdings in foreign stock indices.

The problem is interesting because the asymmetric option payoffs provide the means to protect against adverse market movements. Hence, the inclusion of options as hedging instruments in international portfolio management models has the potential to provide effective means for risk control. In this chapter we develop optimization models for the selection of internationally diversified portfolios including selective hedging decisions in appropriate instruments (options on stock indices and forward currency exchange contracts) in order to control the market and currency risks of the investments. We assess the relative performance of alternative risk hedging strategies through empirical tests. To this end, we also experiment with hedging strategies that impose choices among specific combinations of options in the portfolio.

An essential aspect of the portfolio management model is the specification of uncertainty in the random parameters (i.e., in asset prices, spot exchange rates and resulting option payoffs at the end of the planning period). In the stochastic programming models uncertainty is represented by means of discrete distributions (scenarios) that jointly depict the covariation of these random parameters. We do not impose any assumption concerning the functional form of the distribution of the random parameters. From a statistical analysis of historical data of stock index returns and currency exchange rates we observe that during the period under consideration they exhibited negative skewness and excess kurtosis (see Table 6.1) Jacque-Bera tests on these data series indicate that the normality hypothesis can not be accepted in most cases. This motivates us to generate scenarios without imposing a specific distribution for the random variables. Instead, we rather generate scenarios based on the empirical distribution of asset returns and exchange rates. The scenarios are generated so that the first four marginal moments and correlations of the random variables match their respective historical values over of the previous 10 years.

The stochastic optimization programs minimize the downside risk of the international portfolio's value at the end of the holding period. They also include a parametric constraint to enforce a minimum target level on the portfolio's expected return. The scenarios of asset returns, exchange rates and corresponding option prices are critical inputs to the optimization programs that determine jointly the asset holdings across markets amd the appropriate risk hedging decisions. Our models employ the conditional value-at-risk (CVaR) metric [162, 163] that minimizes the conditional expectation of portfolio losses above a prespecified percentile of the distribution (i.e., minimizes the expected losses beyond VaR). Our motivation for applying the CVaR risk measure stems from the observation that returns of international assets and proportional changes of exchange rates exhibit asymmetric distributions and fat tails. As we discussed in section 4.6 CVaR is a coherent risk measure, and is appropriate for asymmetric distributions because it aims to control the tail of the distribution of portfolio losses.

The contributions of our study are as follows: First, we develop and implement flexible models for international asset allocation that account for market risk of stock index holdings and currency risk of foreign investments in a unified manner. The models optimally select specific investments in domestic and foreign markets (stock and bond indices) and jointly determine appropriate risk hedging decisions via positions in options on stock indices (simple options or quantos) and forward currency contracts, so as to minimize the downside risk of the entire portfolio. In all previous studies in the literature the hedging strategy is either prespecified at the portfolio selection stage, or the hedging decisions are considered separately after the portfolio selection is made. Here all relevant decisions are incorporated in the same optimization model so as to take a holistic view of the problem. Hence, we provide an integrated and more effective tool for managing risk exposures of international portfolios.

Second, we price European options on stock indices and incorporate them in scenario-based optimization models for international asset allocation. The pricing method is general and flexible and does not depend on specific assumptions concerning the distributions of the underlying random variables. The options are priced consistently with the postulated scenarios for asset returns and exchange rates, that in this case reflect skewness and excess kurtosis characteristics as supported by historical data. We note particularly our adaptation of the pricing procedure to the case of quantos (see Appendix 6.6) and the novel consideration of such instruments in the context of international portfolio optimization.

These tools provide the means to investigate the performance of alternative risk hedging strategies, including popular strategies that enforce specific combinations of options (eg, strangle, straddle, strip, strap). We observe that the inclusion of options in the portfolio can materially reduce the downside risk. The returns of portfolios that include options have significantly lower tails and exhibit (more) positively skewed distributions in contrast to the distribution of portfolios without options.

Our empirical tests reveal that quantos provide particularly effective instruments for risk hedging purposes. This is due to the integrative nature of these instruments that cover both the market risk of the underlying security and the associated currency risk. Overall, we observe that progressively integrated views towards risk management are increasingly more effective. That is, incremental benefits in terms of reducing risk or generating cumulative profits can be gained as more risk factors are progressively controlled through appropriate hedging strategies that involve options and currency forward contracts. Hence, we demonstrate that integrated consideration of market and currency risks can yield substantial benefits for international investors. This result could possibly generalize to other portfolio management contexts that are governed by multiple risk factors.

The rest of this chapter is organized as follows. In section 6.2 we formulate the optimization models for international portfolio selection. Two variants of the optimization model are present. One considers the use of simple options while the other incorporates quantos. Both models also include currency forward contracts. We also discuss the scenario generation procedure. In section 6.3

we discuss the hedging strategies employed in the empirical tests. In section 6.4 we describe our computational tests, and we analyze the empirical results. Section 6.5 concludes the paper. Finally in Appendix 6.6 we describe the methodology for pricing simple as well as quanto options.

6.2 Stochastic Programming Models for Risk Management

6.2.1 Model Description

We view the problem of portfolio (re)structuring from the perspective of a US investor who may hold assets denominated in multiple currencies. We consider, specifically, portfolios composed of one stock index and multiple bond indices in various countries. These portfolios are exposed to market risk in the domestic and foreign markets, as well as to currency exchange risk. To hedge the market risk in stock indices, the investor may buy (a combination of) options on these securities with a payoff that gives him protection against unfavorable movements in the price of the securities. The problem has a single time horizon. The options are of the European type and their maturity marches the planning horizon. To hedge the currency risk, the investor may enter into currency exchange contracts in the forward market. The amount of the forward currency transactions for each currency must be specified at the time of the decision (t=0) and hence, must be a deterministic quantity that can be implemented as a contract drown on the basis of the currently quoted forward exchange rate for the term of the planning period. These forward currency exchange contracts can hedge at least partially - depending on their relative magnitude against the investor's total exposure in the respective currency - the corresponding currency risk.

The scenario-based portfolio optimization models address the market and currency risk in an integrated manner. Their deterministic inputs are: the initial asset holdings, the current prices of the securities, the current spot exchange rates, the forward exchange rates for a term equal to the planning horizon and the option prices. The scenario dependent data that, together with the associated probabilities, specify the distribution of the random variables at the end of the planning horizon are: the final prices of the securities and the spot exchange rates at the end of the horizon. These, in turn, uniquely determine the option payoffs at the end of the horizon under each scenario. The European options can be purchased at the beginning of the time horizon and their maturity match the problem horizon.

The model's decision variables determine the required asset purchase and sale transactions that yield a revised portfolio. Thus, the optimal holdings of specific securities in each currency are completely specified. Additionally, the models jointly determine the optimal hedging decisions. These include the volumes of the forward currency contracts and the positions in the available options. Positions in specific combinations of options - reflecting certain hedging strategies - are easily enforced with appropriate constraints. The optimization models incorporate institutional considerations (no short sales for assets, transaction costs) and minimize the downside risk of final portfolio value at the level of a desirable target expected return. Thus, several interrelated decisions that were previously examined separately, are cast in the same framework with consequent benefits to the investors.

The models have a one stage investment horizon $[0, T]$. In this study the length of the horizon is one month. All decisions - portfolio rebalancing, forwards contracts and positions in options - are taken at the beginning of the period (t=0) taking into account the scenarios that describe the plausible changes in the values of the random variables during the planning period. The objective is to minimize a measure of risk for the total value of the revised portfolio at the end of the planning period [T]. As the risk measure we employ the conditional value-at-risk (CVaR) for portfolio losses at a prespecified percentile of the distribution. We parametrically impose a lower bound on the expected portfolio return.

The cashflow balance constraints in each country, as well as the final value of the portfolio under each scenario, are expressed differently for each type of options that is incorporated in the portfolio. This is because quanto options are issued in the investor's reference currency (USD), and thus the investor does not transfer funds in any foreign currency to purchase these options. Simple options, on the other hand, are issued in each currency. We present separately the models for each type of options that is considered.

We use the following notation:

Definitions of sets:

C_0	set of currencies (markets), including the base (reference) currency,
$\ell \in C_0$	the index of the base (reference) currency in the set of currencies,
$C = C_0 \backslash \{\ell\}$	the set of foreign currencies (i.e., excluding the base currency),
I_c	set of asset classes denominated in currency $c \in C_0$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term bond index in each country),
κ	the ordinal index of the stock index security in a set of assets I_c
N	the set of scenarios,
JS_c	the set of all available simple options in market $c \in C_0$ (differing in terms of their exercise price),
JQ_c	the set of all available quanto options in market $c \in C$ (differing in terms of their exercise price

Input Parameters (Data):

(a). Deterministic quantities:

b_{ic}	initial position in asset class $i \in I_c$ of currency $c \in C_0$ (in units of face value),
h_c	initially available cash in currency $c \in C_0$ (surplus if +ve, shortage if -ve),
T	is the time horizon (in our case 1 month),
δ	proportional transaction cost for sales and purchases of assets,
d	proportional transaction cost for currency transactions in spot market,
μ	prespecified target expected return of the revised portfolio,
α	the prespecified confidence level (percentile) for the CVaR measure,
π_{ic}	current market price (in units of the respective currency) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$,
e_c	current spot exchange rate for currency $c \in C$,
f_c	currently quoted forward exchange rate for currency $c \in C$ for a term equal to the horizon,
$S_{0,c}$	initial (t=0) price of the stock index S_c in currency $c \in C_0$ (in units of the respective currency),
$\bar{X}_c$	the fixed exchange rate for value translation of quanto on the stock index S_c of foreign market $c \in C$ (usually, $\bar{X}_c = f_c, \quad \forall c \in C$,
K_j	the strike price of an option (in units of the respective currency in the case of a simple option $j \in JS_c,\ c \in C_0$, or in units of the base currency in the case of a quanto option $j \in JQ_c,\ c \in C$,
$cs(S_c, K_j)$	price of European simple call option $j \in JS_c$ on stock index S_c of country $c \in C_0$ with exercise price K_j and maturity T,
$ps(S_c, K_j)$	Price of European simple put option $j \in JS_c$ on stock index S_c of country $c \in C_0$ with exercise price K_j and maturity T,
$cq(S_c, K_j)$	price of European quanto call option $j \in JQ_c$ on the underlying $\bar{X}_c S_c$, of foreign currency $c \in C$ with exercise price K_j and maturity T,
$pq(S_c, K_j)$	Price of European quanto put option $j \in JQ_c$ on the underlying $\bar{X}_c S_c$, of foreign currency $c \in C$ with exercise price K_j and maturity T.

(b). Scenario dependent quantities:

- p_n obsective probability of occurrence of scenario $n \in N$,
- $\bar{p_n}$ a risk-neutral probability associated with scenario $n \in N$ (used in the option pricing procedure),
- e_c^n spot exchange rate of currency $c \in C$ at the end of the horizon under scenario $n \in N$,
- π_{ic}^n market price (in units of the respective currency) per unit of face value of security $i \in I_c$ at the end of the horizon under scenario $n \in N$,
- $S_{n,c}$ price of the stock index S_c in currency $c \in C$ at the end of the planning period, under scenario n.

All exchange rate parameters $(e_c, f_c, \bar{X}_c, e_c^n)$ are expressed in units of the base currency per one unit of the foreign currency $c \in C$. Of course, the exchange rate of the base currency to itself is trivially equal to one, $e_\ell = f_\ell = \bar{X}_\ell = e_c^n \equiv 1, \quad \forall n \in N$. The stock index prices S_c are expressed in units of the respective currency $c \in C_0$. The prices cs and ps of simple call and put options respectively, are expressed in units of the local currency $c \in C_0$. Conversely, the prices cq and pq of quanto call and put options, respectively, on foreign stock indices, are expressed in terms of the base currency ℓ.

Computed Parameters:

V_ℓ^0 total value (in units of the base currency) of the initial portfolio

$$V_\ell^0 = h_\ell + \sum_{i \in I_\ell} b_{i\ell}\, \pi_{i\ell} + \sum_{c \in C} e_c \left(h_c + \sum_{i \in I_c} b_{ic}\, \pi_{ic} \right). \tag{6.1}$$

Decision Variables:

(a). Asset purchase and sale decisions, and resulting holdings after portfolio revision:

- x_{ic} units of security $i \in I_c$ of currency $c \in C_0$ purchased,
- v_{ic} units of security $i \in I_c$ of currency $c \in C_0$ sold,
- w_{ic} units of asset $i \in I_c$ of currency $c \in C_0$ in the revised portfolio.

(b). Currency transactions in the spot market:

- x_c^e units of the base currency exchanged in the spot market for foreign currency $c \in C$,
- v_c^e units of the base currency collected from sale of foreign currency $c \in C$,

(c). Forward currency exchange contracts:

- u_c^f units of the base currency collected at the end of the period from forward sale of foreign currency $c \in C$. This transaction is decided at the beginning of the period (t=0).

(d). Variables related to option transactions:

$ncs(S_c, K_j)$	purchases of European simple call options $j \in JS_c$ on stock index S_c of currency $c \in C_0$, with exercise price K_j and maturity T,
$nps(S_c, K_j)$	purchases of of European simple put options $j \in JS_c$ on stock index S_c of currency $c \in C_0$, with exercise price K_j and maturity T,
$ncq(S_c, K_j)$	purchases of European quanto call options $j \in JQ_c$ on the underlying $\bar{X}_c S_c$ of foreign currency $c \in C$, with exercise price K_j and maturity T,
$npq(S_c, K_j)$	purchases of European quanto put options $j \in JQ_c$ on the underlying $\bar{X}_c S_c$ of foreign currency $c \in C$, with exercise price K_j and maturity T.

Unlike the holdings of assets for which short positions are not allowed, the variables corresponding to positions in options are not restricted in the optimization models. Hence, short sales of options are permitted as the variables above are allowed to take negative values. Short sales of options can raise additional funds for investments. However, such a case has not been observed in our empirical tests.

Auxiliary variables:

y_n	auxiliary variables used to linearize the piecewise linear function in the definition of CVaR,
z	the VaR value of portfolio losses (at a prespecified confidence level, percentile α),
V_ℓ^n	the total value of the revised portfolio at the end of the holding period under scenario $n \in N$ (in units of the base currency),
R_n	holding-period return of the revised portfolio under scenario $n \in N$,
$\bar{R}$	expected holding-period return of the revised international portfolio.

We investigate instances of the portfolio management model that incorporate either simple options or quantos, but not both. Hence, we need two slightly different model formulations that differ only in the expression of the cashflows. In the case of simple options, the cashflows (initial purchase of the options and collection of the payoffs at the end of the horizon) occur in the respective currencies. On the contrary, in the case of quanto options, all option-related cashflows (initial purchases as well as payoff collections at the end of the planning period) are in the base currency. Of course, when we use quanto options we also allow the purchase of options on the stock index S_ℓ in the base currency. (A quanto and a simple option on a stock index in the reference currency are of course equivalent, as $\bar{X}_\ell = 1$). We formulate now the two portfolio optimization models.

Portfolio Optimization Model with Simple Options

$$\texttt{min} \quad z + \frac{1}{1-\alpha}\sum_{n\in N} p_n y_n \tag{6.2a}$$

$$\begin{aligned}\texttt{s.t.} \quad & h_\ell + \sum_{i\in I_\ell} v_{i\ell}\pi_{i\ell}(1-\delta) + \sum_{c\in C} v_c^e(1-d) = \sum_{i\in I_\ell} x_{i\ell}\pi_{i\ell}(1+\delta) + \sum_{c\in C} x_c^e(1+d) \\ & + \sum_{j\in JS_\ell}[ncs(S_\ell,K_j) * cs(S_\ell,K_j) + nps(S_\ell,K_j) * ps(S_\ell,K_j)]\end{aligned} \tag{6.2b}$$

$$\begin{aligned}& h_c + \sum_{i\in I_c} v_{ic}\pi_{ic}(1-\delta) + \frac{1}{e_c}x_c^e = \sum_{i\in I_c} x_{ic}\pi_{ic}(1+\delta) + \frac{1}{e_c}v_c^e \\ & + \sum_{j\in JS_c}[ncs(S_c,K_j) * cs(S_c,K_j) + nps(S_c,K_j) * ps(S_c,K_j)] \quad \forall\, c\in C\end{aligned} \tag{6.2c}$$

$$\begin{aligned}& V_\ell^n = \sum_{i\in I_\ell} w_{i\ell}\,\pi_{i\ell}^n \\ & + \sum_{j\in JS_\ell}[ncs(S_\ell,K_j) * max(S_{n,\ell}-K_j,0) + nps(S_\ell,K_j) * max(K_j - S_{n,\ell},0)] \\ & + \sum_{c\in C}\Bigg\{u_c^f + e_c^n\Big[\sum_{i\in I_c} w_{ic}\pi_{ic}^n + \sum_{j\in JS_c}[ncs(S_c,K_j) * max(S_{n,c}-K_j,0) \\ & + nps(S_c,K_j) * max(K_j - S_{n,c},0)] - \frac{1}{f_c}u_c^f\Big]\Bigg\} \quad \forall\, n\in N\end{aligned} \tag{6.2d}$$

$$\begin{aligned}& \sum_{j\in JS_c}[ncs(S_c,K_j) * cs(S_c,K_j) + nps(S_c,K_j) * ps(S_c,K_j)] \le w_{kc}\pi_{kc}, \\ & \quad \forall\, c\in C_0\end{aligned} \tag{6.2e}$$

$$0 \le u_c^f \le \sum_{n\in N} p_n(e_c^n \sum_{i\in I_c} w_{ic}\pi_{ic}^n), \quad \forall\, c\in C \tag{6.2f}$$

$$R_n = \frac{V_\ell^n}{V_\ell^0} - 1, \quad \forall n\in N \tag{6.2g}$$

$$\bar{R} = \sum_{n\in N} p_n R_n \tag{6.2h}$$

$$\bar{R} \ge \mu \tag{6.2i}$$

$$y_n \ge L_n - z \qquad \forall\, n\in N \tag{6.2j}$$

$$y_n \ge 0 \qquad \forall\, n\in N \tag{6.2k}$$

$$L_n = -R_n \qquad \forall\, n\in N \tag{6.2l}$$

$$w_{ic} = b_{ic} + x_{ic} - v_{ic} \quad \forall\, i\in I_c, \quad \forall\, c\in C_0 \tag{6.2m}$$

$$x_{ic} \ge 0 \qquad \forall\, i\in I_c, \quad \forall\, c\in C_0 \tag{6.2n}$$

$$w_{ic} \ge 0 \qquad \forall\, i\in I_c, \quad \forall\, c\in C_0 \tag{6.2o}$$

$$0 \le v_{ic} \le b_{ic} \qquad \forall\, i\in I_c, \quad \forall\, c\in C_0 \tag{6.2p}$$

This formulation minimizes the CVaR risk measure (6.2a) of the portfolio losses at the end of the horizon, while constraining the expected portfolio return. The derivation of the objective function when minimizing the CVaR metric of portfolio losses was given in chapter 4.

Equations (6.2b) and (6.2c) impose the cash balance conditions in every currency; the former for the base currency ℓ and the latter for the foreign currencies $c \in C$. In each case we equate the sources and the uses of funds in the respective currency. Total availability of cash stems from initially available reserves, revenues from the sale of initial asset holdings and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of cash include the total expenditures for the purchase of assets and options and outgoing currency exchanges in the spot market. Note that the entire budget is placed in the available securities i.e., we don't have investments in risk-free interest rate (T-Bills), nor do we have borrowing. These could be simple extensions of the model. Note also that linear transaction costs are charged for transactions of assets as well as for currencies. In order to simplify the notation and the formulation, all currency transactions are made through the base currency. Without loss of generality, we do not allow direct transactions between foreign currencies. When reducing holdings in one asset in favor of assets in a different currency, the proceeds from the sale of assets are transfered between respective currencies always through an intermediate conversion to the base currency.

The final value of the portfolio under scenario $n \in N$ is computed in (6.2d). This equation expresses the total terminal value of the portfolio in units of the base currency. The total terminal value reflects the proceeds from the liquidation of all asset holdings at the corresponding market prices, the option payoffs and the proceeds of forward contracts in foreign currencies. The proceeds of investments in foreign markets (assets and option payoffs) are converted to the base currency by applying the respective spot exchange rates at the end of the horizon, after accounting for outstanding currency forward contracts.

Constraints (6.2e) limit the total expenditure for purchases of simple options in each currency. This expenditure is not permitted to exceed the value of the position in the corresponding stock index. This constraint is imposed in order to ensure that options maybe purchased so as to cover the exposure in the underlying stock index (i.e., for the intended hedging purpose) and not for speculative purposes. Similarly, constraints (6.2f) restrict the forward contracts that are used to hedge the currency risk to be up to the expected value of all assets in the respective foreign currency. We can exclude currency hedging decisions by eliminating the variables for the currency forwards contracts (i.e, setting $u_c^f = 0, \ \forall c \in C$).

Equation (6.2g) defines the return of the portfolio at the end of the horizon under scenario $n \in N$. Equation (6.2h) defines the expected return of the portfolio at the end of the horizon, while equation (6.2i) imposes a minimum target bound (μ) on the expected portfolio return. Constraints (6.2j), and (6.2k) are the definitional constraints for determining `CVaR`, while equation (6.2l) is the definition of portfolio loss as the negative return. Equation (6.2m) enforce balance constraint for each asset, in each market. These equations determine the resulting composition of the revised portfolio after the purchase and sale transactions of assets. Short positions in assets are not allowed. Constraints (6.2n) and (6.2o) ensure that the units of assets purchased, as well as the resulting units in the rebalanced portfolio are nonegative. Finally, constraints (6.2p) restrict the sales of each asset by the corresponding initial holdings.

Portfolio Optimization Model with Quanto Options

$$\text{min} \quad z + \frac{1}{1-\alpha}\sum_{n\in N} p_n y_n \tag{6.3a}$$

$$\begin{aligned}\text{s.t.} \quad & h_\ell + \sum_{i\in I_\ell} v_{i\ell}\pi_{i\ell}(1-\delta) + \sum_{c\in C} v_c^e(1-d) = \sum_{i\in I_\ell} x_{i\ell}\pi_{i\ell}(1+\delta) + \sum_{c\in C} x_c^e(1+d) \\ & + \sum_{j\in JS_\ell} [ncs(S_\ell,K_j) * cs(S_\ell,K_j) + nps(S_\ell,K_j) * ps(S_\ell,K_j)] \\ & + \sum_{c\in C}\left[\sum_{j\in JQ_c} [ncq(S_c,K_j) * cq(S_c,K_j) + npq(S_c,K_j) * pq(S_c,K_j)]\right]\end{aligned} \tag{6.3b}$$

$$h_c + \sum_{i\in I_c} v_{ic}\pi_{ic}(1-\delta) + \frac{1}{e_c}x_c^e = \sum_{i\in I_c} x_{ic}\pi_{ic}(1+\delta) + \frac{1}{e_c}v_c^e \quad \forall\, c\in C \tag{6.3c}$$

$$\begin{aligned} V_\ell^n = & \sum_{i\in I_\ell} w_{i\ell}\,\pi_{i\ell}^n \\ & + \sum_{j\in JS_\ell} [ncs(S_\ell,K_j) * max(S_{n,\ell}-K_j,0) + nps(S_\ell,K_j) * max(K_j - S_{n,\ell},0)] \\ & + \sum_{c\in C}\left[\sum_{j\in JQ_c} [ncq(S_c,K_j) * max(\bar{X}_c S_{n,c}-K_j,0) + npq(S_c,K_j) * max(K_j-\bar{X}_c S_{n,c},0)]\right] \\ & + \sum_{c\in C}\left[u_c^f + e_c^n[\sum_{i\in I_c} w_{ic}\pi_{ic}^n - \frac{1}{f_c}u_c^f]\right] \quad \forall\, n\in N\end{aligned} \tag{6.3d}$$

$$\sum_{j\in JQ_c} [ncq(S_c,K_j) * cq(S_c,K_j) + npq(S_c,K_j) * pq(S_c,K_j)] \le e_c w_{k_c}\pi_{k_c}, \quad \forall\, c\in C \tag{6.3e}$$

$$\sum_{j\in JS_\ell} [ncs(S_c,K_j) * cs(S_\ell,K_j) + nps(S_\ell,K_j) * ps(S_\ell,K_j)] \le w_{k\ell}\pi_{k\ell}, \tag{6.3f}$$

and also (6.2f), (6.2j), (6.2k), (6.2g), (6.2h), (6.2i), (6.2n), (6.2o), (6.2p).

This optimization model differs from the previous one primarily in the constraints that account for the cashflows, as the transactions of the quantos and their payoffs are now in the base currency. Equation (6.3b) imposes the cash balance condition for the base currency ℓ. Again, total availability of cash stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. The uses of cash include the total expenditures for the purchase of assets and simple options on the base stock index, the purchases of quanto options on foreign stock indices, and outgoing currency exchanges in the spot market. Equation (6.3c) imposes the cash balance constraints in foreign currencies $c \in C$. In this case, cash is used only for purchases of assets and outgoing currency exchanges in the spot market; in the case of quanto options there are no purchases of options in foreign currencies.

The final value of the portfolio under scenario $n \in N$ is computed in (6.3c). The total value of the revised portfolio at the end of the horizon accounts for the following: the proceeds from the liquidation of all assets (domestic and foreign) at the respective asset prices, the payoff of the option on the domestic stock index, the payoffs of quantos on foreign stock indices and the currency forwards contract. Again, the proceeds of foreign investments are valued in terms of the base currency by employing the applicable spot exchange rate at the end of the horizon, and after accounting for outstanding currency forward contracts.

Finally, constraints (6.3e) and (6.3f) limit the maximum expenditure for quantos in each foreign market, and simple options in the base currency respectively, to the value of the position in the respective stock index.

Quantos are integrative instruments that can hedge the market risk related to the movements of the underlying assets, jointly with the currency risk. In the stochastic programming models developed above, quanto as well as simple option prices are given as inputs. The pricing method used to value these options is given in Appendix.

6.2.2 Scenario Generation

The scenario generation is a critical step for the entire modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution in the values of the random variables (asset returns and exchange rates) and is consistent with market observations and financial theory. We employ an appropriate scenario generation method that conforms to fundamental financial principles. If the scenario generation does not satisfy non-arbitrage conditions then the solutions of the stochastic programs will be biased and will imply unattainable spurious profits. We generate scenarios such that the arbitrage free conditions are met in the scenario set, and we explicitly test for this with the procedure described in section 5.3.

We used the MSCI monthly return data for the stock indices of USA (USS), Great Britain (UKS), Germany (GRS) and Japan (JPS), and for spot and monthly forward exchange rates (UKtoUS, GR-toUS, JPtoUS). We also used return data for Government bond indices from Datastream International. We use Government bonds with three different maturity bands: 1-3 years (US1, UK1, GR1, JP1), 3-7 years (US3, UK3, GR3, JP3) and 7-10 years (US7, UK7, GR7, JP7). We collected 163 months of data that span the time period 05/1988 through 11/2001.

We analyzed the statistical characteristics of the historical data covering the period 05/1988–11/2001. As we can see from Table 6.1, both the domestic returns of the indices and the proportional changes of spot exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. In addition, several of the random variables exhibit excess kurtosis, implying heavier tails than the normal distribution. Jacque-Bera tests on these data indicated that the normality hypotheses can not be accepted for the majority of them.[1] Thus, we need to generate scenarios for asset returns and exchange rates that comply with historical observations, without relying on the normality assumption.

We rely solely on the observed asset returns and exchange rates in the market. We estimate the first four moments and correlations of the assets returns and proportional changes of exchange rates,

[1]The Jacque-Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jacque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

and we generate scenarios based on these statistical characteristics.

We used the scenario generation method developed by Høyland and Wallace [95] and Høyland et al. [93]. The scenarios are generated so that the first four marginal moments and correlations of the random variables match their historical values. Applying this scenario generation method on historical values of the random variables we obtain scenarios for the following quantities:

r_{ic}^n monthly return (in domestic terms) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$ under scenario $n \in N$,

g_c^n proportional change of spot exchange rate of currency $c \in C$ over one month under scenario $n \in N$.

The resulting scenarios are equiprobable in our tests, but generally they don't have to be.

Using these scenarios we compute the asset prices $\hat{\pi}_{ic}^n$ and exchange rates $\hat{e}_c^n$ for each scenario $n \in N$ as follows:

$$\hat{\pi}_{ic}^n = (1 + r_{ic}^n) \times \pi_{ic}, \quad \forall i \in I_c, \quad \forall c \in C_0 \quad \forall n \in N \tag{6.4}$$

$$\hat{e}_c^n = (1 + g_c^n) \times e_c, \quad \forall c \in C, \quad \forall n \in N \tag{6.5}$$

6.3 Hedging Strategies

From the statistics of the random variables in Table 6.1 we observe that the stock indices and the exchange rates exhibit higher volatilities than the bond indices; they also exhibit heavier than normal tails. This justifies our decision to focus in this study on appropriate instruments and strategies for hedging the stock market risks and the currency risks. The portfolio optimization models of the previous section aim exactly at controlling these risks either separately or jointly.

The models can flexibly accommodate alternative investment strategies. By setting the variables $u_c^f = 0$, $\forall c \in C$ we disallow currency hedging through forward exchange contracts. Similarly, by setting all the variables (ncs, nps) associated with purchases of options equal to zero in model 6.2 we disallow the decisions for hedging the market risks of positions in stock indices. To consider totally unhedged portfolios we eliminate from model 6.2 all the hedging decisions (i.e., option purchases and currency forward exchanges) by fixing their values equal to zero. In this manner, the optimization models provide the means to study the effect of strategies for controlling market risks and currency risks either integrally or in isolation.

Certain strategies that concern specific combinations of options in hedging applications are popular in practice. Here we consider the following option trading strategies: Straddle, Strip, Strap, and Strangle. All of these strategies involve combinations of long positions in call and put options. Each of them generates a different payoff profile. The choice among them is based on the investor's view regarding potential movements in the value of the underlying and his preferences for protection in the case of such movements.

Straddle strategy

This strategy suits investors who espouse the idea to "exit the market in the face of a storm//. It is designed to provide protection in the event of increased volatility and yields payoffs if there is a substantial movement in the price of the underlying security, either downside or upside. A long straddle consists of long positions in one call and one put option on the same underlying asset, at the same strike price, and with the same expiration date - the length of the planning horizon in our case. The buyer pays a premium on the call and a premium on the put in order to have the right, but not the obligation, to buy, or sell, a predetermined amount of the underlying asset (the number of the option contracts) at the exercise price. An investor enters a long straddle position when he anticipates an increase in volatility but is unsure about the direction of the movement, yet he wishes to be covered in the event of sharp changes in the price of the underlying in either direction. For a long straddle position to yield a profit, the underlying stock index must swing sufficiently low or high so as to cover the total cost of the option purchases. There is a linear profit against the movement of the stock index in either direction once the cost of the option premiums is cleared. If the price of the underlying stock index remains close to the strike price of the options the straddle results in a loss; the maximum loss is the sum of the purchase prices of the call and put options that constitute the straddle. However, in the event of a large movement of the stock index in either direction the straddle yields a substantial profit. Thus, the loss from a potential large decrease in the stock index is recovered from a long position in the straddle.

Strip strategy

A long strip consists of a long position in one call and two puts with the same exercise price and expiration date. An investor enters into a long strip position when he expects a large move in the stock index and considers a decrease in the index more likely than an increase. So, the investor again buys protection against large swings in the underlying stock index but gives preferential weight towards coverage against downward moves. Again, for a limited range of changes around the strike price the strip strategy results in a loss. But for large swings in either direction it yields a positive payoff; the payoff is twice as steep for downward moves than for upward moves.

Strap strategy

In a sense, the strap strategy is a mirror image of the strip strategy. A long strap consists of a long position in two calls and one put with the same exercise price and expiration date. An investor enters into a long strap position when he expects large moves in the stock index but considers an increase in the index more likely than a decrease. Again the investor reaps a profit in the event of a sharp movement of the stock index in either direction, but gives preferential emphasis to gains from potential upside moves.

The constituent options in these three strategies are typically at-the-money options; that is, their strike price is equal to the current value of the underlying index. Of course the closer that the strike price of a call or a put option is to the current value of the underlying asset, the more expensive the option is. Hence, these strategies are rather costly. Obviously, the strip and strap are costlier than the straddle as they involve an additional option.

Strangle strategy

Similar to the straddle, a long strangle consists of a long call and a long put option on the same underlying asset and with the same expiration date; but in a strangle the two options have different exercise prices. The strike price of the put is lower than the current price of the underlying, while that of the call is higher than the current price of the underlying. Again, a long straddle yields a profit when there is a substantial move in the stock index in either direction. The index must move farther in a strangle than in a straddle for the strategy to yield a profit. But the downside risk if there is only a small change in the value of the stock index is less with the strangle than with the straddle because the strangle is a cheaper alternative than the straddle as the prices of its constituent options are lower than those in the strangle. With a long strangle, an investor buys coverage against large movements in the stock index in either direction; that is, he covers against volatility. The payoff pattern resulting with a strangle depends on how close together the strike prices of the constituent call and put options are; if both of these strike prices approach the current price of the underlying stock index then the payoffs of the resulting strangle resemble those of the straddle. The farther apart the strike prices of the constituent options are the lower the cost of the strategy, but the farther the stock index must move for the strategy to realize a profit.

The payoff patterns of the four option strategies are illustrate in Figure 6.1. We investigate the effectiveness of all these strategies for risk hedging purposes in international portfolio management. In our tests, we design the strangle strategy by setting the strike price of the constituent call option, respectively the price of the put option, to a level 5% higher, respectively 5% lower, than the current value of the underlying stock index.

6.4 Empirical Results

Having implemented appropriate tools to support systematic analysis, in this section we investigate the effectiveness of alternative strategies to control the most influential risk factors for the international portfolios; namely, the market risks of investments in stock indices (domestic and foreign) and the currency risk. We apply the optimization models so as to study the effects of the following investment strategies:

1. Totally unhedged portfolios. Optimal internationally diversified portfolios are determined in this case, but without any explicit regard to cover against either market or currency risk (i.e., neither options nor currency forward exchange decisions are considered).

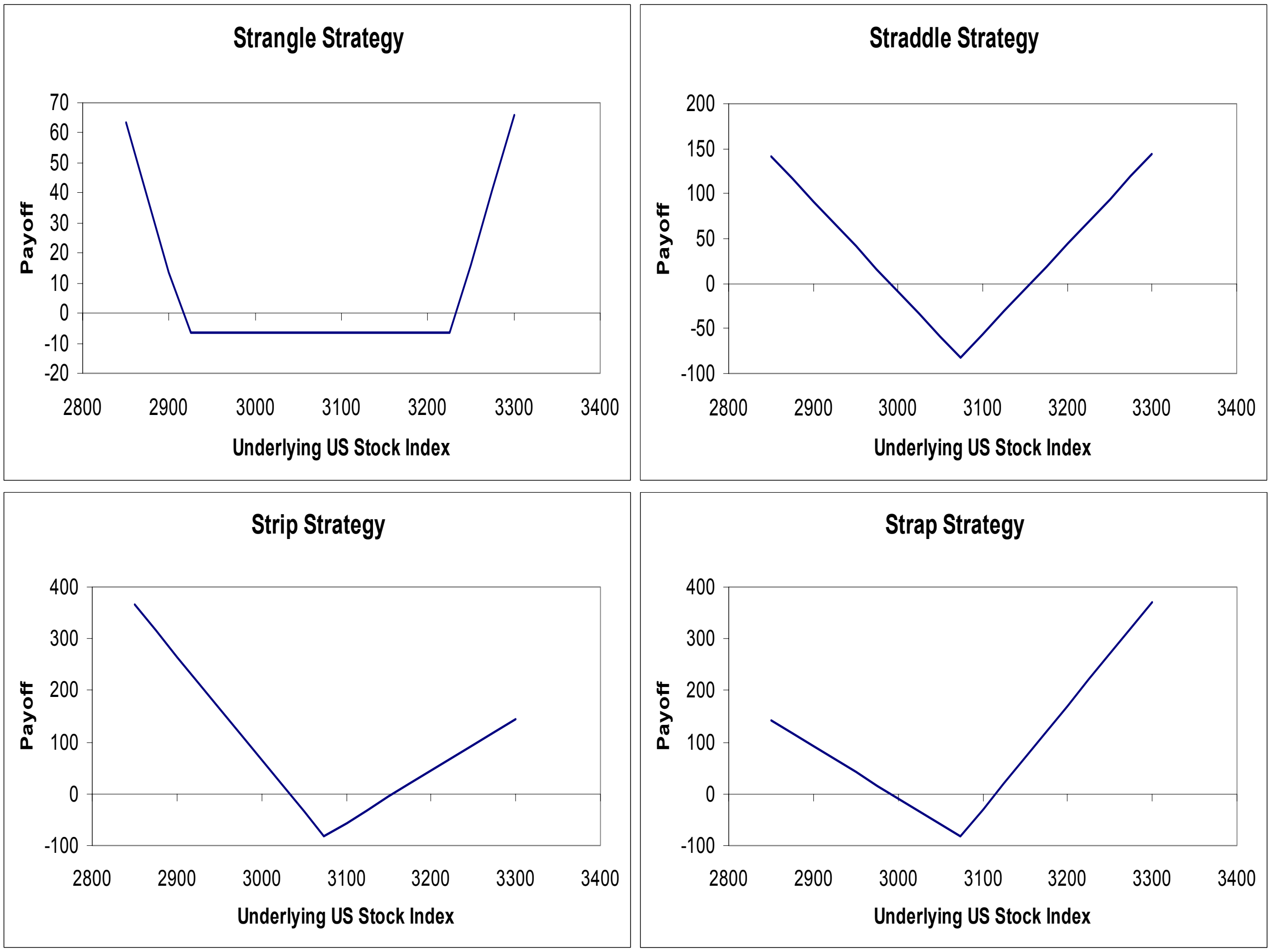


Figure 6.1: Payoff patterns of the option strategies.

2. Control of currency risk only by allowing currency forward exchanges.

3. Control of market risks only by incorporating in the portfolio optimization models simple options on the stock indices.

4. Joint protection against market risk with the use of simple options and against currency risk through currency forward contracts.

5. Use of the integrative options (quantos) to jointly protect against both market and currency risks of positions in stock indices, and coverage of the residual currency risk of the other foreign investments in bond indices through currency forward contracts.

The first strategy constitutes the basic benchmark against which the other strategies that incorporate risk hedging decisions are compared in order to determine the incremental benefits from risk hedging decisions. The second and third cases address separately each of the primary risk factors in order to assess the impact of controlling each type of risk in an international portfolio. The last two cases examine the effectiveness of alternative means for addressing both market and currency risks in an integrated manner. By comparing the results in these cases against those for the previous two cases, we can determine the incremental benefit from controlling both types of risk instead of just either of them alone. In the cases that involve consideration of options - either simple options or quantos - we apply the four option trading strategies that were discussed in the previous section. We study the performance of all these investment strategies both in static as well as in dynamic tests.

Previous empirical works on risk hedging strategies for international portfolios of financial assets considered separately the portfolio structuring decisions and the hedging decisions. For example, previous studies on the impact of currency hedging decisions for international investment portfolios have used either a unitary hedge ratio across all currencies, or currency overlays. In these studies, the currency hedging strategy is either predetermined at the stage of optimal portfolio selection, or the hedging decisions are taken separately following the selection of an international diversified portfolio. This separate consideration of international portfolio management and risk hedging decisions is also the norm in practice. It should obviously be advantageous to optimally determine these interrelated decisions in a single unified framework, as we do in this study.

To our knowledge, the incorporation of selective hedging decisions for both the market and currency risks in the context of normative models for optimal international portfolio management is an entirely novel contribution. The introduction of options as risk hedging instruments in the portfolio optimization procedure is also novel. Here it is additionally considered, in a unified framework, in conjunction with currency forward exchanges. These novel developments provide effective means to address in an integrated manner the problem of optimally managing internationally diversified portfolios while controlling their total exposure to both market and currency risks.

6.4.1 Static Tests

In these tests we examine the in-sample performance of the alternative investment strategies within a postulated scenario set for the random variables. Hence, we investigate the results of a single portfolio selection problem at a specific point in time. Specifically, we consider the international portfolio structuring problem on March 2001; the results at this specific time are typical of the performance of the alternative investment strategies. In the static tests we consider portfolio selection problems that start with a cash endowment in the base currency (USD) only; the initial portfolio does not have any holdings in any other asset.

The setup of the numerical experiments is as follows. The historical data of monthly asset returns (in domestic terms) and corresponding proportional changes in spot exchange rates over the previous ten years (120 observations) are used to determine the statistics of these random variables (i.e., their first four marginal moments and their correlations). These statistics are used to calibrate the scenario generation procedure. Using the moment matching scenario generation procedure of Høyland et al.[93] (see also Høyland and Wallace[95]) we generate 15000 scenarios that capture in terms of a discrete distribution the joint variation of these random variables. On the basis of these scenarios we compute the corresponding payoffs for the simple options and for the quantos under each scenario. The procedures described in Appendix are then applied to price the corresponding options. In this manner we have all the necessary inputs for the stochastic optimization models of section 6.2.

For each of the investment strategies we described at the beginning of this section, we solve the corresponding parametric stochastic optimization model for several different values of target expected return () and we record the results. Thus we trace the risk-return tradeoff profile (efficient frontier) of each strategy for the postulated scenario set. As a measure of risk we use the CVaR of portfolio losses at the 95% threshold.

Figure 6.2 presents the efficient frontiers of optimal international portfolios of stock and bond indices for several investment strategies. We observe that the efficient frontier of totally unhedged portfolios is dominated by all the other efficient frontiers that correspond to risk hedging strategies. Hence, improvements in the risk-reward tradeoffs - in terms of higher expected return for the same level of risk, or conversely, reduced risk for the same level of expected portfolio return - are attained when risk hedging strategies are incorporated in the international portfolio optimization model. This holds for all risk hedging strategies we tested. For the strategies that employ options on stock indices (either simple options or quantos) we present in Figure 6.2 only the results for the strangle strategy. As we demonstrate subsequently, the strangle strategy proved the most effective among the four option trading strategies we tested.

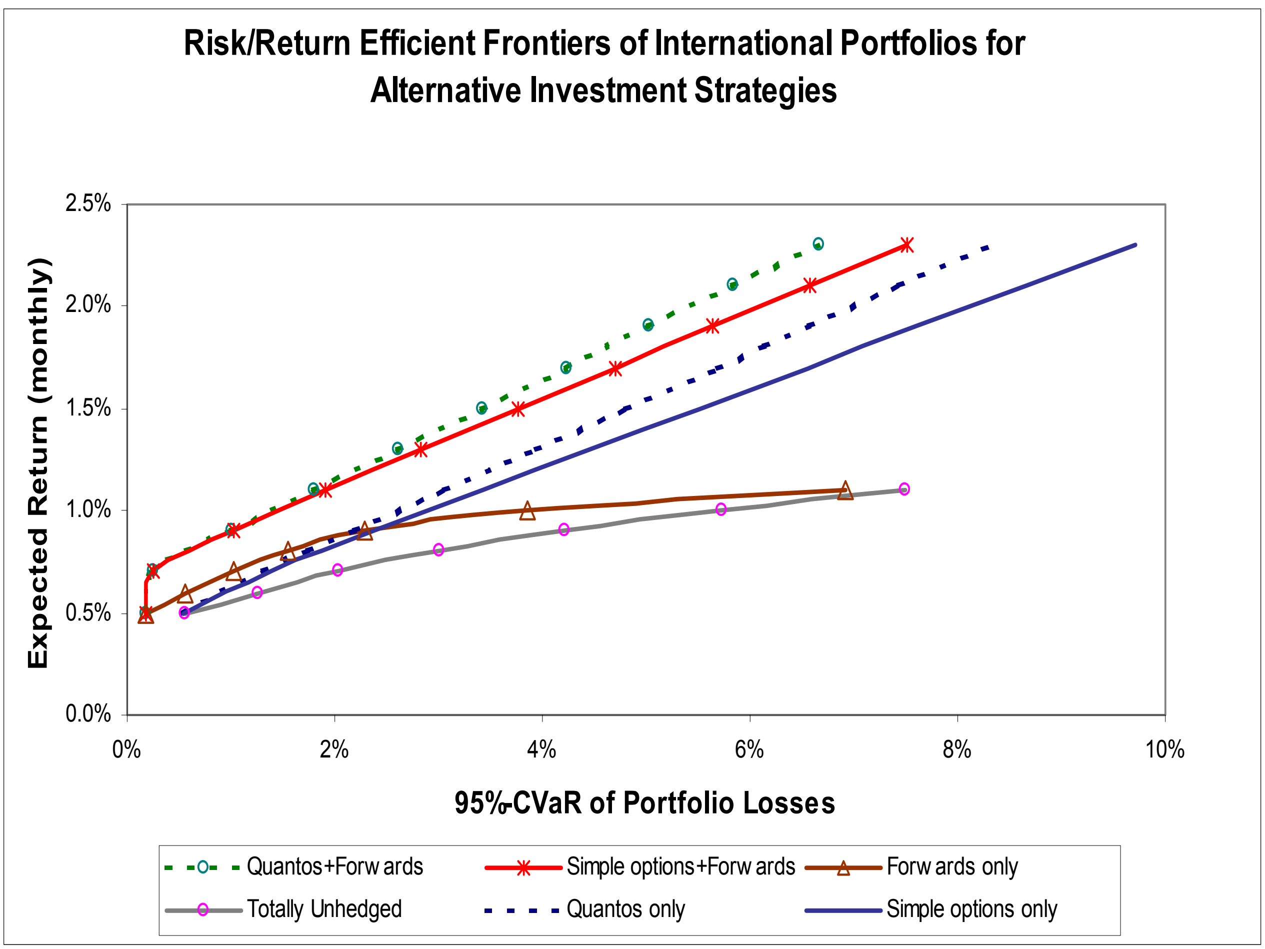


Figure 6.2: Efficient Frontiers of CVaR-optimized international portfolios of stock and bond indices with alternative risk hedging strategies.

We note that the strategies that address the market risk of stock indices (via simple options or quantos) result in an upward rotation of the efficient frontier. The minimum risk portfolio is anchored at the same point in these cases. This is because the optimal minimum risk portfolio solely contains holdings in bond indices, and thus the options do not enter the picture as there are no holdings in the stock indices. However, as higher expected returns are targeted the optimal portfolios gradually switch their positions to stock indices, and in these cases the incorporation of options in the optimal portfolios has a pronounced effect. By controlling the market risk of positions in stock indices through the optimal use of options, significantly higher expected returns can be achieved for the same level of total portfolio risk.

When viewing the impact of risk control of the two major risk factors (market risk of stock indices and currency risk) in isolation, we observe that controlling the market risk is increasingly important for the more aggressive strategies that target higher expected returns. However, for conservative investors controlling the currency risks of all foreign investments yields important benefits. The efficient frontier is shifted to the left when currency risks are addressed through the use of currency forward exchanges, thus reducing the total portfolio risk at each level of target expected return. This includes the minimum risk portfolio as well, as currency forward exchanges are used to cover the currency risk of holdings in foreign bond indices in this case.

The efficient frontiers of strategies that jointly address both market risks of stock index positions through options and currency risks of all foreign investments through currency forward exchanges exhibit the combined effect of the strategies that control just one of these risk factors; that is, both the leftward shift and the upward rotation of the efficient frontiers. Hence, the joint consideration of the market and currency risks in an integrated manner yields significant incremental benefits over the consideration of either of these risk factors in isolation. Lastly, we observe in Figure 6.2 that the use of quantos produces better results than the use of simple options to cover the market risks of stock holdings. This holds true regardless of whether or not currency forward exchanges are used to cover the currency risks of foreign investments, and the benefits are higher in the range of higher expected portfolio return (i.e., for more aggressive investments). The reason is that although both types of options protect against the market risks of stock indices, the quantos additionally protect the investments in these indices against the currency risks. As the proportion of holdings in the stock indices is increased in more aggressive portfolios, the incremental benefit from this additional risk coverage becomes increasingly evident.

Figure 6.3 contrasts the efficient frontiers of strategies that incorporate alternative combinations of options in the portfolios (i.e., staddle, strip, strap and strangle). This figure presents the results for optimal portfolios with simple options as well as those with quanto options. In these tests, selective forward currency exchanges are also used to cover the currency risks. The results indicate that the incorporation of options in the investment opportunity set leads to optimal portfolios with improved efficient frontiers in comparison to unhedged portfolios, regardless of the option trading strategy that is used. Hence, the use of options as hedging instruments in the international portfolios results in substantial improvement of their risk-return profiles. The strangle strategy produces the highest benefits.

Lastly, we examine the effects of option strategies on the distributional characteristics of international portfolios. Figure 6.4 shows the in-sample distributions (i.e, with respect to the postulated scenario set that is used in the optimization models) of returns of optimal portfolios that employ specific risk hedging strategies. Specifically, this figure shows the in-sample distributions of returns for optimal unhedged portfolios, and optimal portfolios that use quanto options and forward currency exchanges to control total risk exposure. We note that the minimization of the CVaR risk metric on portfolio losses - that is used as the objective function in the optimization models - aims to generate portfolios that minimize the downside risk; that is, that have a reduced lower tail of their return distribution. The results of Figure 6.4 highlight exactly two distinguishing characteristics of the optimal optioned portfolios in comparison to optimal unhedged portfolios, regardless of the option trading strategy: (a) the substantial reduction of the downside risk of the portfolio, as evidenced from the reduced left tail of the return distribution, and (b) the more positively skewed return distribution of optimal optioned portfolios. These characteristics are consequences of the asymmetric payoff profiles of the option trading strategies. The protection of the put options against downside risk result in the reduction of the lower tail of the portfolios' return distribution. Similarly, the call options included in the strategies tested induce upside potential as they produce payoffs in the event of upside market moves. These contribute, at least partly, to the generation of more positively skewed return distributions. The strangle strategy produces somewhat more positively skewed portfolio return distributions than the other option trading strategies.

6.4.2 Dynamic Tests

So far we have studied the performance of alternative investment strategies in static tests. That is, we have examined the potential benefits of risk hedging strategies at a single (typical) point in time and against an in-sample discrete distribution (scenario set). Although these tests provide useful insights, they are not sufficient to convincingly compare the relative effectiveness of the alternative strategies for managing international portfolios of financial assets. To this end, in this section we conduct extensive dynamic tests in order to have more substantive comparisons of actual performance. These tests repeatedly apply the models in backtesting experiments using real market data on a rolling horizon basis during the period 05/1998 to 11/2001 (i.e., 43 months).

The simulations start on May 1998 with an initial cash endowment in the investor's reference currency (USD); the initial portfolio does not contain holdings in any other assets. At each month we use the historical observations of the asset returns and spot exchange rates during the previous 10 years in order to calibrate the scenario generation procedure as we explained in the previous section. The moment matching method is then applied to produce 15000 scenarios of asset returns (in domestic terms) and corresponding differentials of spot exchange rates. These scenario-dependent data are used to compute the projected final asset prices, the spot exchange rates and the respective option payoffs at the end of a one-month planning horizon under each scenario. The options are priced on the basis of the postulated scenario set. The scenario-based data are used in conjunction with the observed asset prices, spot and forward exchange rates and the option prices as inputs in the optimization model. The optimization model is solved and the optimal portfolio composition, as well as the forward currency transactions and option purchases, when applicable, are recorded. The clock is then advanced one month, and the actual asset prices and spot exchange rates are revealed. On the basis of these new observations, accounting calculations are carried out so as to settle outstanding investment decisions of the previous month. That is, the currency forward exchanges are settled, as well as potential execution of positions in options, and the resulting cash positions are updated accordingly. Starting with the new portfolio, the procedure is repeated for the next month. A new set of scenarios is computed and the optimization model is resolved using the data of the new scenario set and the new portfolio composition. The ex post realized returns are cumulatively recorded throughout the simulation that runs through November 2001.

These backtesting experiments were executed for several investment strategies that were discussed earlier. In these tests we use a target monthly return ($\mu = 1\%$) so as to induce the optimization models to include positions in stock indices. For conservative targets of expected returns (e.g., the minimum risk case with $\mu = 0\%$) the optimal portfolios are almost exclusively placed in bond indices and thus the options would have no effect.

Figure 6.5 contrasts the ex post realized performance of optimal portfolios for alternative risk hedging strategies. We observe that the totally unhedged portfolios exhibit considerable volatility, as well as declining performance from the beginning of the year 2000. When only currency risk is hedged with forward exchanges the portfolios achieve a much more stable performance; in this case, the portfolios generally produce small stable returns. A severe loss, is experienced only during the crisis of September 11, 2001, but is followed by a quick recovery. So, the consideration of only the currency risk results in much more stable returns, but does not yield substantial incremental profits, at least over this simulation period. On the contrary, we observe substantial additional profits from the outset and throughout the entire simulation period when options are introduced so as to manage the market risks of stock index positions, in addition to currency risk hedging through forward currency contracts. Thus, the impact of the market risk management strategies is substantial. This should be expected as the stock indices exhibit the highest volatility of all investment instruments in the portfolios. The results demonstrate the clear benefits from the joint management of both the market risks and the currency risks. The optimally hedged portfolios again suffer their most severe loss at the time of the September 11, 2001 crisis, but again they recover quickly.

We observe that quantos in conjunction with currency forwards that cover, at least partly, the residual currency risk of investments in bond indices in the portfolios achieve slightly better ex post performance than the portfolios with simple stock options and currency forwards. The difference between the two strategies is small because the investments in stock indices that are picked by the model are much higher in the US stock index (i.e., the base currency) for which currency risk does not play a role, than in foreign stock indices. Their small difference is due to some positions in quantos to cover exposure to the German stock index.

The results indicate that the rewards from the option payoffs, especially in the event of volatile markets, more than offset the cost for buying these options for risk hedging purposes. Note, for example, how the performance of the portfolios is tempered during the periods at which the corresponding unhedged portfolios suffer substantial losses. Moreover, the option trading strategies that have been tested in this study produce additional rewards in the event of large upside moves of the stock indices because of the long positions in call options. Hence, the gains in cases of significant market upswings can be further accentuated.

Figure 6.6 compares the performance of optimal optioned portfolios corresponding to the four option trading strategies. Currency risk is hedged through forwards in these tests. Clearly, the portfolios that contain options on stock indices - whether simple options or quantos - outperform the portfolios that do not contain options (i.e., do not hedge market risks). These results confirm that the inclusion of options as hedging instruments in the portfolios leads to improved performance, irrespective of the option selection strategy that is employed, as the static tests had implied. Among the alternative option trading strategies, strangle yields the best performance. The relative performance of the alternative investment strategies in the backtesting experiments conforms to their respective relative performance in the static tests of the previous section.

The results in Figure 6.5 and 6.6 show that unhedged portfolios achieve the worst performance in the backtesting simulations (lowest cumulative return and highest volatility). Benefits are attained with strategies that control risk exposures, by leading to more stable return paths or, additionally, to higher realized returns. The introduction of options on stock indices, as means to manage market risks, clearly produces substantial benefits in terms of higher cumulative returns. The additional use of currency forwards so as to hedge currency risks yields further incremental benefits and more stability in the realized returns. The overall observation is that superior performance is achieved when both market and currency risks are jointly controlled through appropriate instruments. Portfolios with the integrative options (quantos) demonstrate better performance than corresponding portfolios with simple options on the stock indices, when currency risks are also hedged. These results confirm the anticipated improvements in the performance of international portfolios of financial assets when appropriate strategies for controlling market and/or currency risks are incorporated in the decision process. Moreover, the results illustrate the benefits achieved from the joint control of market and currency risks, and thus demonstrate the advantages of models that take a holistic view towards addressing the total risks from all factors in international portfolio management within a single integrative framework.

We also look at the total volume of portfolio turnovers - expressed in Table 6.2 in terms of the total transaction cost (in USD) for asset purchases and sales (excluding options) during the simulation period 05/1998 - 11/2001. We observe that the total turnover of optimal optioned portfolios is lower than with portfolios without options, indicating a greater stability in the portfolio compositions over time. We have used the following values for the transaction cost parameters: transaction cost for asset purchases and sales $\delta = 0.05\%$, transaction cost for spot currency exchanges $d = 0.01\%$.

Figure 6.7 shows the investments on stock indices and the positions on associated stock options on these indices (that form the strangle strategy) decided by the model. We observe that the model picks US and German stock indices. Moreover, in depreciation periods of the markets (e.g., September 11th), the model decides a large position in options to cover the losses associated with positions in the underlying stock indices.

Figure 6.8 shows the degree of currency hedging in each country (% of foreign investments hedged) when the strangle strategy is used. We observe that when the model decides to invest in a foreign market then most of the times the model also (almost) fully hedges the currency risk of these investments.

Finally, we examine how the results may be affected by the decision maker's risk aversion preferences. We use the parameter for the target expected return () as a proxy measure of risk aversion. Higher levels of target return imply lower risk aversion and lead to more aggressive investments (i.e., selection of riskier stock indices instead of bond indices because of their higher return potential). All previous tests in this section used a monthly return target =1.0% in the optimization models, which is a rather aggressive target. Figure 6.9 shows the realized returns of optimal portfolios for different values of target return. These portfolios use quanto options (with the strangle strategy) and currency forwards to control the market and currency risks, respectively.

For the target return μ=1%, the portfolios in the period of August-September 2001 involved a large position in the US stock index and smaller positions in intermediate- and long-term US government bond indices. Given the large position of the portfolio in the US stock market, it suffered a substantial loss at the time of the September 11 events, even though this loss was subsequently recovered quickly. On the contrary, for more conservative values of the target return the portfolios hold substantial proportions in government bond indices throughout the simulation period and only occasionally include small positions in stock indices. For the minimum risk case (μ=0%) the portfolio at the time of September 11, 2001, included a large allocation in the short-term US government bond index, smaller allocations in the German and Japanese stock indices, and some other smaller positions in other assets. As the returns of the bond indices were not materially affected by the events of September 11, this portfolio escaped any losses at that time, and even achieved gains during and after those events, as can be seen in the results of Figure 6.9. We observe that the minimum risk portfolios (μ=0%) exhibit the more stable returns; generally higher returns are achieved in more aggressive cases (i.e., as the value of target returns is increased) but at the expense of higher volatility.

6.5 Conclusions

In this study we developed and implemented scenario-based optimisation models to control market and currency risk exposures of portfolios of international stock and bond indices. We employed a moment matching procedure to generate scenarios for the underlying random variables (index returns and exchange rates); the scenario generation procedure was calibrated on the basis of historical observations.

Options on the stock indices served as the instruments for hedging market risks, while currency forwards served as the means for controlling currency risks. Additionally, quanto options were considered in order to jointly cover market and currency risk exposures of positions in stock indices. In order to incorporate the options in the stochastic programming models we adapted an option pricing procedure so as to price the options consistently with the postulated scenarios for the underlying assets, while at the same time satisfying the fundamental no-arbitrage condition.

The optimization models provided the necessary tools so as to investigate alternative investment strategies for managing risks in international portfolios. We studied the performance of alternative strategies in extensive empirical tests, both in a static as well as in a dynamic setting.

Empirical results indicate that the market risk affects more than the currency risk the performance of the portfolios, and hedging this risk using options is extremely beneficial. Both static as well as dynamic tests show that increasingly integrated views towards risk are more effective compared to unhedged positions. We observe that hedging both the market and currency risk really does pay. Regardless of the trading strategy of options that is selected, the incorporation of options in international portfolios improves significantly the performance of these portfolios. Although, it is not clear enough which of the alternative options (simple of quantos) is indeed more preferable, since their performance is almost the same, in all cases quantos exhibit slightly better results than simple options, and never worse. Thus, integration of market and currency risk into portfolios hedging strategies does pay. Finally, among the trading strategies of options, portfolios with strangle strategy yields the best ex ante and ex post performance.

Extensions to this approach are possible to allow portfolio managers to adjust this procedure to their specific investment circumstances. First of all, the choice of the risk measure that is used as well as the choice of the scenario generation procedure, are made for illustration purposes. The modelling framework we develop in this paper is not restricted to these choices. Alternative risk measures or utility functions, can be optimized, or they can be constrained in the model. Moreover, alternative scenario generation procedures to moment matching can be employed.

We can price and incorporate currency options to hedge the currency risk, together with options on stock indices. Further restrictions, like liquidity constraints, can be easily incorporated into the model.

The next step in this work is to incorporate derivative securities in multistage stochastic programming models. In that case, investors have to price new options in every stage with different maturities and exercise prices, and decisions must be made, concerning sales or purchases of all instruments, including options.

6.6 Appendix: Option Pricing Procedures

To price the options we use the methodology developed in the previous chapter.

6.6.1 Pricing Simple Options on Stock Indices

This type of option is relevant for the investor who wants to invest in a stock index, desires protection against losses in that asset, but is unconcerned about the translation risk arising from a potential drop in the exchange rate, in case this is an option on a foreign stock index. So, the investment in this option aims to cover market, but not currency risk. To price these options, we adopt the view point of the local-based option writer. The price of the option as well as the payoff, are in units of the respective currency. The option's pay-off under scenario n is:

$$CS^n(S_c, K_j) = max[S_{n,c} - K_j, 0], \quad \forall c \in C_0, \quad j \in JS_c \tag{6.6}$$

where K_j is the strike price in units of the respective currency. The underlying asset for this option is the stock index S_c. We assume that stock indices do not pay dividends.

We use the procedure described above to determine the set of risk neutral probabilities $\bar{p}'_{cn}$ associated with the set of scenarios, for each stock-index separately in the respective country. Once we find the risk-neutral probabilities, we can price the options. The "fair" value of the call option today, is the expected payoff in the risk-neutral measure of the option over all scenarios , discounted by the local riskless rate:

$$cs(S_c, K_j) = e^{-r_c T} \sum_{n=1}^{N} \bar{p}'_{cn}[max(S_{n,c} - K_j, 0)] \quad \forall c \in C_o, \quad j \in JS_c \tag{6.7}$$

And for put options, again we discount the expected payoff by the riskless rate. That is:

$$ps(S_c, K_j) = e^{-r_c T} \sum_{n=1}^{N} \bar{p}'_{cn}[max(K_j - S_{n,c}, 0)] \quad \forall c \in C_o, \quad j \in JS_c \tag{6.8}$$

6.6.2 Pricing Quanto Options on Foreign Stock Indices

A quanto, often known by the more formal name of guaranteed exchange rate contract, is effectively a foreign exchange contract incorporated into an underlying foreign equity option which allows the investor to lock in a prespecified foreign exchange rate and eliminate currency risk. Thus, quantos are contracts that control market and currency risk in an integrated manner. This is particularly useful if the investor fears currency devaluation or when he believes that spot rates will be lower than forward rates suggest. Typically, the forward rate for term equal to option's maturity is used as the predetermined exchange rate. Reiner [160], Jamshidian [104] and Kat and Roozen [118] value quanto contracts that involve both foreign exchange and stock price risk, when both the underlying asset and the exchange rate follow Geometric Brownian Motion.

The investors' pay-off under scenario n, in units of the base currency now, for a call option, is that of a foreign equity call times a fixed exchange rate:

$$CQ^n(S_c, K_j) = \bar{X}_c max[S_{n,c} - K_j, 0] = max[S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0]$$

$\bar{X}_c$ is the rate at which the translation will be made and the two equivalent forms of the pay-off arise from the choice of expressing the strike in foreign or domestic terms (obviously the strike K_j is in units of foreign currency). We choose $\bar{X}_c$ to be the forward exchange rate with the same maturity T. The underlying asset for this option is the foreign stock price multiplied by the fixed exchange rate, i.e., $S_c\bar{X}_c$.

To price the quanto options, we discount by the riskless rate of the base currency this time, the expected payoff of the option over all scenarios, in the risk-neutral measure. In this case, we find the risk-neutral distribution $\bar{P}'$ for the US based investor (under which all underlying assets, $S_c\bar{X}_c, \forall c \in C$, are martingales) and we price the quanto options on each one of the foreign stock indices using these probabilities. All the cashflows, prices and payoffs are in the base currency. Thus, for a call quanto:

$$cq(S_c, K_j) = e^{-rT} \sum_{n=1}^{N} \bar{p}'_n [max(S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0)], \quad \forall c \in C, \quad j \in JQ_c \tag{6.10}$$

And for a put quanto:

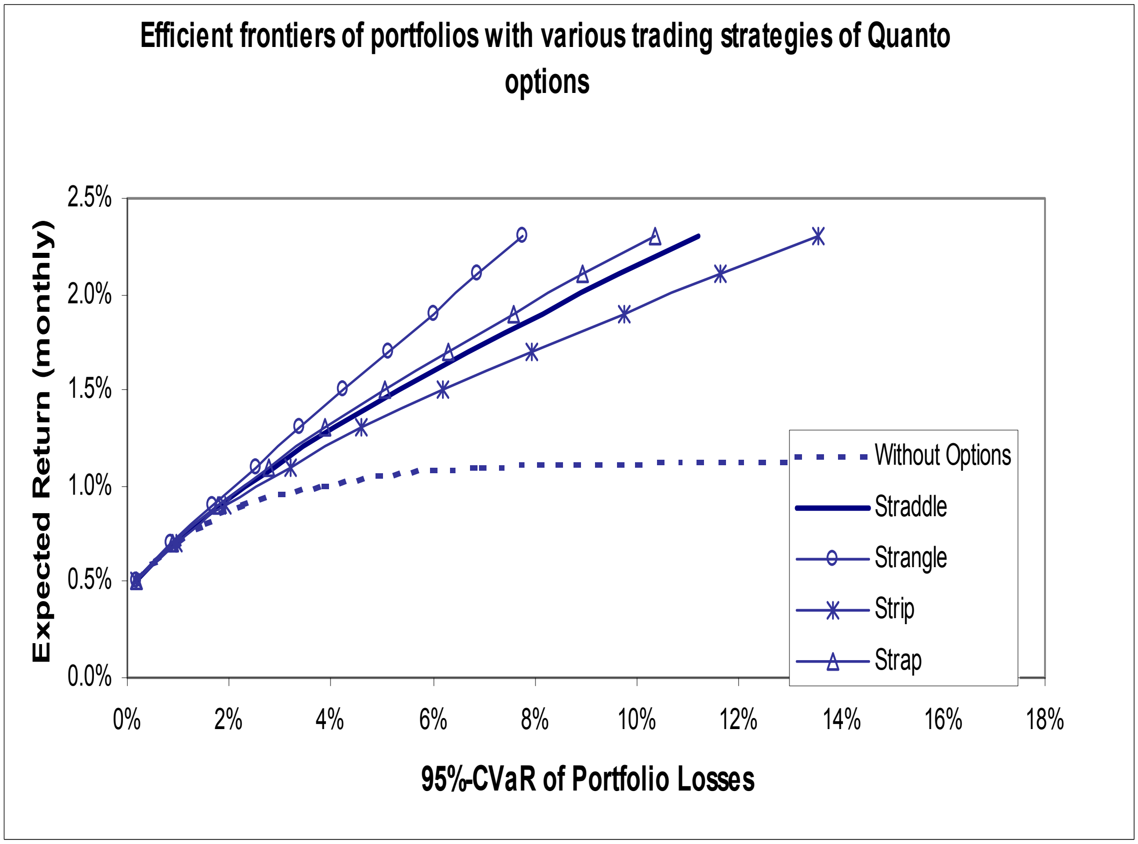
$$pq(S_c, K_j) = e^{-rT} \sum_{n=1}^{N} \bar{p}'_n [max(S_{n,c}\bar{X}_c - K_j\bar{X}_c, 0)], \quad \forall c \in C, \quad j \in JQ_c \tag{6.11}$$

Statistical Characteristics of Monthly Domestic Returns of Assets					
Asset Class	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
USS	1.211%	4.097%	-0.413	3.649	6.11
UKS	1.025%	4.254%	-0.018	3.371	0.606
GRS	1.167%	5.807%	-0.561	4.301	13.92
JPS	-0.170%	6.881%	0.017	3.836	2.45
US1	0.567%	0.504%	-0.006	2.765	0.739
US3	0.654%	1.118%	-0.112	2.626	1.028
US7	0.710%	1.670%	-0.090	2.912	0.295
UK1	0.680%	0.714%	0.995	7.461	150.815
UK3	0.741%	1.349%	0.510	4.631	29.45
UK7	0.793%	1.880%	0.134	3.390	1.533
GR1	0.493%	0.480%	0.288	4.451	31.24
GR3	0.552%	0.930%	-0.255	3.162	4.58
GR7	0.583%	1.372%	-0.663	3.887	22.87
JP1	0.283%	0.481%	0.663	4.708	12.98
JP3	0.416%	1.121%	-0.099	4.401	26.37
JP7	0.503%	1.678%	-0.519	5.434	47.62
Statistical Characteristics of Monthly Proportional Spot Exchange Rate Changes					
Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
UKtoUS	-0.116%	2.894%	-0.755	5.672	102.39
GRtoUS	-0.124%	3.091%	-0.215	3.399	5.69
JPtoUS	0.030%	3.615%	0.942	6.213	105.27

Table 6.1: Statistical characteristics and Jacque-Bera test for normality of historical monthly data of domestic returns of assets and proportional changes of spot exchange rates over the period 05/1988–11/2001.

Portfolios	Without options	Simple options	Quanto options
Turnover	15359	14411	14117

Table 6.2: Total cost for asset transactions (in USD) during the simulation period (05/1998- 11/2001)



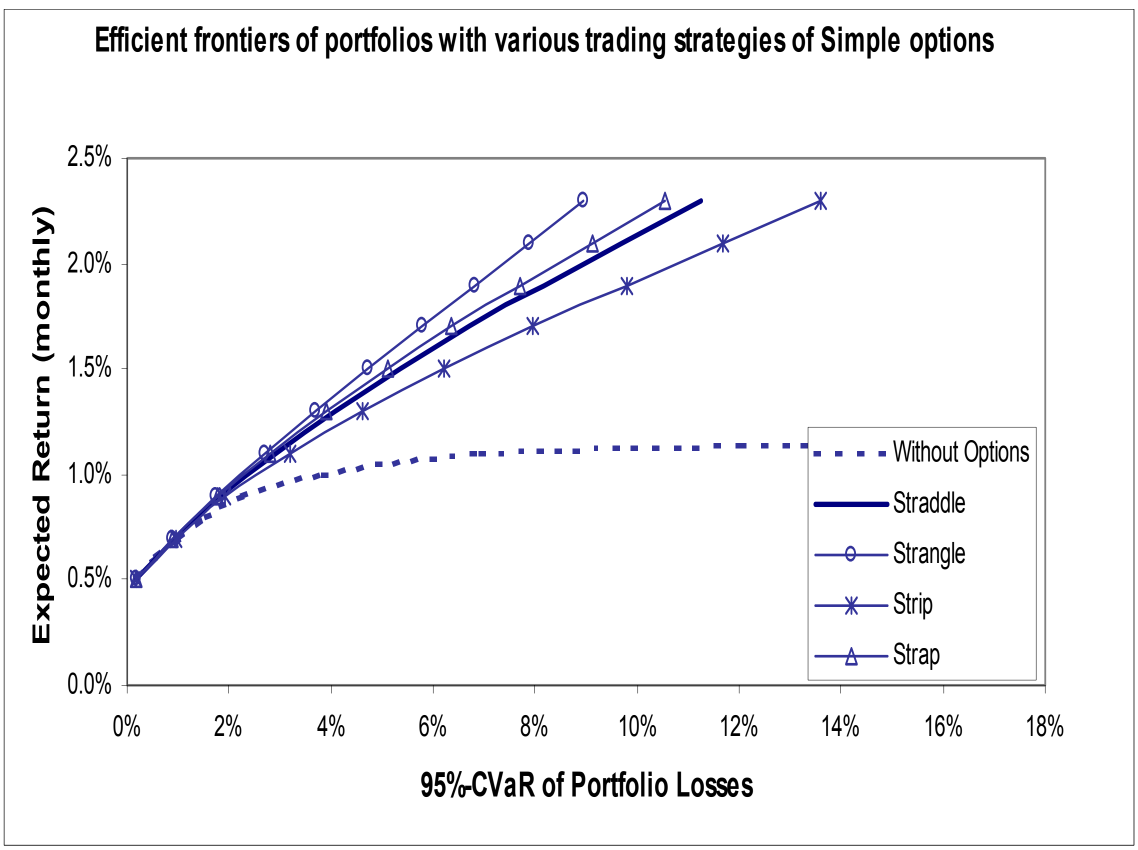


Figure 6.3: Efficient frontiers of CVaR-optimized international portfolios of stock and bond indices with alternative strategies for simple options and quantos.

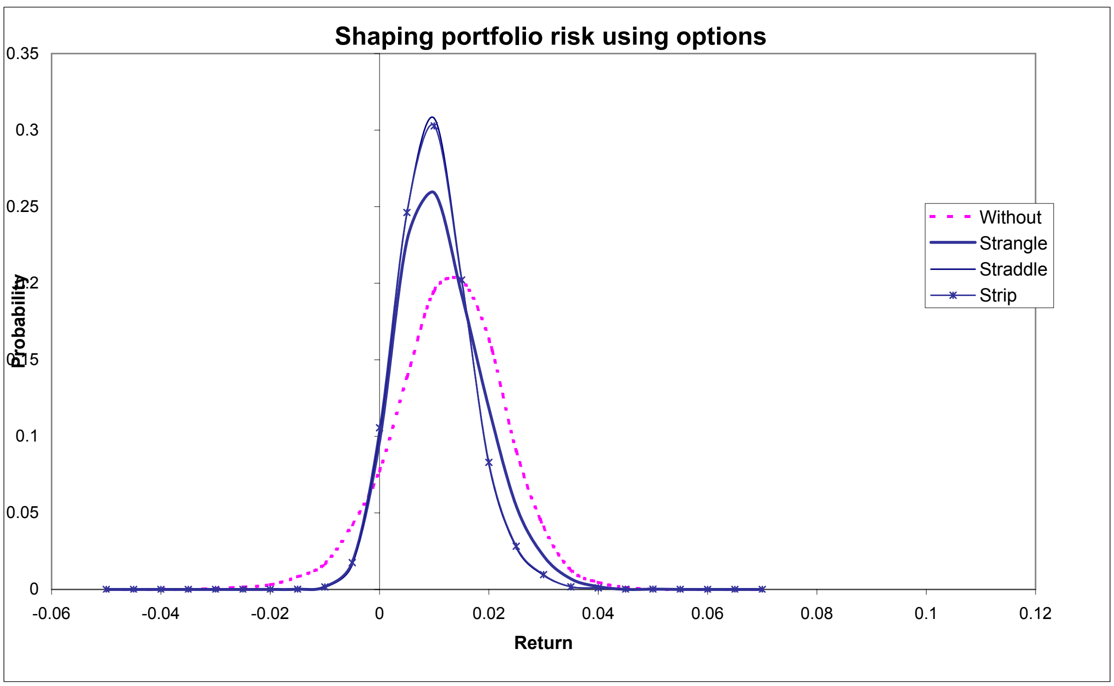


Figure 6.4: Comparison of portfolio return distributions of portfolios without options vs portfolios with alternative option trading strategies (March 2001).

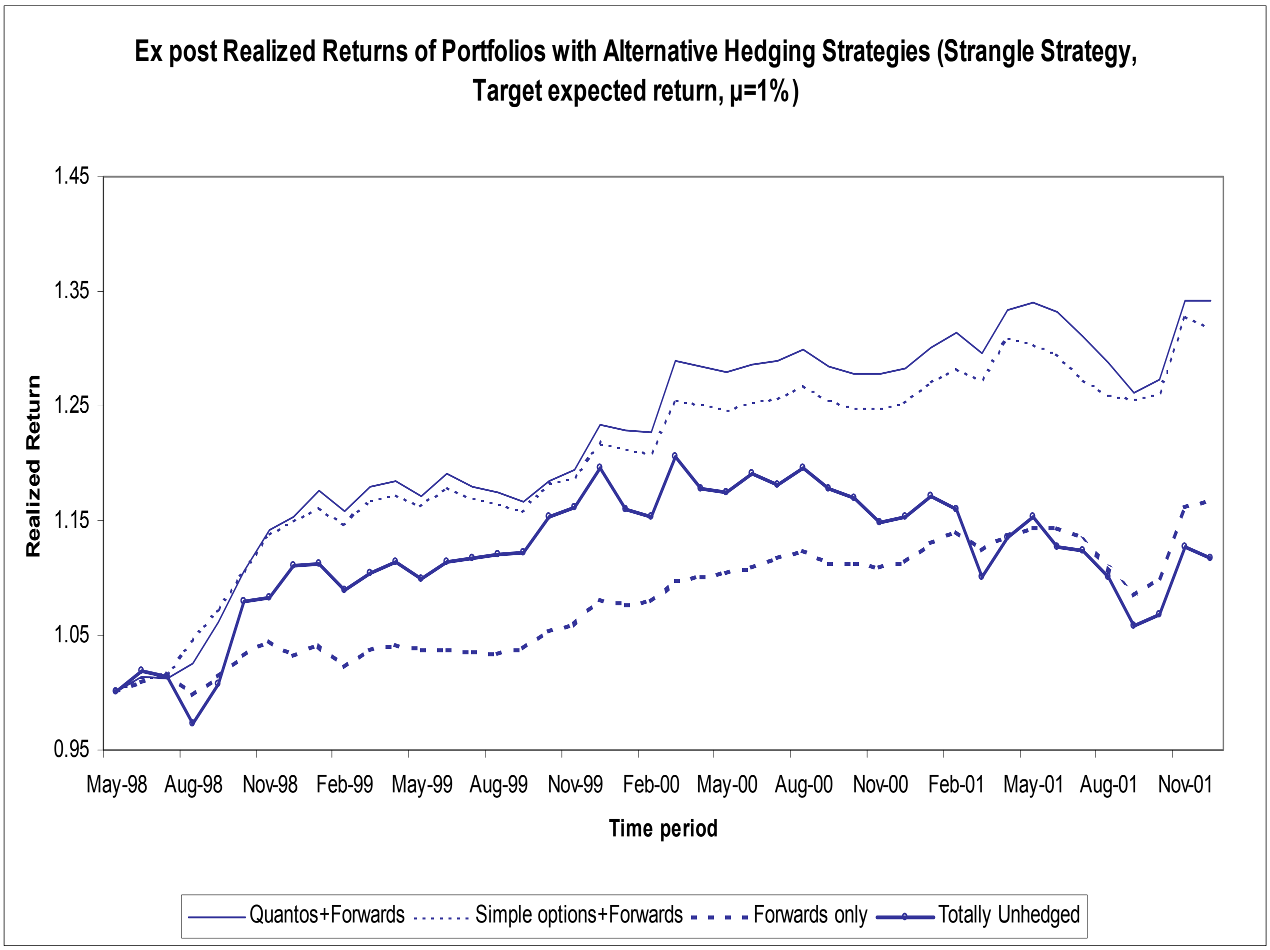
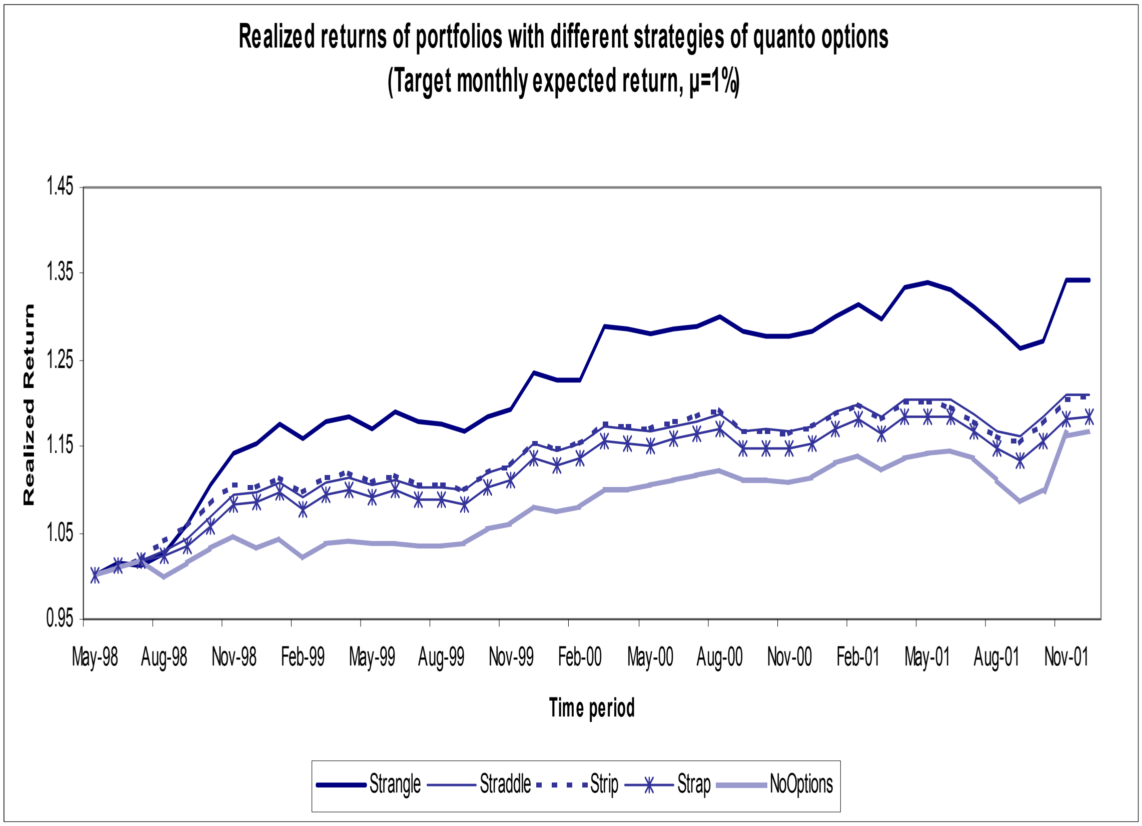


Figure 6.5: Ex-post realized returns of CVaR-optimized international portfolios of stock and bond indices including alternative strategies for hedging risks with simple options or quantos, and forward currency exchanges. The Strangle trading strategy is used for the options.



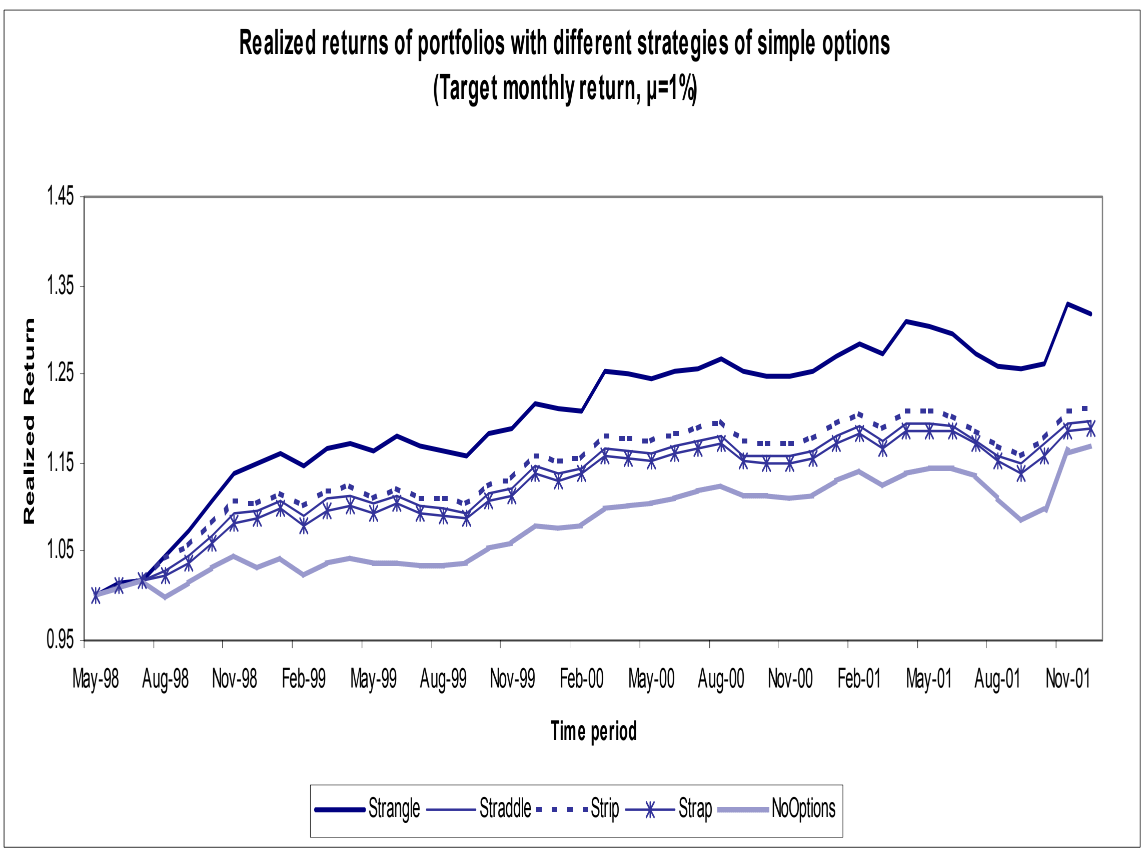
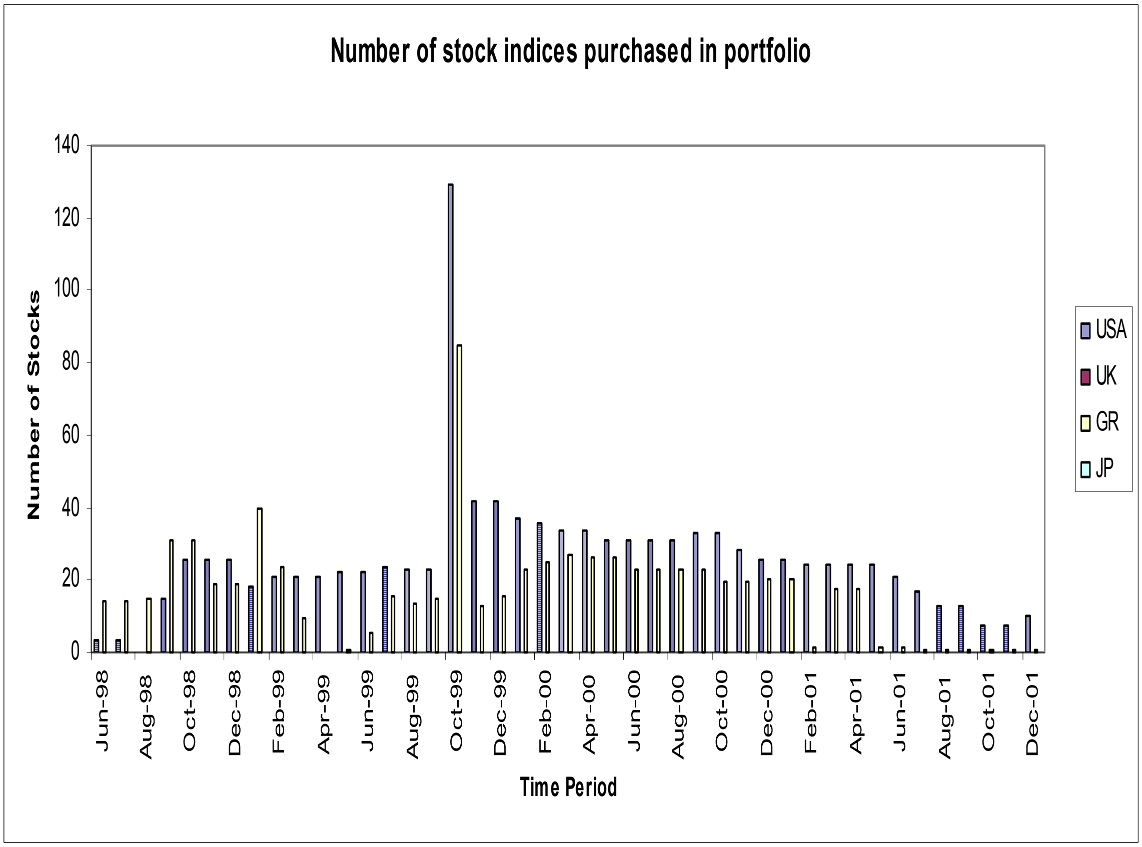


Figure 6.6: Ex-post realized returns of CVaR-optimized international portfolios of stock and bond indices with alternative option trading strategies.



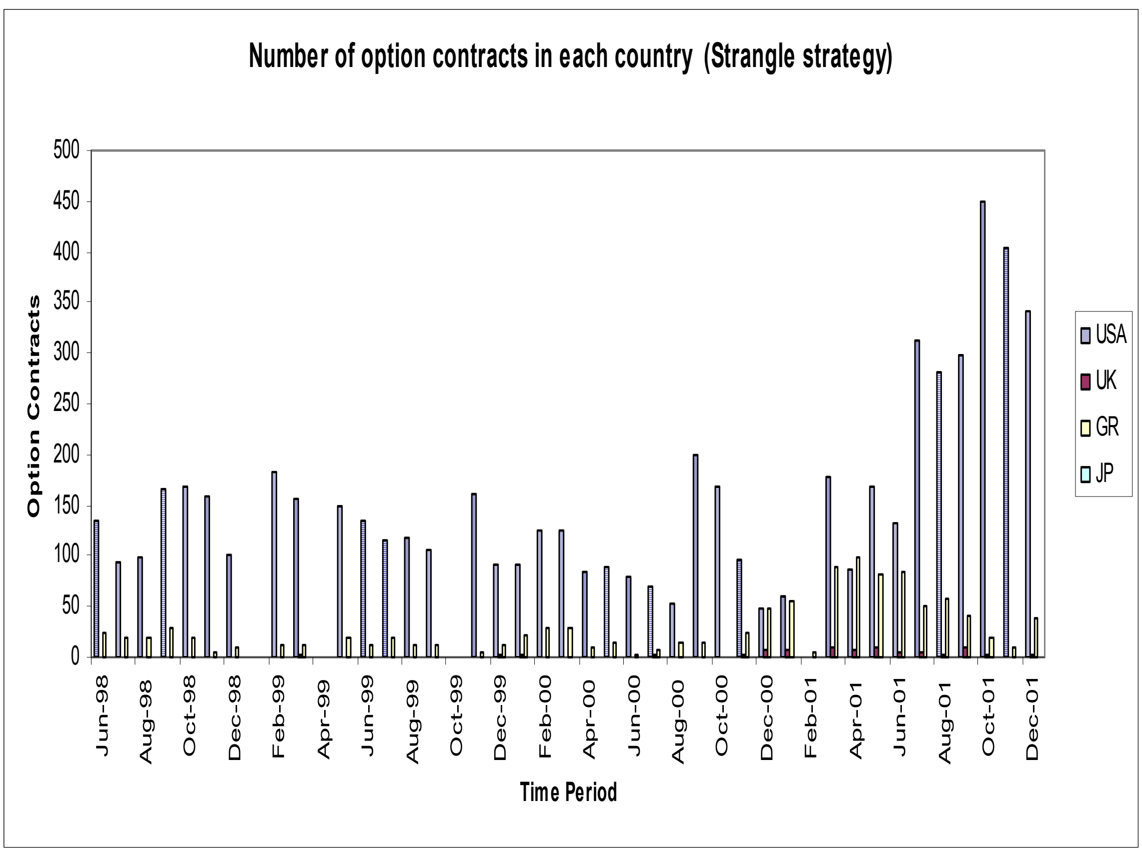


Figure 6.7: Optimal positions in stock indices and quanto options on these indices (Strangle strategy)

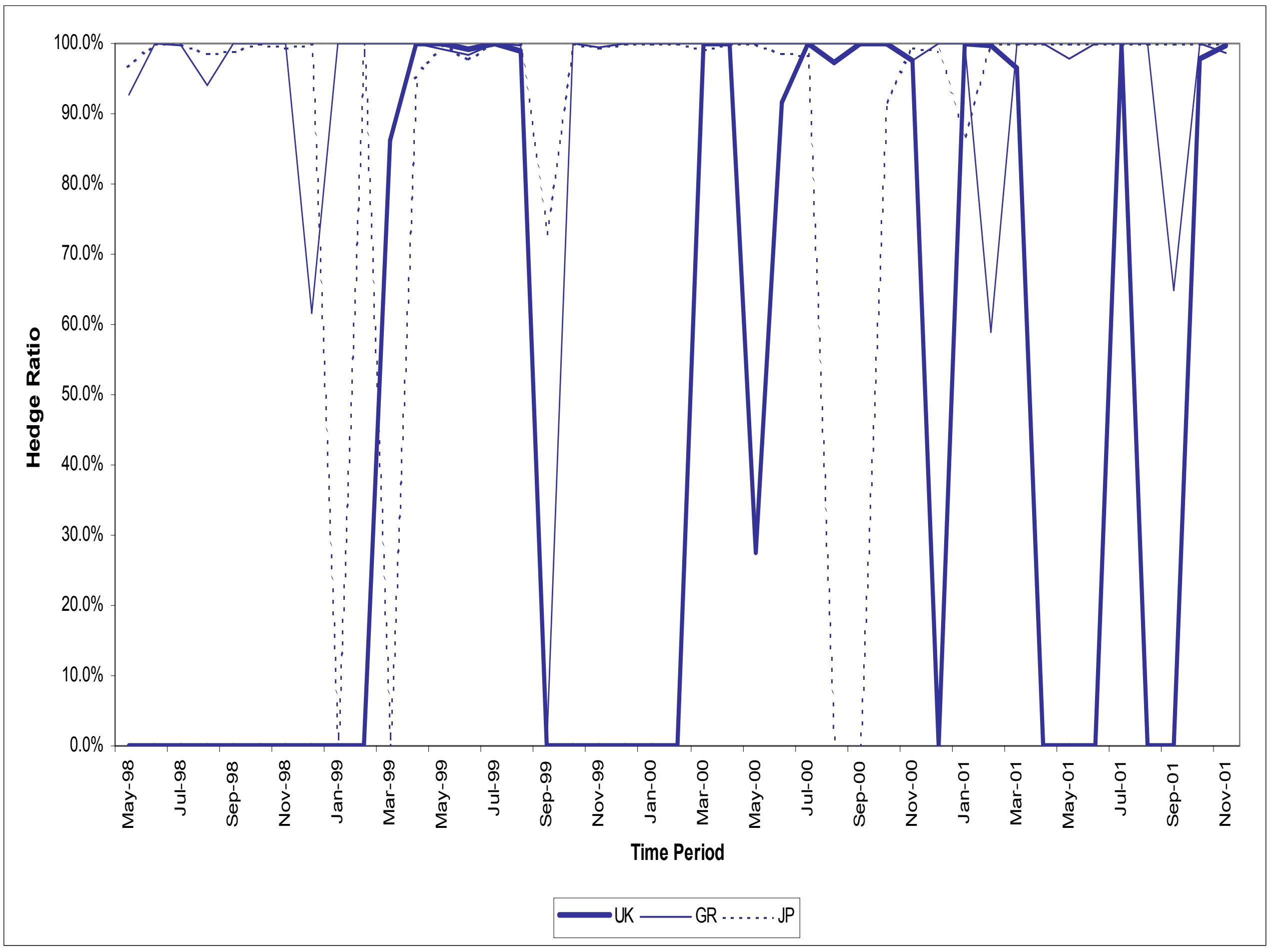


Figure 6.8: Hedge ratios of foreign investments in backtesting experiments (quanto options, strangle strategy).

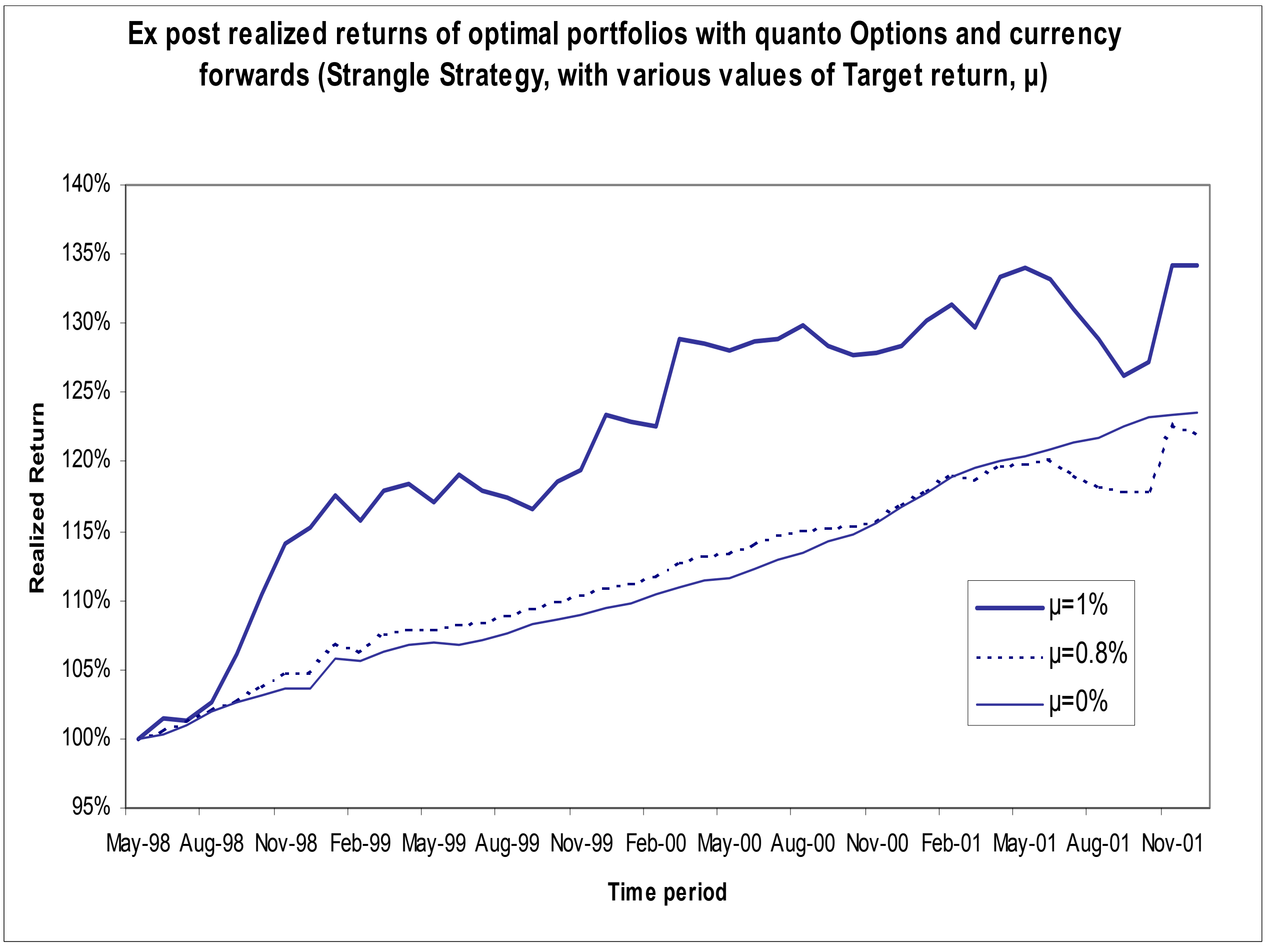


Figure 6.9: Ex-post realized returns of CVaR optimization models for internationally diversified portfolios of stocks bonds and simple Options. Comparison of different Target Returns.

Chapter 7

Models for Controlling the Currency Risk with Options and Forwards

In this chapter we investigate the use of currency options as means of controlling the currency risk of foreign investments in the context of international portfolios. Due to the asymmetric nature of their payoffs, options are particularly suitable instruments for risk management. Currency options can cover against losses from potential unfavorable changes in exchange rates, while preserving the upside potential.

We adapt the multistage stochastic programming models so as to incorporate decisions for optimal positions in currency options. We apply a valuation procedure presented in chapter 5 to price the currency options consistently with the discrete scenario sets of asset prices and exchange rates; the option pricing procedure for currency options is presented in the Appendix of this chapter. We investigate the effectiveness of decision strategies that employ currency options to control currency risk exposures in portfolios of international financial assets. To this end, we empirically compare such strategies against the use of currency forward contracts as means of controlling currency risk. The goal is to identify appropriate instruments and decision strategies for risk management in international portfolios. Besides individual options, we also consider trading strategies that involve combinations of options and have specific payoff characteristics.

The alternative hedging strategies are compared in terms of their ex ante potential in static tests, as well as in terms of their ex post realized performance in dynamic backtesting simulations. We find that the use of a single put option per currency leads to inferior results compared to currency forward contracts. However, appropriate option-trading strategies (e.g., BearSpread) result in superior performance. Moreover, we contrast the performance of a multistage stochastic programming model against its myopic, single-stage counterpart. We find that in all tests, and regardless of the hedging strategy, the multistage model exhibits superior performance than the single-stage model. This clearly demonstrates the additional benefit from employing a multistage, dynamic model rather than a myopic decision model.

7.1 Introduction

The objective of this work is to investigate a series of methodological questions concerning optimal international diversification and the associated problem of controlling currency risk. Currency risk is an important aspect in international portfolio management problems. In the previous chapter we studied models that employed various types of stock index options to hedge against the market risks of international portfolios. Options can also be used to hedge currency risk. A key question addressed in this chapter is whether forward contracts — that were used so far in the thesis — are effective currency hedging instruments, or whether superior performance can be achieved by using combinations of currency options with desirable payoff patterns.

Thus, in this chapter we focus on optimal ways to hedge the currency risk exposure of international portfolios. This research aims to identify the most appropriate instruments for currency hedging purposes, as well as effective strategies (using these instruments) that minimize the downside risk of international portfolios.

Currency hedging decisions concern the choice of either foreign exchange forward contracts or currency options. A foreign exchange forward contract is an agreement between two parties (the investor and a bank) to buy (or sell) a certain amount of foreign currency at a future date at an exchange rate specified at the time of the agreement. Foreign exchange forward contracts are sold by major commercial banks and typically have fixed short-term maturities of one, six and nine months.

While currency forwards are a simple and cost-effective way to alter the variability of revenues from foreign currency sources, they may not be equally effective for all types of foreign exchange rate risk management problems. These contracts fix both the rate as well as the amount of a foreign exchange transaction, and thus protect the value of a certain foreign amount against potential drops in the exchange rate. If the amount of foreign revenues is known with certainty, then an equivalent currency forward contract will completely eliminate the currency risk. In the case of foreign asset holdings, their final value is stochastic due to the uncertainty of their returns. The important limitation of currency forwards is that by fixing the exchange rate they deprive the opportunity for gains in the event that the exchange rate rises. One could argue that mitigating risk should be the primary consideration, while potential benefits from favorable exchange rate movements should be of a secondary concern. But it is the entire risk-return tradeoff that should guide portfolio management decisions.

Options provide alternative means to control risks. Currency options give the holder the right, but not the obligation, to buy or sell a fixed amount of foreign currency at a specific rate at a future time. The appropriate options for currency hedging purposes are put options, which give the holder the right to sell a foreign currency. Put options make profit when the exchange rate falls bellow the exercise price (allowing the holder to sell foreign currency at a higher rate). Currency options, like forwards, are affected by interest rate spreads. Still, options have natural advantages for hedgers. An exporter could "shell" his future foreign exchange receipts by purchasing a currency put. A portfolio manager could protect his foreign asset holdings by buying currency options that form a particular trading strategy, to cover each currency exposure in the portfolio.

The advantage of a put option is the protection that a hedger needs from losses in the event of a significant drop in the exchange rate, but with no sacrifice of potential benefits from upside movements in a foreign currency. A currency option does not have to be exercised if the spot exchange rate moves in favor of the option holder, who can then take the benefit of a favorable currency movement by not exercising the option. Forward contracts tie the option holder to the forward rate, giving rise to so-called "hedging losses" that reflect the foregone benefit from potential improvements in the exchange rate. In other words, a hedge through a forward contract leaves the holder no worse off, but no better off either. On the contrary, a currency put is exercised only if the exchange rates drop substantially; the exercise of the option then offsets the effect of the adverse movement in the exchange rate. But potential gains in the event of favorable movements in the exchange rate are not affected, as the option is not exercised in that case. This asymmetry of payoffs and risk is unique among financial instruments. The closest analogy is an insurance policy, which offers protection from adverse circumstances while leaving the holder free to benefit from favorable circumstances.

As one might expect, this unique advantage does not come free; options cost money to purchase. Depending on volatility, and desired risk coverage, options may be cheap or dear. As should be expected, a put option is cheaper the lower its exercise price is in comparison to the current (spot) exchange rate. There is an obvious tradeoff between cost and risk coverage. It is necessary to weigh the cost and expected return that options would yield against the changed profile of risks.

Although many studies examined the use of forward contracts to hedge currency risk, the empirical evidence for the use of options is limited. In the majority of the international asset allocation studies cited in the literature review of section 2.2, currency hedging is performed using forward or future contracts. Conover and Dubofsky [49] work on the use of American options. They examine empirical results from the implementation of portfolio insurance strategies employing currency spot and future options. They find that protective puts using future options are generally dominated by both protective puts that use options on spot currencies and by fiduciary calls on futures contracts. Wong [188] examines the optimal hedging decision of a competitive exporting firm under hedgeable exchange rate risk and non-hedgeable price risk. The firm avails itself of a rich set of risk-sharing instruments including currency options and futures. He finds that a long position in put options is the firm's optimal hedging strategy. Chang and Wong [46] study how a risk-averse multinational firm can employ derivative securities on related currencies to reduce its foreign risk exposure. They find that when the exchange rates are correlated, the firm would optimally use currency options for hedging purposes. Lien and Tse [134] compare the hedging effectiveness of currency options versus futures on the basis of lower partial moments (LPM). They find that currency futures is always a better hedging instrument than currency options. The only situation in which options outperform futures occurs when the individual hedger is optimistic (with a large target return) and not too concerned about large losses. Steil [176] applies an expected utility analysis to derive optimal contingent claims for hedging foreign transaction exposure as well as optimal forward and option hedge alternatives. Using quadratic, negative exponential and positive exponential utility functions, Steil shows that currency options have little, if any, useful role to play in hedging contingent foreign exchange transaction exposures. Hull and White [96] analyze ways in which banks and other financial institutions can hedge their risks when they write non-exchange-traded foreign currency options. They find that Delta+Gamma hedging performs well when the traded options have a fairly constant implied volatility and a short time to maturity. Delta+Sigma hedging outperforms other hedging alternatives when the traded options being used have a non-constant implied volatility and a long time to maturity.

Another issue addressed in the literature is that of the asymmetric nature of foreign exchange exposure due to the hedging strategies implemented by the companies. Booth [31] examines the role of transaction costs and the asymmetry produced in the firm's profit function in an attempt to provide a more realistic analysis of hedging strategies, specifically currency put options, that provide downside protection while allowing upside potential. Booth suggests that hedging instruments with asymmetric payoffs, such as currency options, appear to be most useful.

The results of these studies are rather conflicting. In some of the aforementioned studies currency options constitute effective currency hedging instruments; in others, currency options are dominated by forward contracts. Our empirical results show that a hedging strategy consisting of a single put option per currency leads to inferior performance compared to forward contracts. But combinations of currency put options can lead to significant performance improvements.

The primary aim of this chapter is to examine the hedging role of currency options and to identify appropriate trading strategies of these options by comparing empirically their performance against the use of currency forward contracts. To this end, we adopt the multistage stochastic programming model for international portfolio management that was developed in chapter 4 by introducing currency options to the investment opportunity set at each decision stage. The model accounts for the effects of portfolio (re)structuring decisions over a multi-period horizon, including positions in currency options among its permissible decisions. A number of modeling questions need to be resolved. The model's rebalancing decisions must account for the discretionary exercise of expiring options at each decision point. Moreover, currency options must be suitably valued at each decision stage of the multistage stochastic program.

The multistage stochastic program is a flexible model that takes a longer-term view of the international portfolio management problem and accounts for decision dynamics, including the optimal selection of currency options for risk hedging purposes. This model provides an indispensable and practical decision support tool for international portfolio management. Using the multistage stochastic program as a testbed, we are able to compare empirically the performance of alternative hedging strategies that use either currency options (individual or combinations) or currency forward contracts.

Multi-stage models help decision makers adopt more effective decisions. The decisions are based on longer-term potential benefits and avoid myopic reactions to short-term movements that may lead to losses. The suitability of options to deal with asymmetric distributions of exchange rates, and the multi-period nature of international portfolio management problems, led us to pursue the development of flexible models for hedging international portfolios, expecting that dynamic model should clearly outperform its myopic counterpart.

We empirically examine the improvements in the performance of portfolios when we extend the models into a multistage setting. Studies of stochastic programming applications did not provide unambiguous evidence as to whether there are clear benefits from multistage stochastic programs relative to single-period ones to warrant the additional effort of modelling and solving these more complex models.

We develop, implement and test, stochastic programming models for managing international portfolios that minimize the tail risk of portfolio losses over a multi-period planning horizon, measured by the CVaR risk metric. The novel feature of the current models concerns the inclusion of currency options (either individual or combinations that constitute specific strategies) for hedging purposes, in the context of dynamic international portfolio management models. The scenarios of asset returns, exchange rates, and corresponding currency options prices are critical inputs to the stochastic optimization program. The model determines sequences of portfolio rebalancing decisions as the scenarios evolve over time, and specifies jointly the particular investments in each market and the level of currency hedging through the purchase of currency options.

The first contribution of this study is the inclusion of currency options as hedging tools in stochastic programming models for managing portfolios of international financial assets. Despite the advocacy of currency options for foreign exchange risk management, their incorporation in practical portfolio management models had remained largely unexplored. We investigate alternative trading strategies of currency options and empirically examine their performance using real market data.

The second contribution is the development and empirical validation of stochastic programming models for managing international portfolios that include currency options. We demonstrate through empirical tests the superiority of the dynamic model over its single-period counterpart. We show that regardless of the trading strategy of options, the multistage model results in superior performance.

To summarize, we first price currency options at the nodes of a scenario tree. These prices are used as input to single- and multistage stochastic programming models. The pricing method takes into account that the distribution of exchange rates is not normal but exhibits nonzero skewness and excess kurtosis. We incorporate in the portfolio management models appropriate trading strategies for currency options as means for mitigating currency risk. The models determine jointly the optimal portfolio composition and the level of currency hedging through options or forward contracts.

We use the dynamic stochastic programming model as a basis to empirically compare the relative effectiveness of currency options and forward contracts to hedge the currency risk of international portfolios. We analyze the effect of such hedging instruments on the ex post return performance of international portfolios of stock and bond indices, in dynamic backtesting experiments.

The rest of this chapter is organized as follows. In section 7.2 we present the formulation of the optimization models for international portfolio selection. In section 7.3 we discuss the hedging strategies employed in the empirical tests. In section 7.4 we describe the computational tests and we discuss the empirical results. Section 7.5 concludes the paper. Finally, in the Appendix we describe the methodology for pricing currency options.

7.2 The International Portfolio Management Model

The problem of portfolio (re)structuring is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. These portfolios are exposed to market and currency exchange risk. To cope with the market risk, the portfolio is diversified internationally across markets. International diversification exposes the foreign investments to currency risk. To hedge the currency risk, the investor may enter into currency exchange contracts in the forward market, or buy individual currency put options or in specific combinations, thus forming a particular trading strategy.

The aim of the international portfolio management model is to determine the optimal portfolio composition that dominates all others in terms of having the minimum shortfall risk at each level of expected return. We explore single-stage, as well as multistage stochastic programming models to manage international investment portfolios. The multistage model specifies a sequence of buying and selling decisions at discrete points in time (monthly intervals). Decisions are made at the beginning of each time interval. The portfolio manager starts with a given portfolio and with a set of postulated scenarios about future states of the economy represented in terms of a scenario tree (e.g., see Figure 4.1), as well as corresponding currency option prices depended on the postulated scenarios. This information is incorporated into a portfolio restructuring decision. The composition of the portfolio at each decision point depends on the transactions that were decided at the previous decision point. The portfolio value depends on the outcomes of asset returns and exchange rates realized in the interim period and, consequently, on the discretionary exercise of currency options whose purchase was decided at the previous decision point. Another portfolio restructuring decision is then made at that node of the scenario tree based on the available portfolio, the subsequent outcomes of the random variables, and the available currency options (depending on their estimated prices).

In this section we develop scenario-based stochastic programming models for managing investment portfolios of international stock and government bond indices. The models address the problems of currency risk management and optimal portfolio selection in an integrated manner. Their deterministic inputs are: the initial asset holdings, the current prices of the securities, the current spot exchange rates, the forward exchange rates for a term equal to the decision interval or the currency option prices (depending on which instruments are used to hedge the currency risk). We must also specify scenario dependent data that, together with the associated probabilities, represent the discrete distribution of the random variables at any stage. The prices of the securities and the spot exchange rates at any node of the scenario tree are generated according to the moment-matching procedure described in chapter 4. These, in turn, uniquely determine the option payoffs at any node of the scenario tree.

The model's decision variables determine the asset and option purchase and sale transactions that yield the revised portfolio. Positions in specific combinations of currency options — reflecting certain hedging strategies — are easily enforced with appropriate constraints. The optimization models incorporate practical considerations (no short sales for assets, transaction costs) and minimize the tail risk of final portfolio value at the end of the planning horizon for a given level of target expected return.

We first price the currency options at the root node and at any intermediate node, using a method that is described in the Appendix of this chapter. These prices are used as input to the optimization models together with the postulated scenarios of asset returns and exchange rates. The options are European, may be purchased at any decision node and have a maturity of one period. Thus, at every decision node of the scenario tree, the selected options in the current portfolio on that node may be exercised, and new option contracts may be purchased. The models determine jointly the portfolio positions (not only across the different markets, but also to the specific securities within each markets), and the levels of currency hedging in each market via forward contracts or currency options.

The cashflow balance constraints for each currency, as well as the final value of the portfolio under each scenario, are computed differently for each kind of currency hedging instruments that are considered in the portfolio. We present separately the model associated with the use of each type of these instruments.

We use the following notation:

Definitions of sets:

C_0	set of currencies (countries), including the base (reference) currency,
$\ell \in C_0$	the index of the base (reference) currency in the set of currencies,
$C = C_0 \backslash \{\ell\}$	the set of foreign currencies (i.e., excluding the base currency),
I_c	set of assets denominated in currency $c \in C_0$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term government bond index in each country),
$\boldsymbol{N}$	is the set of nodes of the scenario tree,
$n \in \boldsymbol{N}$	is a typical node of the scenario tree ($n = 0$ denotes the root node at $t = 0$),
$\boldsymbol{N_t} \subset \boldsymbol{N}$	is the set of distinct nodes of the tree at time period $t = 0, 1, ..., T$,
$\boldsymbol{N_T} \subset \boldsymbol{N}$	is the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios,
$S_n \subset \boldsymbol{N}$	is the set of immediate successor nodes of node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n.
J_c	set of available currency options (differing in terms of exercise price).

Input Parameters (Data):

(a). Deterministic quantities:

b_{ic}	initial position in asset $i \in I_c$ of currency $c \in C_0$ (in units of face value),
h_c^0	initially available cash in currency $c \in C_0$ (surplus if +ve, shortage if -ve),
T	length of the time horizon (number of decision periods),
δ	proportional transaction cost for sales and purchases of assets,
d	proportional transaction cost for currency transactions in the spot market,
μ	prespecified target expected return over the planning horizon,
α	the prespecified confidence level (percentile) for the CVaR measure,
π_{ic}^0	current market price (in units of the respective currency) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$,
e_c^0	current spot exchange rate for foreign currency $c \in C$,
f_c^0	currently quoted one-month forward exchange rate for foreign currency $c \in C$,
K_j	the strike price of an option $j \in J_c$, in currency $c \in C$ (in units of currency c).

(b). Scenario dependent parameters:

p_n — objective probability of occurrence of node $n \in \boldsymbol{N}$,

e_c^n — spot exchange rate of currency $c \in C$ at node $n \in \boldsymbol{N}$,

f_c^n — one month forward exchange rate for foreign currency $c \in C$ at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$,

π_{ic}^n — price of asset $i \in I_c$, $c \in C_0$ on node $n \in \boldsymbol{N}$ (in units of domestic currency c),

$cc^n(e_c^n, K_j)$ — price of European call currency option $j \in J_c$ on exchange rate of currency $c \in C$, at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity of one month,

$pc^n(e_c^n, K_j)$ — price of European put currency option $j \in J_c$ on exchange rate of currency $c \in C$, at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity of one month.

All exchange rate parameters $(e_c^0, f_c^0, e_c^n, f_c^n)$ are expressed in units of the base currency per one unit of the foreign currency $c \in C$. Of course, the exchange rate of the base currency to itself is trivially equal to one, $f_\ell^n = e_\ell^n \equiv 1, \quad \forall n \in \boldsymbol{N}$. The prices cc and pc of currency call and put options respectively, are expressed in units of the base currency ℓ.

Computed Parameters:

V_ℓ^0 — total value (in units of the base currency) of the initial portfolio.

$$V_\ell^0 = h_\ell^0 + \sum_{i \in I_\ell} b_{i\ell}\, \pi_{i\ell}^0 + \sum_{c \in C} e_c^0 \left(h_c^0 + \sum_{i \in I_c} b_{ic}\, \pi_{ic}^0 \right) \tag{7.1}$$

Decision Variables:

(Decisions are made at the root and at intermediate nodes, thus $\forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$)

(a). Instrument purchase, sale, and hold quantities (all quantities are in units of face value):

- x_{ic}^n — units of asset $i \in I_c$ of currency $c \in C_0$ purchased,
- v_{ic}^n — units of asset $i \in I_c$ of currency $c \in C_0$ sold,
- w_{ic}^n — resulting units of asset $i \in I_c$ of currency $c \in C_0$ in the revised portfolio.

(b). Currency transfers in the spot market:

- $x_{c,e}^n$ — units of the base currency exchanged in the spot market for foreign currency $c \in C$,
- $v_{c,e}^n$ — units of the base currency collected from a sale of foreign currency $c \in C$.

(c). Forward currency exchange contracts (when we use forwards instead of options):

- $u_{c,f}^n$ — amount of base currency collected from sale of currency $c \in \boldsymbol{C}$ in the forward market (i.e., amount of forward contract, in units of the base currency). A negative value indicates a purchase of the foreign currency in the forward market. These decisions are made at node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, but the actual transaction is executed at the end of the respective period, i.e., at the successor nodes S_n.

(d). Variables related to currency options transactions:

- $ncc_{c,j}^n$ — purchases of European call currency option $j \in J_c$ on the exchange rate of currency $c \in C$, with exercise price K_j and maturity of one month,
- $npc_{c,j}^n$ — purchases of European put currency option $j \in J_c$ on the exchange rate of currency $c \in C$, with exercise price K_j and maturity of one month.

When currency options are used to hedge unfavorable exchange rate movements, only long positions in the respective trading strategies of currency options are allowed.

Auxiliary variables:

- y_n auxiliary variables used to linearize the piecewise linear function in the definition of CVaR of portfolio losses at the end of the planning horizon, $n \in \boldsymbol{N_T}$,
- z the VaR value of terminal portfolio losses (at a prespecified confidence level, percentile α),
- V_ℓ^n the total value of the portfolio at the end of the holding period at leaf node $n \in \boldsymbol{N_T}$ (in units of the base currency),
- R_n return of the international portfolio over the planning horizon at node $n \in \boldsymbol{N_T}$,
- $\bar{R}$ expected return of the international portfolio over the planning horizon.

We consider either forward contracts or currency options in the optimization models to hedge against the currency risk of the international portfolios, but not both. Hence, we need two slightly different formulations of the optimization model, which differ in the cashflow balance constraints and the computation of the final portfolio value.

7.2.1 Model Formulation

The model can now be formulated for each type of currency hedging instruments as follows:

Portfolio Optimization Model with Currency Options

$$\min \quad z + \frac{1}{1-\alpha}\sum_{n\in \boldsymbol{N_T}} p_n y_n \tag{7.2a}$$

$$\text{s.t.} \quad h_\ell^0 + \sum_{i\in I_\ell} v_{i\ell}^0 \pi_{i\ell}^0(1-\delta) + \sum_{c\in C} v_{c,e}^0(1-d) = \sum_{i\in I_\ell} x_{i\ell}^0\pi_{i\ell}^0(1+\delta) + \sum_{c\in C} x_{c,e}^0(1+d)$$

$$+ \sum_{c\in C}\left\{\sum_{j\in J_c}\left[npc_{c,j}^0 * pc^0(e_c^0, K_j)\right]\right\} \tag{7.2b}$$

$$h_c^0 + \sum_{i\in I_c} v_{ic}^0\pi_{ic}^0(1-\delta) + \frac{1}{e_c^0}x_{c,e}^0 = \sum_{i\in I_c} x_{ic}^0\pi_{ic}^0(1+\delta) + \frac{1}{e_c^0}v_{c,e}^0, \quad \forall\, c\in C \tag{7.2c}$$

$$h_\ell^n + \sum_{i\in I_\ell} v_{i\ell}^n\pi_{i\ell}^n(1-\delta) + \sum_{c\in C} v_{c,e}^n(1-d) + \sum_{c\in C}\left\{\sum_{j\in J_c}\left[npc_{c,j}^{p(n)} * max(K_j - e_c^n, 0)\right]\right\}$$

$$= \sum_{i\in I_\ell} x_{i\ell}^n\pi_{i\ell}^n(1+\delta) + \sum_{c\in C} x_{c,e}^n(1+d) + \sum_{c\in C}\left\{\sum_{j\in J_c}\left[npc_{c,j}^n * pc^n(e_c^n, K_j)\right]\right\},$$

$$\forall\, n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\} \tag{7.2d}$$

$$h_c^n + \sum_{i\in I_c} v_{ic}^n\pi_{ic}^n(1-\delta) + \frac{1}{e_c^n}x_{c,e}^n = \sum_{i\in I_c} x_{ic}^n\pi_{ic}^n(1+\delta) + \frac{1}{e_c^n}v_{c,e}^n,$$

$$\forall\, c\in C,\ \forall\, n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\} \tag{7.2e}$$

$$V_\ell^n = \sum_{i\in I_\ell} w_{i\ell}^{p(n)}\pi_{i\ell}^n + \sum_{c\in C}\left\{e_c^n\left[\sum_{i\in I_c} w_{ic}^{p(n)}\pi_{ic}^n\right] + \sum_{j\in J_c}\left[npc_{c,j}^{p(n)} * max(K_j - e_c^n, 0)\right]\right\},$$

$$\forall n\in \boldsymbol{N_T} \tag{7.2f}$$

$$\sum_{j\in J_c} npc_{c,j}^n \le \sum_{i\in I_c} w_{ic}^n\pi_{ic}^n, \quad \forall c\in C, \quad \forall n\in \boldsymbol{N}\setminus \boldsymbol{N_T} \tag{7.2g}$$

$$R_n = \frac{V_\ell^n}{V_\ell^0} - 1, \qquad \forall n\in \boldsymbol{N_T} \tag{7.2h}$$

$$\bar{R} = \sum_{n\in \boldsymbol{N_T}} p_n R_n \ge \mu, \tag{7.2i}$$

$$y_n \ge L_n - z, \qquad \forall n\in \boldsymbol{N_T} \tag{7.2j}$$

$$y_n \ge 0\,, \qquad \forall n\in \boldsymbol{N_T} \tag{7.2k}$$

$$L_n = -R_n\,, \qquad \forall n\in \boldsymbol{N_T} \tag{7.2l}$$

$$w_{ic}^0 = b_{ic} + x_{ic}^0 - v_{ic}^0, \qquad \forall\, i\in I_c\,,\ \forall\, c\in C_0 \tag{7.2m}$$

$$w_{ic}^n = w_{ic}^{p(n)} + x_{ic}^n - v_{ic}^n, \qquad \forall\, i\in I_c\,,\ \forall\, c\in C_0,\ \forall\, n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\} \tag{7.2n}$$

$$x_{ic}^n \ge 0, \quad w_{ic}^n \ge 0\,, \qquad \forall i\in I_c\,,\ \forall c\in C_0,\ \forall\, n\in \boldsymbol{N}\setminus \boldsymbol{N_T} \tag{7.2o}$$

$$0 \le v_{ic}^0 \le b_{ic}, \qquad \forall i\in I_c\,,\ \forall c\in C_0 \tag{7.2p}$$

$$0 \le v_{ic}^n \le w_{ic}^{p(n)}, \qquad \forall i\in I_c\,,\ \forall c\in C_0,\ \forall\, n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\} \tag{7.2q}$$

This formulation of the model minimizes the CVaR (7.2a) of portfolio losses at the end of the horizon, while constraining the expected portfolio return (7.2i). Due to the highly skewed return distributions of options, the application of the traditional mean-variance framework for portfolio optimization is inappropriate when options are considered. The aim of using currency options is to provide an effective means for currency hedging so as to minimizes the tail risk of the international portfolios. The tail (excess shortfall) risk is measured here by the Conditional-Value-at-Risk (CVaR) metric. The motivation for applying the CVaR risk measure has been explained in previous chapters.

Unlike currency forward contracts, currency options entail an initial payment (cost of the option) at the time a decision is made to enter into an option contract. The payment of these costs for the purchase of options needs to be considered in cash balance constraints. Similarly, the conditional payoffs of the options — in the cases they are exercised — must also be properly accounted in the cash balance conditions at the respective expiration dates. We consider only European options; so, they can be exercised only at their expiration date (maturity). Specifically, we use options with a single-period maturity (one month in our implementation). So, options purchased at some decision stage mature at exactly the next decision period, at which time they are either exercised, if they are profitable, or are simply left to expire.

The exercise prices of the options are specified exogenously and are not part of the decision process. However, by considering multiple options with different strike prices on the same currency, we provide the model substantial flexibility to chose the most appropriate options at each decision stage. The optimal positions in option contracts, as well as the optimal portfolio rebalancing decisions, are considered in a unified manner at each non-terminal node of the scenario tree. The prices of the options at each node of the tree are computed according to the valuation procedure presented in the appendix. The corresponding payoffs at the immediate successor nodes on the tree are also computed and entered as input to the optimization program. Thus, (7.2a) – (7.2q) is a linear program.

Equations (7.2b) and (7.2c) impose the cash balance conditions in every currency at the first decision stage; the former for the base currency ℓ and the latter for the foreign currencies $c \in C$. In each case, we equate the sources and the uses of funds in the respective currency. The availability of funds stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of funds include the total expenditures for the purchase of assets and currency options (for the cash equation of the base currency only), and outgoing currency exchanges in the spot market. Note that the entire budget is placed in the available securities; that is, we don't have investments in risk-free interest rate (T-Bills), nor do we have borrowing. These could be simple extensions of the model. As in all models in previous chapters, all currency transactions are made through the base currency. We do not allow direct transactions between foreign currencies.

Equations (7.2d) and (7.2e) impose the cash balance conditions in every currency at subsequent decision states. Now the total availability of cash comes from exogenous reserves, if any, revenues from the sale of asset holdings in the portfolio at hand, and potential payoffs from the exercise of currency option contracts decided at the predecessor node. Again, the uses of cash include the purchase of assets and currency options with maturity one period forward, and outgoing currency exchanges in the spot market. The cash flows (purchases and payoffs) associated with currency options enter only the cash balance equations of the base currency.

The final value of the portfolio at leaf node $n \in \boldsymbol{N_T}$ is computed in (7.2f). This equation expresses the total terminal value of the portfolio in units of the base currency. The total terminal value reflects the proceeds from the liquidation of all final asset holdings at the corresponding market prices and the payoffs of currency put options expiring at the end of the horizon. The revenues in foreign currencies are converted to the base currency by applying the respective spot exchange rates at the end of the horizon.

Constraints (7.2g) limit the total number of currency put options that can be purchased on each foreign currency. The total position in put options of each foreign currency is restricted by the total value of assets that are held in the respective currency after the portfolio revision. The idea is that currency puts are used for hedging purposes only, and can cover up to the foreign exchange rate exposure of the portfolio held at the respective decision state.

Equation (7.2h) defines the return of the portfolio during the planning horizon at leaf node $n \in \boldsymbol{N}$. Constraint (7.2i) imposes a minimum target bound, μ, on the expected portfolio return over the planning horizon. Constraints (7.2j) and (7.2k) are the definitional constraints for determining `CVaR`, while equation (7.2l) defines portfolio losses as negative returns. Equations (7.2m) enforce the balance constraint for each asset, at the first decision stage, while equations (7.2n) similarly impose the balance constraint for each asset, at subsequent decision states. These equations determine the resulting composition of the revised portfolio after the purchase and sale transactions of assets, the first for the root node and the second for the remaining nodes of the scenario tree. Short positions in assets are not allowed, so, constraints (7.2o) ensure that the units of assets purchased, as well as the resulting holdings in the rebalanced portfolio are nonegative. Finally, constraints (7.2p) and (7.2q), restrict the sales of each asset by the corresponding holdings in the portfolio at the time of a rebalancing decision.

Starting with an initial portfolio and using a representation of uncertainty for the asset prices and exchange rates by means of a scenario tree, as well as the prices and payoffs of the currency put options at each decision node of this scenario tree, the multistage portfolio optimization model determines optimal decisions under the contingencies of the scenario tree. The portfolio rebalancing decisions at each node of the tree specify not only the allocation of funds across markets but also the position in each asset. Moreover, currency options contracts are appropriately determined so as to hedge the currency risk exposure of the foreign investments during the holding period (i.e., until the next portfolio rebalancing decision). We employ the model so as to analyze empirically the effectiveness of trading strategies based on currency options in controlling the currency risk of international portfolios.

Portfolio Optimization Model with Currency Forward Contracts

$$\min \quad z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N_T}} p_n y_n \tag{7.3a}$$

$$\text{s.t.} \quad h_\ell^0 + \sum_{i \in I_\ell} v_{i\ell}^0 \pi_{i\ell}^0 (1-\delta) + \sum_{c \in C} v_{c,e}^0 (1-d) = \sum_{i \in I_\ell} x_{i\ell}^0 \pi_{i\ell}^0 (1+\delta) + \sum_{c \in C} x_{c,e}^0 (1+d) \tag{7.3b}$$

$$h_c^0 + \sum_{i \in I_c} v_{ic}^0 \pi_{ic}^0 (1-\delta) + \frac{1}{e_c^0} x_{c,e}^0 = \sum_{i \in I_c} x_{ic}^0 \pi_{ic}^0 (1+\delta) + \frac{1}{e_c^0} v_{c,e}^0, \quad \forall\, c \in C \tag{7.3c}$$

$$h_\ell^n + \sum_{i \in I_\ell} v_{i\ell}^n \pi_{i\ell}^n (1-\delta) + \sum_{c \in C} v_{c,e}^n (1-d) + \sum_{c \in C} u_{c,f}^{p(n)} =$$

$$\sum_{i \in I_\ell} x_{i\ell}^n \pi_{i\ell}^n (1+\delta) + \sum_{c \in C} x_{c,e}^n (1+d), \quad \forall\, n \in \boldsymbol{N_T} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{7.3d}$$

$$h_c^n + \sum_{i \in I_c} v_{ic}^n \pi_{ic}^n (1-\delta) + \frac{1}{e_c^n} x_{c,e}^n (1-d) =$$

$$\sum_{i \in I_c} x_{ic}^n \pi_{ic}^n (1+\delta) + \frac{1}{e_c^n} v_{c,e}^n + \frac{1}{f_c^{p(n)}} u_{c,f}^{p(n)}, \quad \forall\, c \in C, \quad \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{7.3e}$$

$$V_\ell^n = \sum_{i \in I_\ell} w_{i\ell}^{p(n)} \pi_{i\ell}^n + \sum_{c \in C} \left[u_{c,f}^{p(n)} + e_c^n \Big[\sum_{i \in I_c} w_{ic}^{p(n)} \pi_{ic}^n - \frac{1}{f_c^{p(n)}} u_{c,f}^{p(n)} \Big] \right]$$

$$\forall\, n \in \boldsymbol{N_T} \tag{7.3f}$$

$$0 \le u_{c,f}^n \le \sum_{i \in I_c} w_{ic}^n \Big(\sum_{m \in S_n} p_m e_c^m \pi_{ic}^m \Big), \quad \forall\, c \in C, \quad \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{7.3g}$$

and also constraints (7.2h) – (7.2q).

Positions in short-term currency forwards shell the value of foreign investments against potential reductions in exchange rates. However, as we noted earlier, by fixing the exchange rate for forward transactions, these contracts also forgo potential gains in the event of favorable movements in the exchange rates. This is the penalty "paid" for the protection against downside risk. We consider currency forward contracts with a single-period term (one month in our implementation). Hence, forward contracts decided in one period are executed in the next decision period. Positions in foreign currency forward contracts are introduced as decision variables ($v_{c,f}^n$) at each decision state of the multistage portfolio optimization program. Hence, these decisions are determined jointly with the corresponding portfolio rebalancing decisions in an integrated manner. Forward positions in different currencies are not explicitly connected. The model is allowed to chose different coverage of the foreign exchange exposures in the different currencies (i.e., different hedge ratios across currencies). This reflects a flexible selective hedging approach.

The primary differences between this model formulation and the previous model that employed currency options concern the cash balance constraints, and the valuation of the portfolio at the end of the planning horizon. Equation (7.3b) imposes the cash balance condition for the base currency ℓ, in the first stage. Again, total availability of cash stems from initially available reserves, revenues from sales of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. The uses of cash include the total expenditures for the purchase of assets and outgoing currency exchanges in the spot market. Equation (7.3c) similarly imposes the initial cash balance constraints for foreign currencies $c \in C$.

Equations (7.3d) and (7.3e) impose the cash balance conditions for every currency at subsequent decision states. These equations additionally account for the cashflows associated with currency forward contracts that were decided at the predecessor decision state.

The final value of the portfolio at the leaf node $n \in \boldsymbol{N_T}$ is computed by equation (7.3f). Again, the total terminal value of the portfolio is expressed in units of the base currency. The total terminal value reflects the proceeds from the liquidation of the final asset holdings at the corresponding market prices and the proceeds of outstanding forward contracts in foreign currencies. The revenues in foreign currencies are converted to the base currency by applying the respective spot exchange rates at the end of the horizon, after settling the outstanding forward contracts.

Constraints (7.3g) limit the currency forward contracts. The amount of a forward contract in a foreign currency is restricted by the expected value of all asset holdings in the respective currency after the revision of the portfolio at that state. This ensures that forward contracts are used only for hedging, and not for speculative purposes. The right-hand side of (7.3g) reflects the expected value of the respective foreign positions at the end of the holding period. The expectation is taken over the discrete outcomes of the successor nodes (S_n) at the decision state $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$.

7.2.2 Scenario Generation

The scenario generation is a critical step for the entire modelling process. A set of representative scenarios is needed that adequately depicts the anticipated evolution of the underlying financial primitives (asset returns and exchange rates) and is consistent with market observations and financial theory. The currency options are priced consistently with the postulated scenario sets according to the procedure described in the appendix. We use the moment-matching scenario generation method, developed by Høyland and Wallace [95] and Høyland et al. [93], that has been presented in previous chapters. The outcomes of the domestic asset returns and exchange rates at each stage of the scenario tree are generated so that their first four marginal moments and correlations match their respective historical values. Thus, the outcomes on the scenario tree reflect the empirical distribution of the random variables as implied by historical observations.

We analyze the statistical characteristics of the historical data of exchange rates covering the period 05/1988–11/2001 that were used in static and dynamic tests. As we can see from Table 7.1, the monthly variations of spot exchange rates exhibit skewed distributions; they also exhibit considerable variance in comparison to their mean. They also exhibit excess kurtosis, implying heavier tails than the normal distribution. Jacque-Bera tests on these data indicate that normality hypotheses cannot be accepted. [1] This motivated us to employ the moment-matching scenario generation procedure and also to adopt an option valuation approach that accounts for the higher moments exhibited by

[1] The Jacque-Bera statistic has a $\mathcal{X}^2$ distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.991 and 9.210, respectively. Therefore, the normality hypothesis is rejected when the Jacque-Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

exchange rate fluctuations.

Statistical Characteristics of Monthly Proportional Spot Exchange Rate Changes					
Exchange Rate	Mean	Std. Dev.	Skewness	Kurtosis	Jacque-Bera Statistic
UKtoUS	-0.116%	2.894%	-0.755	5.672	102.39
GRtoUS	-0.124%	3.091%	-0.215	3.399	5.69
JPtoUS	0.030%	3.615%	0.942	6.213	105.27

Table 7.1: Statistical characteristics and Jacque-Bera statistic for normality of historical monthly changes of spot exchange rates over the period 05/1988–11/2001.

7.3 Currency Hedging Strategies

The question concerning effective ways to hedge the currency risk exposure of international financial portfolios has troubled researchers. Over the years, there has been considerable controversy on this question. See, for example, the literature reviewed in sections 2.2, 3.1 and 7.1. In favor of not hedging is the observation that, historically, changes in exchange rates have had fairly low correlations with foreign stock and bond returns. This lack of a systematic relationship could, in principle, lower portfolio risk. Another argument is that over a long horizon, currency movements cancel out — the mean-reversion argument. In other words, exchange rates have expected return of zero in the long run. On the other hand, for active portfolio managers who are concerned with shorter-term horizons, it is important to take into account the impact of currency movements on the risk-return characteristics of international portfolios. Moreover, currency returns tend to be episodic; in other words, there can be sufficient volatility in exchange rates in the short run that needs to be controlled with appropriate hedging means. More importantly, currency movements tend to exhibit some degree of persistence (volatility clamping). For these reasons, appropriate hedging strategies are being actively sought by researchers and practitioners to improve the performance of international portfolios.

A key question is the identification of effective instruments and appropriate strategies for controlling the currency risk in international portfolios. In dynamic portfolio management settings this becomes an especially challenging problem. The selection of incorrect hedging instruments and strategies can be just as costly as using totally unhedged positions. In this chapter we attempt to shed some light to the problem through extensive empirical experiments. We use the multistage stochastic programming model as a framework to assess the relative performance of alternative strategies that employ either currency forward contracts or currency options for hedging purposes.

A currency forward contract constites an obligation to sell (or to buy) a certain amount of a foreign currency at a specific future date, at a predetermined exchange rate. Therefore, the forward contract eliminates the downside risk for the amount of the transaction, but at the same time it forgoes the upside potential in the event of rising exchange rates. By contrast, a currency put option provides insurance against downside risk, while retaining upside potential as the option is simply not exercised if the exchange rate rises. So, currency forward contracts can be considered as more rigid hedge tools in comparison to currency options.

In this study we experiment with two different trading strategies involving currency options, that have different payoff profiles.

Using a Single Put Option

By buying a European put currency option, the investor acquires the right, but not the obligation, to sell a certain amount of foreign currency at a specified rate (exercise price) at the end of the option's expiration period. The maximum loss the investor may incur is the premium paid to buy the currency option; this cost is incurred when the option expires without being exercised. The option yields a profit if the exchange rate falls. Of course a small fall in the value of the foreign currency will not guarantee a profit, because the currency must fall sufficiently below the strike price to cover the premium before any profit is earned. This premium (option price) is lowest for deep "out-of-the-money" puts — whose strike price is much lower than the current exchange rate — and highest for deep "in-the-money" puts — whose strike price is much higher than the current exchange rate.

In the numerical tests we consider put options for each foreign currency with three different strike prices ("in-the-money, ITM, "at-the-money", ATM, and "out-of-the-money", OTM). These three options constitute the set of available options, J_c, for each foreign currency $c \in C$. All the options have a one-month maturity that matches the duration of each holding period in the model. The ATM options have strike prices equal to the respective spot exchange rates at the time of issue. As decisions are considered at all (non-leaf) nodes of the scenario tree, the strike prices of the ATM options are equal to the scenario dependent exchange rates specified for the corresponding node of the scenario tree. The ITM and OTM options are similarly specified. The ITM and OTM put options have strike prices that are 5% higher, respectively 5% lower, than the corresponding spot exchange rates at the respective decision state. These levels of option strike prices have been chosen fairly arbitrarily. Obviously, a larger set of options with different strike prices and cost can be easily included in the model.

The model is allowed to take only long positions in these put options. Thus, we additionally have the non-negativity constraints for the positions in options in the model 7.2:

$$npc_{c,j}^{n} \geq 0, \quad \forall j \in J_c, \ \ \forall c \in C, \ \ \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$$

Using a BearSpread of Put Options

A BearSpread is a strategy composed of two put options with the same expiration date. The strategy involves a long ITM put and a short OTM put. The strike prices of the constituent options are set as described above.

The maximum payoff (in downturns) of this strategy is the difference between the two strike prices, minus the net premium for the two options. The break-even point is calculated as the difference between the higher strike price of the ITM option and the net premium. The maximum loss is again the net premium for the positions in the two options.

Let $npc^n_{c,ITM}$ and $npc^n_{c,OTM}$ be the long position in the ITM and the short position in the OTM currency put option, respectively, constituting a BearSpread position in foreign currency $c \in C$. To incorporate the BearSpeard strategy in the optimization model 7.2, we additionally impose the following constraints:

$$\begin{aligned} npc^n_{c,ITM} &+ npc^n_{c,OTM} = 0, \quad \forall c \in C, \quad \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \\ npc^n_{c,ITM} &\geq 0, \quad \forall c \in C, \quad \forall n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \end{aligned}$$

The first constraint ensures that the long and short positions in the respective put options have the same magnitude, while the second constraint ensures that the long position is in the ITM option.

The payoff profile of the BearSpread is contrasted in Figure 7.1 to that of a long position in an OTM currency put option. We investigate empirically the relative performance of these strategies in an international portfolio management problem. The aim of the numerical tests is to identify the most suitable instruments and hedging strategies that enhance the performance of international diversified portfolios.

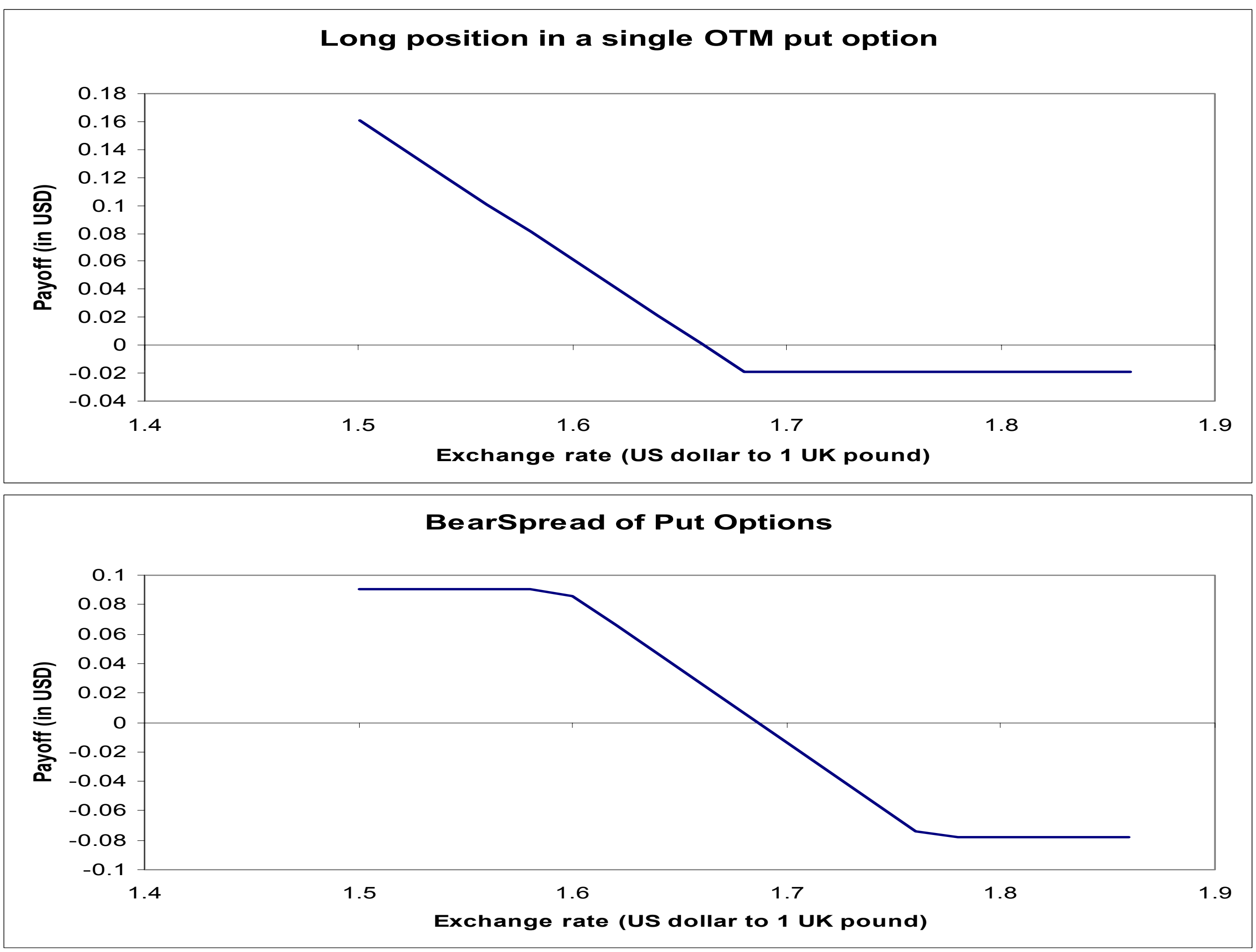


Figure 7.1: Payoff patterns of currency option strategies.

7.4 Empirical Results

As past research suggest, currency risk is a main source of overall risk of international diversified portfolios (given that the market risk associated with these positions is partly hedged through international diversification). Thus, controlling the currency risk is an important task for controlling and improving the performance of international investments. This study examines the effectiveness of controlling the currency risk for international diversified mixed asset portfolios (stock and bond indices) via different hedge tools. Several hedging strategies, using either forward contracts or currency options are evaluated and compared each other.

We solved single-stage as well as two-stage instances of the models described above. The major research studies are the following:

- Forwards vs currency options for currency hedging,
- Alternative hedging strategies with currency options,
- Single-stage vs two-stage models with currency options.

First, we investigate whether currency hedging through forward contracts is an effective way of hedging the market risk, or currency options are superior hedging instruments. We compare the performance of forwards versus currency options both in static, as well as in dynamic tests. In all tests we use selective hedging strategies.

Second, we analyze the alternative performance of different trading strategies of multiple currency options. There are a number of different trading strategies of these options. We are concentrated on those strategies that hedge against unfavorable exchange rate movements. Giving access to such currency options, and given each time the specific trading strategy using combinations of these options on each exchange rate, the model [6.2] decides the optimal exposure to the market risk (the positions in stocks) and the optimal position in hedge (the position in currency options) in order to achieve the minimum downside risk at the level of the desirable expected return. Forward positions are not allowed.

Finally, we extent the model in a multistage setting. The model permits rebalancing in each intermediate stage, where existing currency options are exercised, and new option contracts are purchased. We investigate the possible improvements in the performance of international portfolios using multistage models, over their single-stage counterparts.

In this section we present both risk-return efficient frontiers, as well as results from dynamic tests. These dynamic tests consist of backtesting experiments over a rolling horizon of 43 months, from 05/1998 to 11/2001. At each month we use the historical data from the previous ten years to calculate the four marginal moments and correlations of the random variables, that will be the data to the scenario generation procedure. This procedure generates a scenario tree of the general for that has been presented in chapter 4. Moreover, we price the respective currency options on each particular node of this scenario tree, using the methodology described in the appendix of this chapter. Then the scenario tree and the option prices are used as inputs to the portfolio optimization problem. Each month we solve the respective optimization model (single or multistage) and record the optimal portfolio (the one decided in the first stage). The clock is advanced one month and the realized return of this optimal portfolio is calculated from the observed market values of the assets, the observed exchange rates at that time and the payoff from currency options. The same procedure is then repeated next month and the ex post realized returns are compounded. We run multiple such backtesting experiments, using the CVaR risk measure to control for risk in all cases.

7.4.1 Efficient Frontiers

In this part we examine the impact of forwards and currency options, by comparing the risk/return characteristics of using each one of these instruments to hedge against the currency risk of the international porftolios, with the totally unhedged portfolio selection. Thus, we examine the effects of hedging, by comparing hedging strategies in terms of efficient frontiers. We analyze the effects of adding currency options in portfolios consisting of international stock and bond indices. Figure 7.2 contrasts the efficient frontiers of international portfolios on August 2001 using the CVaR risk measure, of hedged portfolios using currency options or forward contracts, versus totally unhedged portfolios. We consider two different strategies with currency options: (1)a single put option (either "at-the-money", "in-the-money" or "out-of-the-money" (2)a BearSpread strategy of put options.

The first observation is that risk-return efficient frontiers of currency hedged portfolios (using either forwards or currency options) clearly dominate the efficient frontiers of portfolios without hedging. It is clear that selectively hedged portfolios are preferable to unhedged ones, as they yield lower risk for the same level of expected return. Although unhedged portfolios can reach almost the range of expected return, they exhibit efficient frontiers that extend into a range of higher levels of risk. For all levels of risk, the international frontier represent higher levels of expected return whenever the investor hedged the currency exposure. It is also obvious that hedging the currency risk provides higher benefits, in terms of higher expected returns relative to the unhedged case for the medium and high risk portfolios, rather than for the low risk portfolios. The potential gain from risk reduction is increasing for more aggressive targets of expected portfolio returns.

The second observation is that in ex ante tests, the forward contacts are better hedging instruments than currency options. Regarding the different hedging instruments and strategies, Figure 7.2 shows that, while put options (regardless of the trading strategy) result in dominant efficient frontiers compared to the unhedged case, the forwards exhibit the largest shift upwards of the frontiers relative to currency options. That is, for any value of target expected return, the optimal hedged portfolios with forwards exhibit a lower level of risk. Although currency options improve the performance of international portfolios over the totaly unhedged positions, forward contracts perform even better in static tests.

Among the trading strategies using currency options, we observe that "out-of-the-money" options exhibit frontiers that are closed enough to those obtained from forward contracts, specially in the maximum target expected return. Considering frontiers, we also observe that the BearSpread trading strategy follows, but the differences from the first two are increasing for more aggressive targets of expected portfolio returns. Finally, portfolios with “in-the-money” or “at-the-money” options give almost indistinguishable risk/return efficient frontiers.

In all cases single-stage models have been used.

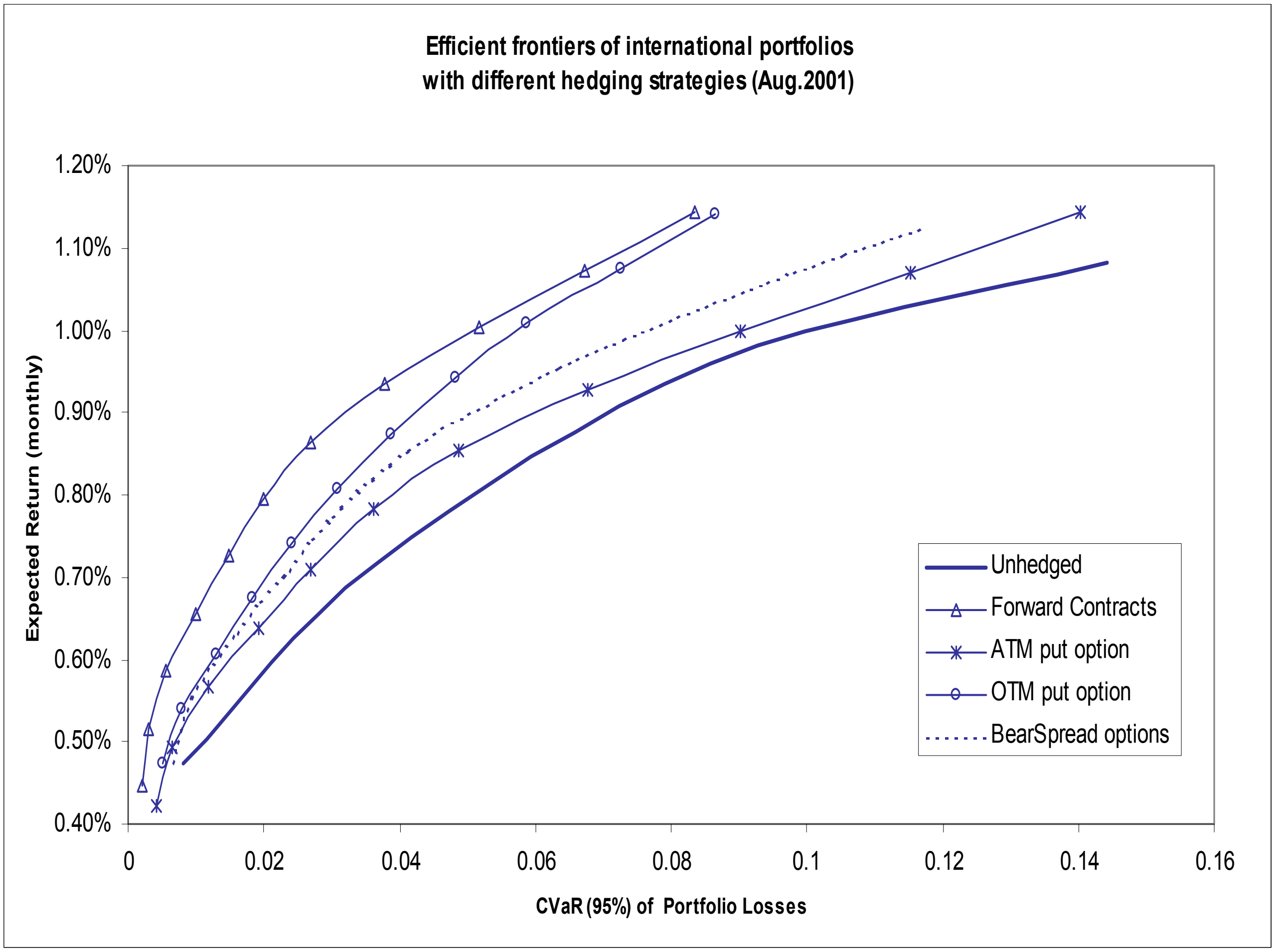


Figure 7.2: Efficient frontiers of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and currency hedging instruments. Comparison of forwards vs currency options.

7.4.2 Dynamic Tests: Comparative Performance of Portfolios with Currency Options

So far we observe that in static tests forward contracts perform better than the currency options. We did backtesting experiments on a rolling horizon basis for a more substantive comparison between portfolios with different types of currency options and hedging strategies.

Single-Stage Models

First, we compare the ex post performance of portfolios with various hedging strategies and instruments in a single-stage setting.

Figure 7.3 contrasts the ex post performance of portfolios with forward contracts versus the performances of portfolios with different strategies of put options, the first graph corresponds to the minimum risk case — i.e., when the models simply minimize the CVaR risk measure at the end of the planning horizon, without imposing any target expected portfolio return at that period, while for the second graph the target expected return during the planning horizon is $\mu = 1\%$. The first observation is that in the minimum risk case of these dynamic tests, forward contracts perform slightly better than single put options, regardless of the exercise price of these options (i.e., "in-the-money", "at-the-money" or "out-of-the-money"). This result indicate the effectiveness of forwards in hedging the currency risk. In this minimum risk case, the models do not pick a large number of currency options, resulting in low currency hedge ratios. When "in-the-money" and "at-the-money" options are considered, the model picks a small number of currency options. The model do not pick any "out-of-the-money" options, thus these portfolios are totally unhedged (their performance is very close with that of totaly unhedged positions in the minimum risk that we have observed in the first graph of Figure 4.3 of chapter 4). Generally, in the minimum risk case, forward contracts and single put options resulted in essentially the same ex post performance.

This Figure also presents the ex post performance of portfolios that use a combination of currency put options that form the BearSpread strategy. We observe a large improvement in the performance of portfolios when we hedge the currency risk using a BearSpread of put options. We observe large returns of the portfolio that uses the BearSpread strategy, over the period between September and October 1999. At that time the US dollar to 1 Japanese yen exchange rate rises significantly. The model decides investments in Japanese Government bond indices and large positions in currency options that form the BearSpread strategy. Thus, in the appreciation period of the foreign currency (depreciation of the reference currency) the options are abandoned, while the model takes advantage of this favorable exchange rate movement. Forward contracts lock at a prespecified exchange rate, and cannot benefit from these favorable currency movements. Thus, when exchange rates are fairly stable, the performance of forwards or currency options is almost the same. But only the options can result in significant payoffs from favorable exchange rate movements, while hedge the opposite direction.

The Figure also shows the performance of more aggressive models. In this case we observe that portfolios with forward contracts exhibit the more stable performance, while currency options offer greater fluctuations. We also observe that the BearSpread strategy of put options exhibit the best ex post performance among the currency hedging instruments. The losses followed the market crisis of September 11th are extremely high regardless of the hedging strategy, an observation that is expected since we only manage the currency risk, while the portfolios are still exposed to market risk.

Multi-Stage Models

We extend the models into a multistage setting. Figure 7.4 again contrasts the ex post performance of portfolios with forward contracts versus the performance of portfolios with different kinds of put options. The first graph corresponds to the minimum risk case, while for the second graph the target expected return during the planning horizon is $\mu = 2\%$. We observe a slightly different pattern in a multistage setting. Although again forward contacts perform better than "at-the-money" and "out-of-the-money" put options, the figure shows a large improvement in the performance of "in-the-money" put options (both in the minimum risk and for more aggressive portfolios). Although portfolios with "in-the-money" put options exhibit larger fluctuations compared to the other strategies, they result in higher realized return. In this multistage setting, the model picks larger number of these options relative to the number decided by the respective single-stage model. Capturing the dynamics of exchange rates, these options can benefit from favorable exchange rate movements that are observed in the periods between September and October 1999, and November and December 2000. The multistage model picks Japanese and German government bonds in these periods, and hedge these positions using currency options on the relative exchange rates, while the single-stage model invests in government bonds in Great Britain and Germany. As we said above, the US dollar to 1 Japanese yen exchange rate rises significantly, leading in considerable gains from these investments when the spot exchnage rate is used for the translation of the foreign to the base currency. Finally, we observe again that the BearSpread of put options results in the best ex post performance. This strategy can benefit from any exchange rate movement, while exhibiting stable performance. Overall, the results indicate that although forward contracts are more effective in hedging the currency risk compared to put options alone, appropriate combinations of put options lead to considerable performance improvements, as they can take advantage of propitious movements of the underlying.

Next, we turn to a comparative assessment of single- and two-stage models for international portfolio management. We examine the performance of the models, in dynamic tests with real market data. Figure 7.5 compares the performance of single versus multistage models with currency options or forward contracts. The first graph corresponds to the experiments with minimum risk measured by the CVaR, without any constraint in the target expected return. The second graph corresponds to more aggressive portfolios (target expected return $\mu = 1\%$ for single-stage models and $\mu = 2\%$ for two-stage models).

The first observation is that in the minimum risk case the models offer stability in the performance of portfolios, while the more aggressive portfolios exhibit large fluctuations, thus offering greater risk. In Figure 7.5 we observe that when forward contracts are used to hedge the currency risk, the multistage model gives slightly better results (an observation we made in chapter 4). When currency options are used, the performance improvements of the multistage models are exceptional. In this Figure we show the performance of portfolios with "in-the-money" put options. The additional realized returns of multistage models with these options are superb, compared to the relative single-stage model. Performance improvements of the multistage model are observed also when we use the

BearSpread strategy of put options. In any case, regardless of the trading strategy of options that is used, multistage models offer considerably greater performance over the single-stage counterparts. The same conclusion can be inferred from Figure 7.5. In all tests, multistage models give the superior performance, while they are never worse. Thus, rebalancing the optioned portfolios in intermediate stages is extremely beneficial. The additional information in the representation of the uncertainty when we extend in to a two-stage setting, improves considerably the performance of the models. Empirical studies on the superiority of multistage stochastic programming model over the myopic one are virtually nonexistent in the literature. The results in this study show superior efficacy of such models in dynamic tests. The asymmetric nature of options together with the multi-period nature of stochastic programming models constitute effective risk management tools for international investments.

Finally, figure 7.6 shows the degrees of currency hedging in each country (% of foreign investments hedged), using either forward contracts (first graph) or currency options that form the BearSpread strategy (second graph), in the multistage setting (based on first-stage decisions for currency hedging). The differences are significant. We observe that when the model with currency options decides to invest in a foreign market, the investments are almost always approximately fully hedged, with the hedge ratio to be between 90% and 100%. When the hedge ratio goes to zero (investments in Japan) the model decides not to invest in the particular market. The patterns are quite different when we use forward contracts. In this case the hedge ratios take any value between zero and one.

7.5 Conclusions

In this chapter we investigate alternative hedging strategies for managing the currency risk of international investment portfolios. We analyze in multi-period setting, the effectiveness of alternative currency hedging instruments and strategies to hedge against the currency risk of international portfolios.

Empirical results indicate that forward contracts are more effective hedging instruments than single put options. Both static as well as dynamic tests show that the performance of portfolios with forward contacts is better than that of put options, except when "in-the-money" put options are used in multistage models. But appropriate combinations of put options, like BearSpread, lead to performance improvements. These strategies of put options can benefit from favorable exchange rate movements, resulting in small periods with high expected return (when they capture the movement of the underlying asset) and large periods with stable performance. Forwards, on the other hand, give more stable performance, but since they lock in a prespecified forward rate, they cannot take advantage of any movements of the underlying.

Finally, we observe a large improvement in the performance of portfolios when we move to multistage models, regardless of the trading strategy and the hedging instruments that are used. In every case, multistage models offer the best ex post performance. This is a very important result, since it strengthens the argument in favor of multistage models for portfolio management, compared to their single-stage counterparts.

The next step in this work is to incorporate derivative securities in multistage stochastic programming models for hedging against multiple risk factors, thus capturing interactions among them. We can price and incorporate currency options to hedge the currency risk, together with options on stock indices to hedge the market risk in an integrated manner. Further restrictions, like liquidity constraints, can be easily incorporated into the model. In that case, investors have to price new options in every stage with different maturities and exercise prices, and decisions must be made, concerning sales or purchases of all instruments, including options.

7.6 Appendix: Pricing Currency Options

To price currency options we employ a valuation procedure developed by Corrado and Su [52], based on an idea of approximating the density of the underlying by a series expansion that was introduced by Jarrow and Rudd [106]. The fundamentals of this valuation method were outlined in chapter 5. Backus et al. [12] have extended this work to price currency options; we adapt their methodology. We want to price a currency call option on the exchange rate e, at an arbitrary node $n_t \in \boldsymbol{N_t} \setminus \boldsymbol{N_T}$ of the scenario tree, at stage $t = 0, ..., T-1$. The currency option has a maturity of one period; i.e., it mature at period $t+1$. n_t is the root node of the subtree that branches out to the successor nodes $n \in S_{n_t}$, of n_t. This is a single-stage subtree that defines the possible outcomes in the value of the underlying during the option's lifetime. Thus, the option is priced on the basis of the distribution represented by the subtree.

We use the following notation on this subtree

e_t	is the underlying exchange rate at node n_t,
$\hat{e}_{t+1}$	is the underlying exchange rate at the maturity of the option (random variable),
r_t^d	is the riskless rate in the base currency for the term of the option,
r_t^f	is the riskless rate in the foreign currency for the term of the option,
K	is the exercise price of the currency options, (USD to 1 unit of the foreign currency).

Let the appreciation rate of the spot exchange rate e for the term of the option, starting from node n_t, be:

$$x_{t+1} = \ell n(\hat{e}_{t+1}) - \ell n(e_t) = \ell n(\frac{\hat{e}_{t+1}}{e_t}) \tag{7.5}$$

Then

$$\hat{e}_{t+1} = e_t exp(x_{t+1}) \tag{7.6}$$

and the conditional distribution of $\hat{e}_{t+1}$ depends on that of x_{t+1}.

The price of a European call option on currency e with strike price K, on node n_t is

$$cc_t(e_t, K) = E_t[M_{t,t+1}(\hat{e}_{t+1} - K)^+] = E_t[M_{t,t+1} max(\hat{e}_{t+1} - K, 0)] \tag{7.7}$$

where $M_{t,t+1}$ is a stochastic discount factor, assumed to be independent of the underlying stochastic exchange rate e. In the "risk-neutral" setting, the call price depends on the conditional distribution of x_{t+1}:

$$\begin{aligned} cc_t(e_t, K) &= exp(-r_t^d)E_t\big[(e_{t+1} - K)^+\big] \\ &= exp(-r_t^d)\int_{\ell n(K/e_t)}^{\infty}(e_t exp(x) - K)f(x)dx \end{aligned} \tag{7.8}$$

where $f(.)$ is the conditional density of x_{t+1}. Integrating, we can find the price of the currency call option. But typically, this conditional density is not analytically available.

In a Gram-Charlier series expansion, a normal density is augmented with additional terms capturing the effects of skewness and kurtosis in the underlying random variable. The resulting truncated series may be viewed as the normal probability density function multiplied by a polynomial that accounts for the effects of departure from normality. The Gram-Charlier series uses the moments of the objective distribution. The underlying theory is described by Johnson, Kotz and Balakrishnan [110] and Kolassa [125]. A Gram-Charlier expansion represents an approximate density function for a standardized random variable that differs from the standard normal in having nonzero skewness and kurtosis. If the one period log-change (x_{t+1}) in the spot exchange rate e has conditional mean μ and standard deviation σ, the standardized variable is:

$$\omega = \frac{x_{t+1} - \mu}{\sigma}. \tag{7.9}$$

A Gram-Charlier expansion defines an approximate density for ω by

$$f(\omega) = \varphi(\omega) - \gamma_1 \frac{1}{3!}D^3\varphi(\omega) + \gamma_2 \frac{1}{4!}D^4\varphi(\omega) \tag{7.10}$$

where

$$\varphi(\omega) = \frac{1}{\sqrt{2\pi}}exp(-\omega^2/2) \tag{7.11}$$

is the standard normal density, D^j denotes the jth derivative of what follows.

Using the Gram-Charlier expansion in (7.10), Backus et al. [12] solved the pricing equation (8.6) and obtained the following result for the price of a currency call option:

$$\begin{aligned} cc_t(e_t, K) &= e_t exp(-r_t^f)N(d) - K exp(-r_t^d)N(d - \sigma) \\ &+ e_t exp(-r_t^d)\varphi(d)\sigma[\frac{\gamma_1}{3!}(2\sigma - d) - \frac{\gamma_2}{4!}(1 - d^2 + 3d\sigma - 3\sigma^2)] \end{aligned} \tag{7.12}$$

where

$$d = \frac{\ell n(e_t/K) - (r_t^f - r_t^d) + \sigma^2/2}{\sigma} \tag{7.13}$$

$\varphi(.)$ is the standard normal density, $N(.)$ is the cumulative distribution of the standard normal, γ_1 and γ_2 are the Fisher parameters for skewness and kurtosis, and μ_i is the i^{th} central moment:

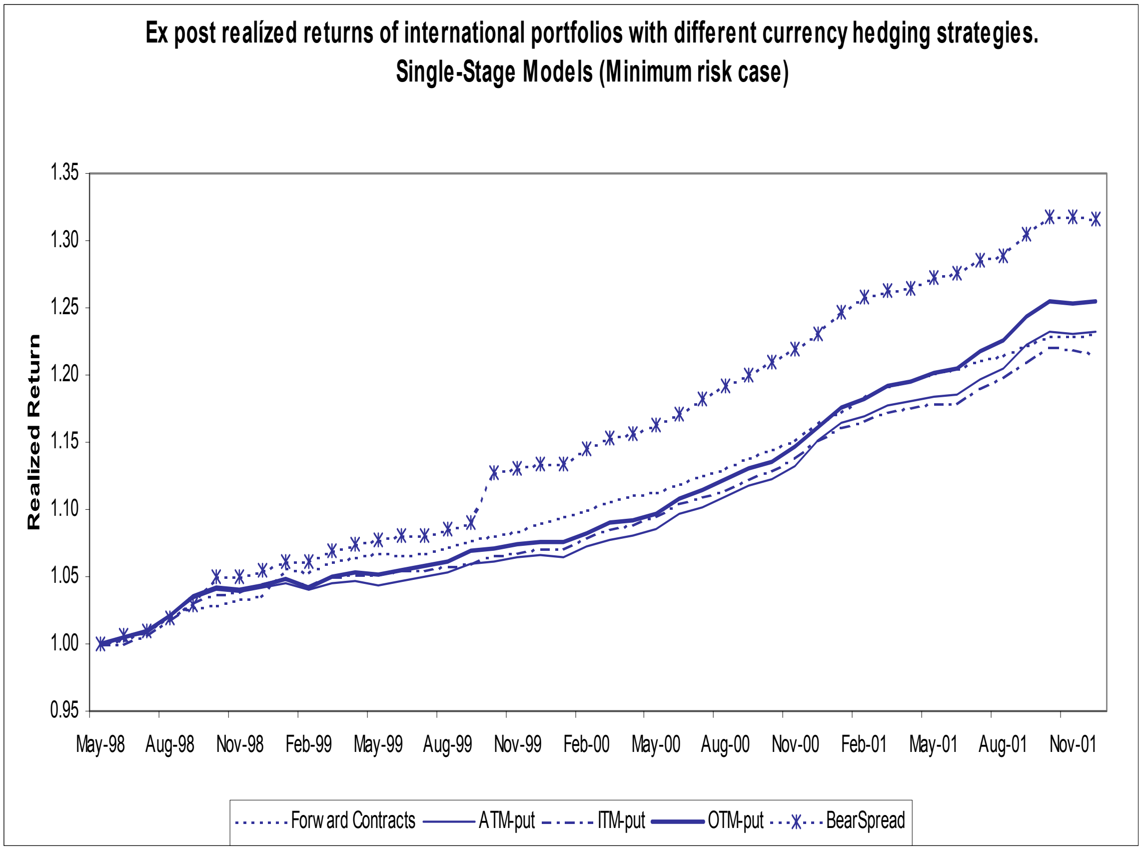
$$\begin{aligned} \gamma_1 &= \frac{\mu_3}{\mu_2^{3/2}} \quad \text{and} \quad \gamma_2 = \frac{\mu_4}{\mu_2^2} \\ \varphi(\omega) &= \frac{1}{\sqrt{2\pi}}exp(-\omega^2/2) \end{aligned}$$

The price of a put option with the same term and strike price K is given by the put-call parity condition:

$$pc_t(e_t, K) = cc_t(e_t, K) + Kexp(-r_t^d) - e_t \tag{7.14}$$

To apply this pricing procedure, we first calculate the first four moments of the underlying exchange rate at node $n_t \in \boldsymbol{N} \setminus \boldsymbol{N_T}$ based on the postulated outcomes for the successor nodes $n \in S_{n_t}$. These estimates of the moments, together with the other parameters which are deterministic (current spot exchange rate e_t, exercise price K, interest rates r_t^f and r_t^d), are used as inputs to equation (7.12) to price the currency call option. The price of the corresponding currency put option is computed by (7.14).

This method of expanding the density of the underlying, has been applied by Abken, Madan and Ramamurtie [1, 2], Brenner and Eom [38], Knight and Satchell [124], Longstaff [136], and Madan and Milne [138].



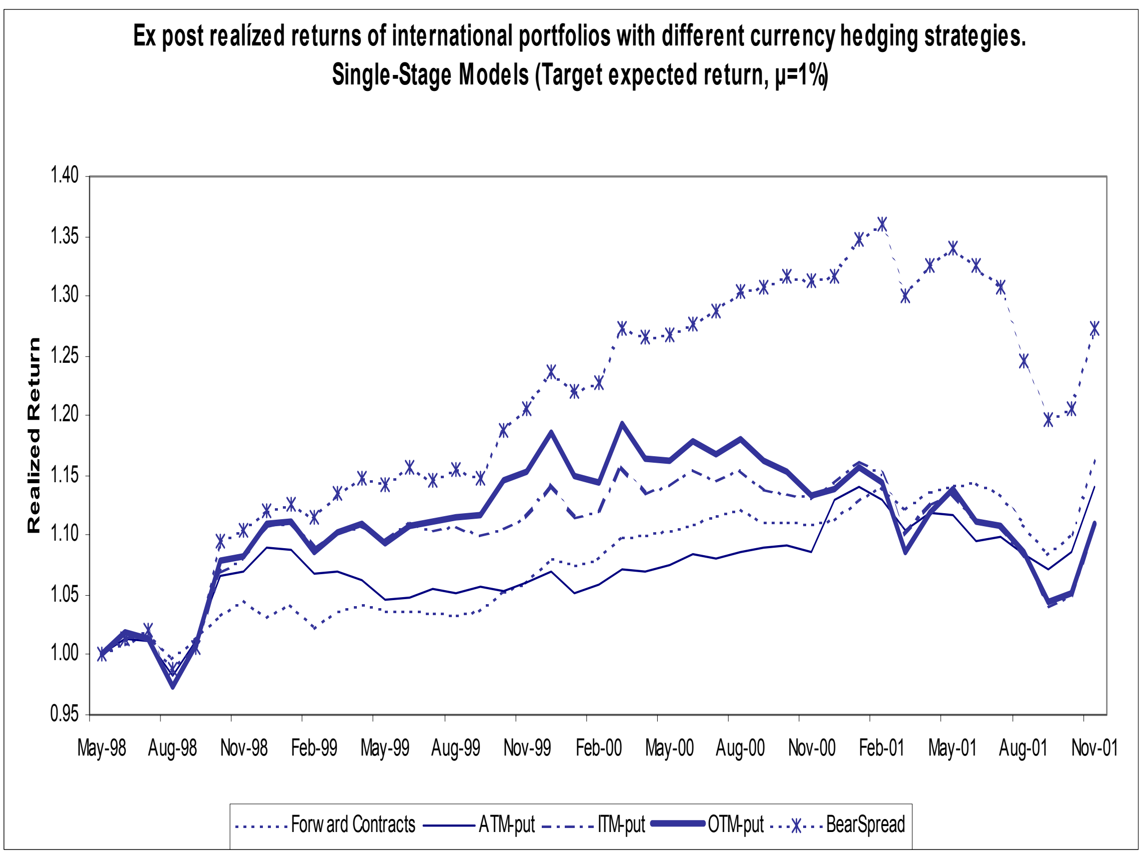
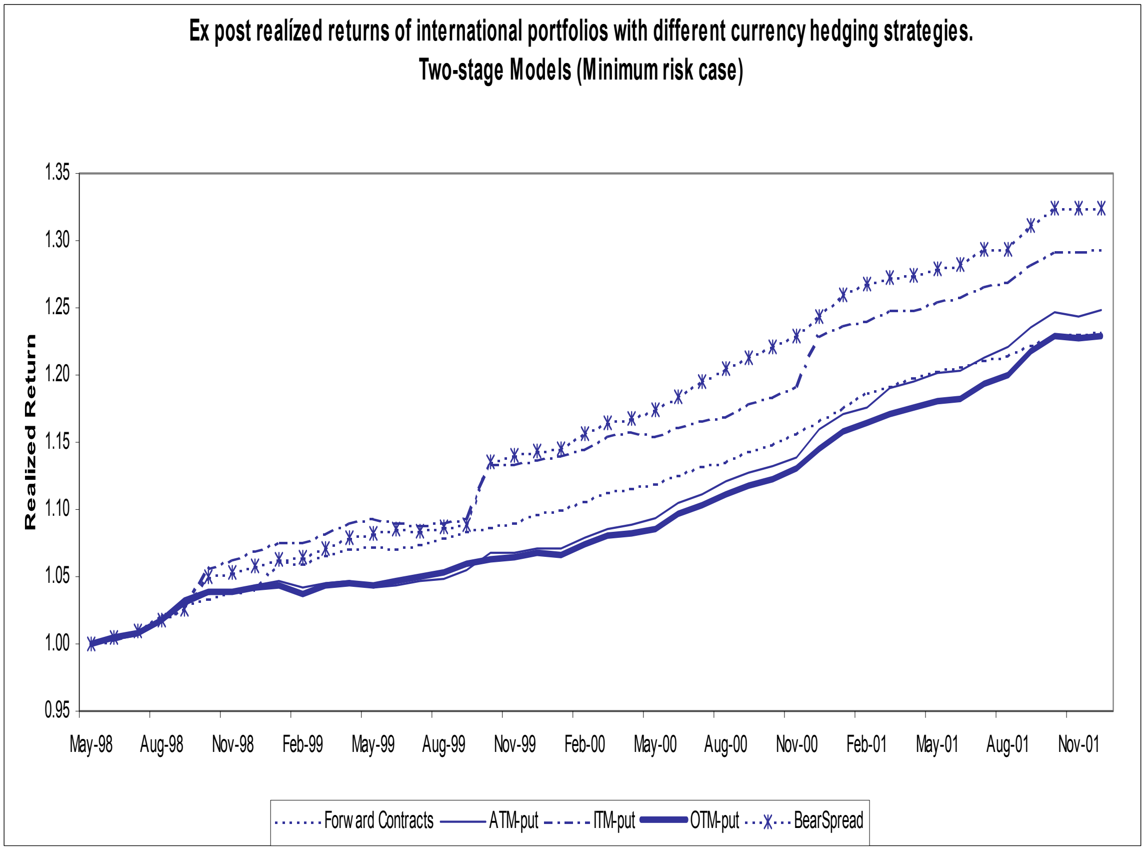


Figure 7.3: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and currency hedging instruments. Comparison of forwards vs currency options. The first graph corresponds to the minimum risk case, while for the second graph the target expected return during the planning horizon is $\mu = 1$. Single-Stage Models.



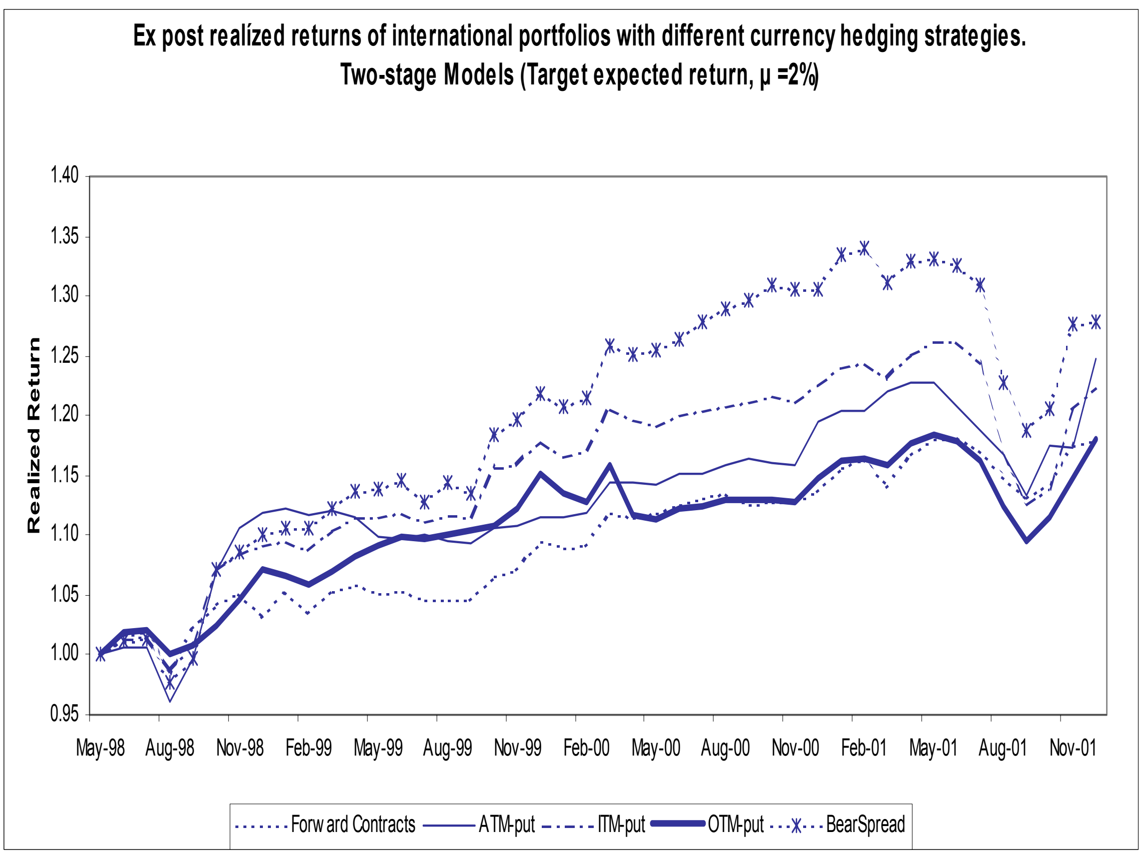
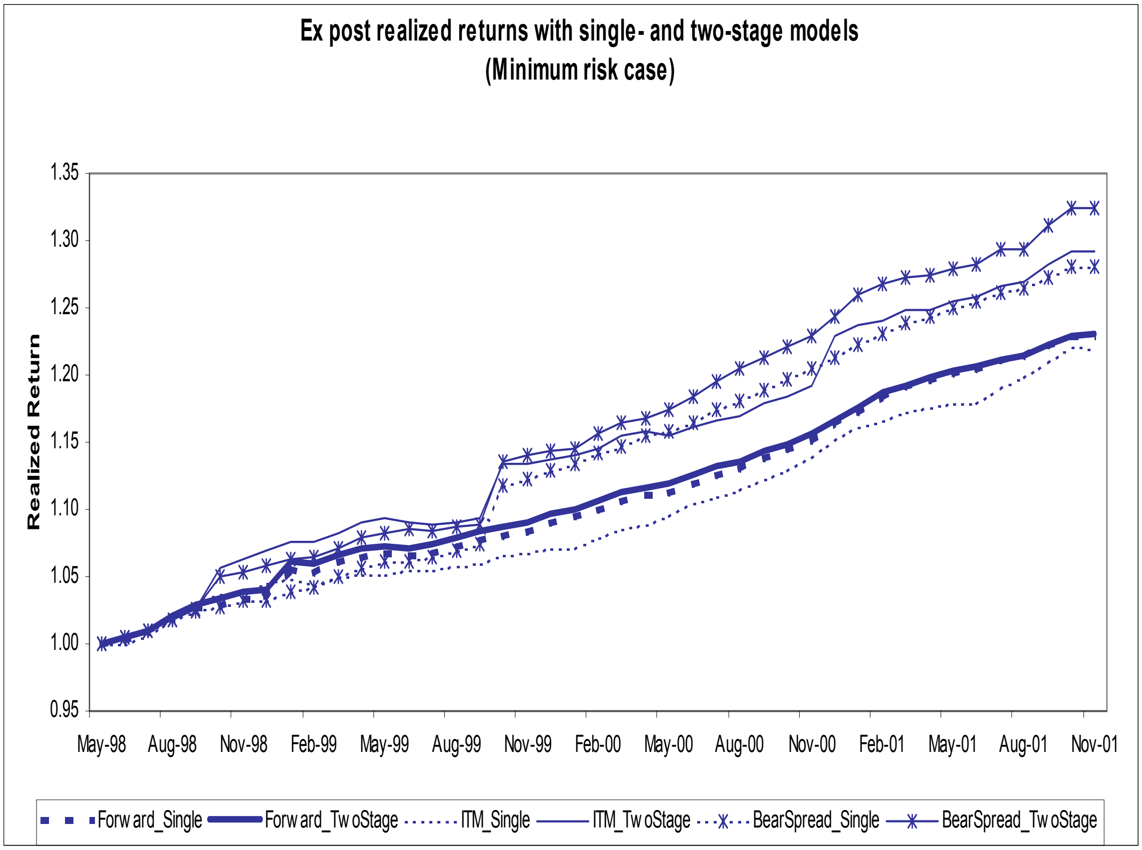


Figure 7.4: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and currency hedging instruments. Comparison of forwards vs currency options. Single-Stage Models.



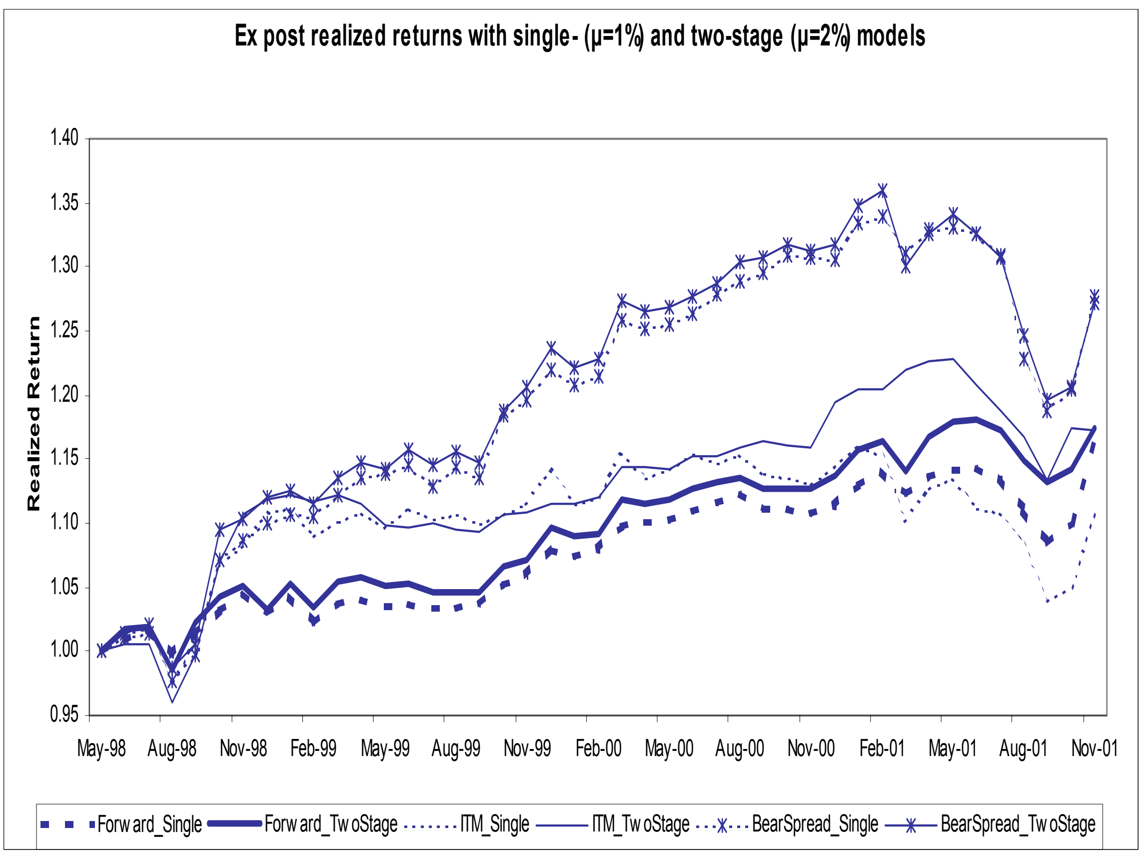
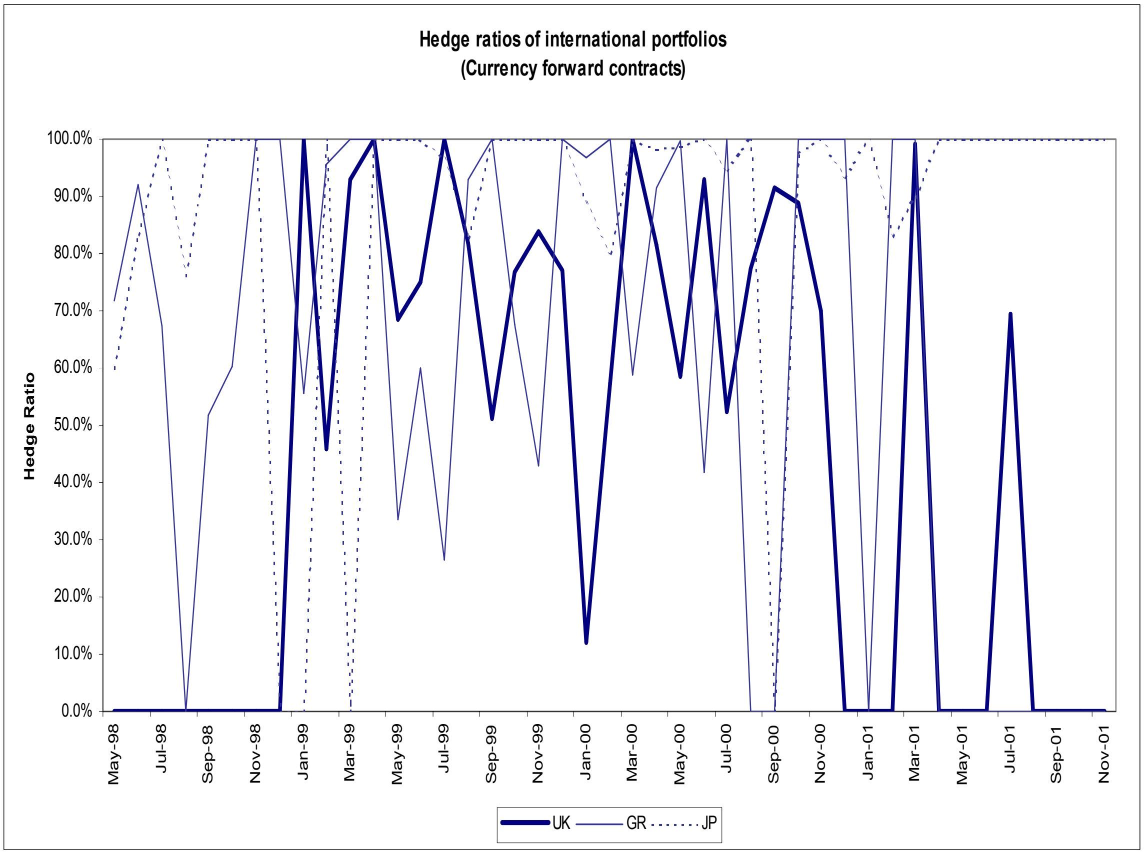


Figure 7.5: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks bonds, and currency hedging instruments. Single vs Multi-Stage Models.



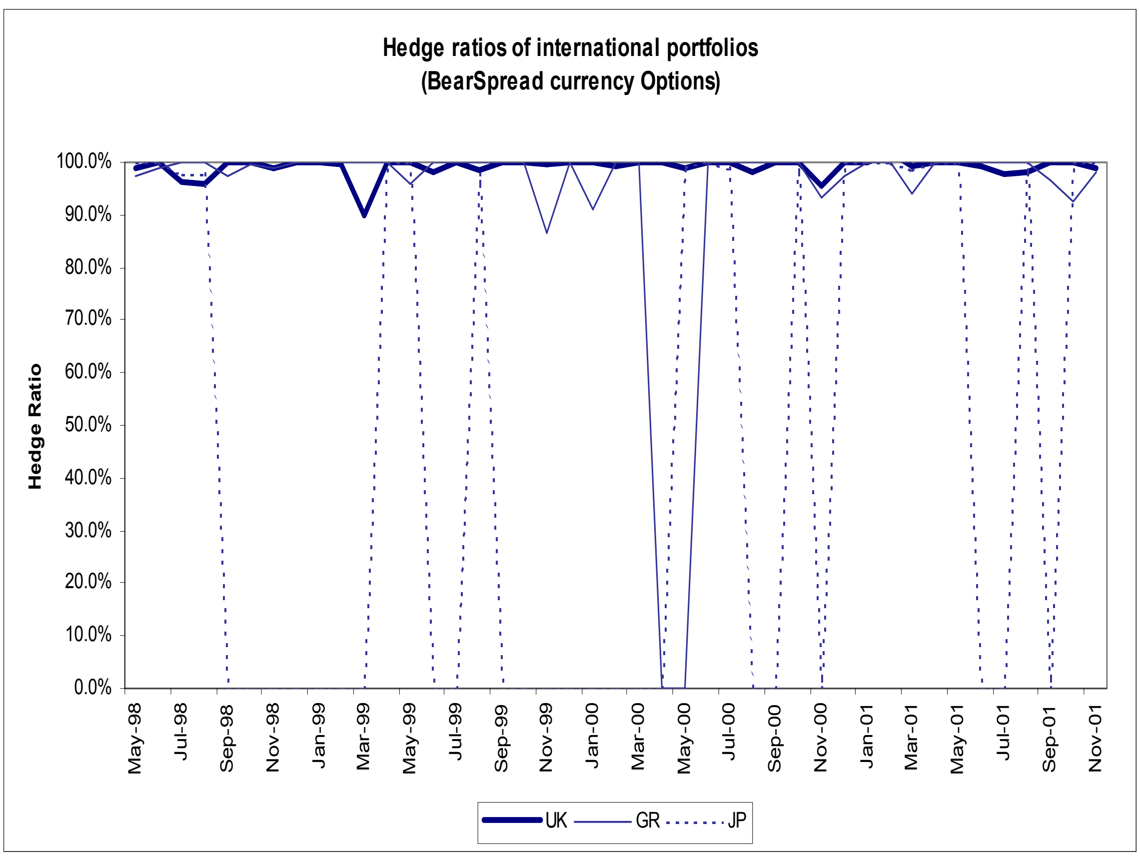


Figure 7.6: Optimal hedge rations against each currency. The first graph corresponds to hedging with Forward contracts, while the second graph corresponds to hedging with a BearSpread strategy of put currency options. In both cases, multi-Stage models have been ran.

Chapter 8

Integrated Stochastic Programming Models to Jointly Control Market and Currency Risks in International Portfolios

In chapter 4 we developed a general international portfolio management model in a multiperiod decision framework. We implemented a stochastic programming model that consider successive portfolio rebalancing decisions at multiple time periods. In chapter 5 we adapted methodologies to price options and incorporate them in scenario-based stochastic programming models. The pricing methods have been implemented and tested using empirical option prices data. Options are appropriate instruments in the risk management framework developed in this thesis, as due to their asymmetric payoff nature they can hedge against unfavorable movements of the underlying asset. Next, in chapter 6 we showed how various kinds of European stock options can be used to hedge against the market risk of international portfolios. Moreover, in chapter 7 currency options have been incorporated in the models in order to hedge the currency risk of international portfolios. In both cases, we showed that the incorporation of options (either stock or currency options) yields significant performance improvements of international portfolios compared to portfolios without options.

In this chapter we combine and extend the developments of all previous chapters in the context of an integrated risk management framework. We show that increasingly integrated views towards total risk management using options are more effective compared to consideration of constituent risk components in isolation. We implement multistage stochastic programming models that incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively. These models shape the overall portfolio risk profile accounting for both the market and the currency risks, thus contributing directly to the objective of total risk management. Empirical results indicate the superiority of this integrated risk management scheme. The simultaneous use of stock and currency options yields the best ex post performance of international portfolios. Moreover, the multistage model clearly outperforms the myopic (single-stage) variant of the model. The integrated modelling framework of this chapter provides a complete basis to comprehensively address all relevant decisions in international portfolio management in a unified manner.

8.1 Introduction

Despite the evolution in the use of derivatives, the issue of hedging the multiple risk factors that affect the performance of international portfolios using options in an integrated framework, has not been tackled previously. What we do model in this chapter, is the hedging or risk management advantage of having additional markets, the options markets beyond the asset markets. We show the effectiveness of using these additional contract markets for hedging purposes. For the international portfolio manager it allows to hedge the volatility of asset returns and exchange rates that he would otherwise have to face for the entire holding period.

In this work we are trying to take into account the dynamic interaction of portfolio components relative to time and combinations of market and currency movements. Multi-stage stochastic programming models together with specific options are suitable for this task. Most money is made or lost because of market and currency movements, and the cause is mishedging. This motivates us to analyze the hedging power of options, and develop multistage stochastic programming models for managing international portfolios that integrate the market and the currency risk by deciding the optimal hedging scheme with options against the risk factors.

Given a choice, investors will not trade options when they are confident about the direction of the market. The reverse of this scenario, though, is that at times investors cannot forecast the direction of the market prior to certain events (which is almost always the case in financial markets), but expect high volatility. In this case, the best choice is to hedge the exposure to market and currency risk using options. This work proves that using options, the investors are able to customize sophisticated strategies for hedging purposes. Options trading also allows complex strategies to be achieved at relatively low cost. Thus options trading expands and improves the risk management tools.

The two important key features of options are first, their insurance protection and second, their upside potential. Through paying a premium to buy the options, a fixed exercise price would be guaranteed. On the other hand, hedging the market risk with stock options will eliminate any chance of losses from a fall in the market, while the only outflow of funds would be the premium payment. Hedging the currency risk with currency options will again eliminate any chance of currency losses. Therefore, if the market or the currency movements are in the favor of the investor and upside potential is available, the option will be abandoned and the holder have the chance to enter into a spot deal. If the movements are against the investor, the options will be exercised at the prespecified exercise prices. Thus the options profile exposes an "asymmetric risk". The most the portfolio investor can lose is the option premium and the most that he can profit is limited only by how far the markets and the currencies move.

In this chapter we adopt the multistage stochastic programming model for international portfolio management developed in chapter 4 by introducing stock and currency options to the investment opportunity set at each decision stage. We optimize the shortfall risk of the total exposure in domestic and foreign markets at the end of the holding period. The scenarios of asset returns and corresponding option prices are used as inputs to optimization programs that determine jointly the levels of hedging (the optimal positions in options) and the investments in particular assets. The model accounts for the effects of portfolio (re)structuring decisions over a multi-period horizon, including positions in options among its permissible decisions. The model's rebalancing decisions must account for the discretionary exercise of expiring options at each decision state. Moreover, stock and currency options must be suitably valued at each decision state of the multistage stochastic program. The non-linear payoff characteristic of options lead to complex, distinctly asymmetric portfolio return distributions with significant moments beyond the mean and the variance. This motivate us to optimize the conditional value-at-risk (CVaR) metric, which is suitable for asymmetric portfolio return distributions.

The inclusion of stock options in international portfolios is intended to cover the relevant market risk exposures from positions in the stock indices. Since the portfolio is international, we additionally allow the model to buy currency options in order to hedge the currency risk associated with foreign investments. Our aim is to control the total portfolio risk using options and to achieve a desirable balance between portfolio risk and expected return. Our optimization models take a holistic view of the risk management problem for the international portfolios. The models incorporate in a unified framework the decisions for investments in various assets across countries and the positions in the appropriate hedging instruments (i.e., options on stock indices and options on exchange rates). These decisions have typically been considered separately in practice. We consider again two kinds of stock options: simple options on the domestic or foreign stock indices and quantos on foreign stock indices.

To show the hedging power of particular options, we start with totally unhedged portfolios, and combining the results from the two previous chapters, we show that the inclusion of either stock or currency options improves considerably the performance of international portfolios. Then, gradually, we move to a more integrative risk management framework where multiple kinds of options are used to hedge against various risk factors in an integrated framework, thus considering any interactions among them. We show that increasingly integrated views towards risk, using options, are more effective compared to the naked position. Finally the models are extended into a multistage setting. We show that this holistic multi-period framework constitute the most effective risk management scheme. This framework captures the decision dynamics over multi-period planning, and considers simultaneously the hedging against multiple factors of risk using options, thus capturing any interdependencies among these risk factors.

Given the increasing recognition of risk management, and the hedging power of options, our contribution is the development of an integrated framework where options are used to hedge the exposure against each one of the risk factors. The options schemes proposed in this study work best — and most efficiently — when we consider simultaneous hedging against the risk factors. Stochastic programming models for international portfolio management derive optimal positions in international assets and optimal hedge levels (positions in currency and stock options). This integrated multi-period risk management framework with options is another contribution. We develop single and multistage models that: First, price the stock and currency options at any decision state by taking in to account that the distribution of the underlying assets is not normal but exhibit skewness and excess kurtosis. Second, incorporate appropriate trading strategies of options in international portfolios for hedging purposes, taking into account the cost of purchasing these options. Third, specify jointly the optimal portfolio composition and the optimal level of hedging through options. This integrated framework allow the models to take into account any interactions among the movements in asset prices and exchange rates.

The rest of this chapter is organized as follows. In section 8.2 we present the formulation of the optimization models for international portfolio management. In section 8.3 we describe our computational tests, and we discuss the empirical results derived both from single-stage as well as multistage models. Finally, section 8.4 concludes the chapter. The pricing of quanto options is given in the Appendix.

8.2 The International Portfolio Management Models

We apply single-stage, as well as two-stage stochastic programs to model an international portfolio management problem. The problem of portfolio (re)structuring is viewed from the perspective of a US investor who may hold assets denominated in multiple currencies. All currency exchanges are executed with respect to the base currency. Thus, to reposition his investments from one market (currency) to another, the investor must first convert to base currency the proceeds of foreign asset sales in the market in which he reduces his presence and then purchase the foreign currency in which he wishes to increase his investments. The current spot exchange rates of foreign currencies to USD apply in the currency exchange transactions.

The investor's portfolio is exposed to market risk in the domestic and foreign markets, as well as to currency exchange risk. To hedge the market risk the investor buys options on stock indices (simple options or quantos). To hedge the currency risk, the investor may buy currency options. Options can be purchased at each decision state, forming a particular trading strategies (different for stock and currency options). We first price the options at any decision state, using the method described in chapter 5 (the option pricing formula is expressed as the sum of three parts: a Black-Scholes option price, plus separate adjustments for nonnormal skewness and kurtosis). We extend this methodology to price quanto options. The pricing method for quantos is given in the Appendix of this chapter. These prices are used as inputs to the optimization models together with the postulated scenarios for the underlying assets (asset returns and proportional changes of exchange rates). The options are European, are purchased at the beginning of the time horizon and at any intermediate stage (the decision on the optimal number of options purchased in every intermediate node is scenario dependent), with maturity the horizon of one period (one month). Thus, at every decision state, the existing options in the portfolio may be the exercised (or abandoned), and new option contracts could be purchased. The portfolio manager starts with a given portfolio and with a set of postulated scenarios about future states of the economy represented in terms of a scenario tree (e.g., see Figure 4.1), as well as corresponding stock and currency option prices depended on the postulated scenarios. This information is incorporated into a portfolio restructuring decision. The composition of the portfolio at each decision point depends on the transactions that were decided at the previous decision point. The portfolio value depends on the outcomes of asset returns and exchange rates realized in the interim period and, consequently, on the discretionary exercise of all options whose purchase was decided at the previous decision point. Another portfolio restructuring decision is then made at that node of the scenario tree based on the available portfolio, the subsequent outcomes of the random variables, and the available options (depending on their estimated prices). At the end of the horizon (one month in case of the single model, two months for the two-stage model) we compute the scenario-dependent value of each investment using its projected price under the respective scenario node. The USD-equivalent value is determined by applying the corresponding estimate of the appropriate spot exchange rate to USD at the end of the period under the same scenario node.

At each decision state, the model decides the optimal portfolio composition in domestic and foreign markets as well as the number of options (stock as well as currency options) in order to form optimal hedging positions against the risk factors. The optimization models incorporate practical considerations (no short sales for assets, transaction costs), specific trading strategies of the options (exogenously determined), and determine portfolios that are optimal i.e., achieve the minimum tail risk at the level of the desirable expected return.

The scenario generation procedure (the moment matching scenario generation method, described in previous chapters) yields scenarios for the asset prices and exchange rates that depict a tree structure. A the root node ($n = 0$), security prices, and spot exchange rates are known. At each future trading date (decision state) a finite number of scenario nodes of the economy is possible. Scenario nodes of the tree store the new information that arrives at the corresponding state.

The cashflow balance constraints in each country, as well as the final value of the portfolio under each scenario, are expressed differently for each type of stock options that is incorporated in the portfolio. This is because quanto options are issued in the investor's reference currency (USD), and thus the investor does not transfer funds in any foreign currency to purchase these options. Simple options, on the other hand, are issued in each currency. We present separately the models for each type of options that is considered.

We use the following notation:

Definitions of sets:

C_0	set of currencies (markets), including the base (reference) currency,
$\ell \in C_0$	the index of the base (reference) currency in the set of currencies,
$C = C_0 \backslash \{\ell\}$	the set of foreign currencies (i.e., excluding the base currency),
I_c	set of asset denominated in currency $c \in C_0$ (these consist of one stock index, one short-term, one intermediate-term, and one long-term governement bond index in each country),
κ	the ordinal index of the stock index security in a set of assets I_c
$\boldsymbol{N}$	is the set of nodes of the scenario tree,
$n \in \boldsymbol{N}$	is a typical node of the scenario tree ($n = 0$ denotes the root node at $t = 0$),
$\boldsymbol{N_t} \subset \boldsymbol{N}$	is the set of distinct nodes of the tree at time period $t = 0, 1, ..., T$,
$\boldsymbol{N_T} \subset \boldsymbol{N}$	is the set of leaf (terminal) nodes at the last period T, that uniquely identify the scenarios,
$S_n \subset \boldsymbol{N}$	is the set of immediate successor nodes of node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$. This set of nodes represents the discrete distribution of the random variables at the respective time period, conditional on the state of node n.
JS_c	the set of available simple stock options in market $c \in C_0$ (differing in terms of their exercise price),
JQ_c	the set of available quanto options in market $c \in C$ (differing in terms of their exercise price),
JC_c	the set of available currency options in market c, (differing in terms of their exercise price).

Input Parameters (Data):

(a). Deterministic quantities:

- b_{ic} initial position in asset $i \in I_c$ of currency $c \in C_0$ (in units of face value),
- h_c^0 initially available cash in currency $c \in C_0$ (surplus if +ve, shortage if -ve),
- δ proportional transaction cost for sales and purchases of assets,
- T the time horizon
- d proportional transaction cost for currency transactions in the spot market,
- μ prespecified target expected return over the planning horizon,
- α the prespecified confidence level (percentile) for the CVaR measure,
- π_{ic}^0 current market price (in units of the respective currency) per unit of face value of asset $i \in I_c$ in currency $c \in C_0$,
- e_c^0 current spot exchange rate for foreign currency $c \in C$,
- f_c currently quoted one-month forward exchange rate for foreign currency $c \in C$,
- $\bar{X}_c$ the fixed exchange rate for value translation of quanto on the stock index S_c of foreign market $c \in C$ (usually, $\bar{X}_c = f_c, \quad \forall c \in C$),
- K_j the strike price of an option ($j \in JS_c$ for simple options, $j \in JQ_c$ for quantos and $j \in JC_c$ for currency options) in the same units as the underlying asset of the option.

(b). Scenario dependent quantities:

- p_n objective probability of occurrence of scenario node $n \in \boldsymbol{N}$,
- e_c^n spot exchange rate of currency $c \in C$ at node $n \in \boldsymbol{N}$,
- π_{ic}^n market price (in units of the respective currency) per unit of face value of security $i \in I_c$ on node $n \in \boldsymbol{N}$
- $cs^n(S_c^n, K_j)$ price of European simple call option $j \in JS_c$ on stock index S_c^n of currency $c \in C_0$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity one period,
- $ps^n(S_c^n, K_j)$ price of European simple put option $j \in JS_c$ on stock index S_c^n of currency $c \in C_0$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity one period,
- $cq^n(S_c^n, K_j)$ price of European quanto call option $j \in JQ_c$ on the underlying $\bar{X}_c S_c^n$, of foreign currency $c \in C$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity one period,
- $pq^n(S_c^n, K_j)$ price of European quanto put option $j \in JQ_c$ on the underlying $\bar{X}_c S_c$, of foreign currency $c \in C$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise price K_j and maturity one period,
- $cc^n(e_c^n, K_{c,j})$ price of European call currency option $j \in JC_c$ on exchange rate of currency $c \in C$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise K_j and maturity one period,
- $pc^n(e_c^n, K_{c,j})$ price of European put currency option $j \in JC_c$ on exchange rate of currency $c \in C$, on node $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$, with exercise K_j and maturity one period.

All exchange rate parameters (e_c, e_c^n, f_c^n) are expressed in units of the base currency per one unit of the foreign currency $c \in C$. Of course, the exchange rate of the base currency to itself is trivially equal to one, $e_\ell = e_c^\ell \equiv 1, \quad \forall n \in \boldsymbol{N}$. The prices cs and ps of call and put simple stock options are expressed in units of the respective currency. The prices cq and pq of call and put quanto options as well as the prices cc and pc of currency call and put options respectively, are expressed in units of the base currency ℓ.

Computed Parameters:

V_ℓ^0 total value (in units of the base currency) of the initial portfolio.

$$V_\ell^0 = h_\ell^0 + \sum_{i \in I_\ell} b_{i\ell}\, \pi_{i\ell}^0 + \sum_{c \in C} e_c^0 \left(h_c^0 + \sum_{i \in I_c} b_{ic}\, \pi_{ic}^0 \right) \tag{8.1}$$

Decision Variables All the variables are decided at the root or at any intermediate node, thus $n \in \boldsymbol{N} \setminus \boldsymbol{N_T}$):

(a). Asset purchase and sale decisions, and resulting holdings after portfolio revision:

- x_{ic}^n units of asset $i \in I_c$ of currency $c \in C_0$ purchased,
- v_{ic}^n units of asset $i \in I_c$ of currency $c \in C_0$ sold,
- w_{ic}^n units of asset $i \in I_c$ of currency $c \in C_0$ in the revised portfolio.

(b). Currency transactions in the spot market:

- $x_{c,e}^n$ units of the base currency exchanged in the spot market for foreign currency $c \in C$,
- $v_{c,e}^n$ units of the base currency collected from a sale of foreign currency $c \in C$.

(c). Option related variables:

- $ncs_{c,j}^n$ purchases of European simple call options $j \in JS_c$ on stock index S_c^n of currency $c \in C_0$, with exercise price K_j and maturity one period,
- $nps_{c,j}^n$ purchases of of European simple put options $j \in JS_c$ on stock index S_c of currency $c \in C_0$, with exercise price K_j and maturity one period,
- $ncq_{c,j}^n$ purchases of European quanto call options $j \in JQ_c$ on the underlying $\bar{X}_c S_c^n$ of foreign currency $c \in C$, with exercise price K_j and maturity one period,
- $npq_{c,j}^n$ purchases of European quanto put options $j \in JQ_c$ on the underlying $\bar{X}_c S_c^n$ of foreign currency $c \in C$, with exercise price K_j and maturity one period.
- $ncc_{c,j}^n$ purchases of European call currency options $j \in JC_c$ on the exchange rate of currency $c \in C$, with exercise price K_j, and maturity one period
- $npc_{c,j}^n$ purchases of European put currency options $j \in JC_c$ on the exchange rate of currency $c \in C$, with exercise price K_j, and maturity one period.

Auxiliary variables:

y_n	auxiliary variables used to linearize the nondifferentiable function in the definition of CVaR of portfolio losses at the end of the planning horizon, $n \in \boldsymbol{N_T}$,
z	the VaR value of terminal portfolio losses (at a prespecified confidence level, percentile α),
V_ℓ^n	the total value of the portfolio at the end of the holding period at node $n \in \boldsymbol{N_T}$ (in units of the base currency),
R_n	return of the international portfolio over the planning horizon, at node $n \in \boldsymbol{N_T}$,
$\bar{R}$	expected return of the international portfolio over the planning horizon.

As in chapter 6, we investigate instances of the portfolio management model that incorporate currency options with either simple options or quantos, but not both. Hence, we need two slightly different model formulations that differ only in the expression of the cashflows. In the case of simple options, the cashflows (initial purchase of the options and collection of the payoffs at the end of the horizon) occur in the respective currencies. On the contrary, in the case of quanto options, all option-related cashflows (initial purchases as well as payoff collections at the end of the planning period) are in the base currency. Of course, when we use quanto options we also allow the purchase of simple options on the stock index S_ℓ in the base currency. (A quanto and a simple option on a stock index in the reference currency are of course equivalent, as $\bar{X}_\ell = 1$). We formulate now the two portfolio optimization models.

Portfolio Optimization Model with Simple and Currency Options

$$\texttt{min} \quad z + \frac{1}{1-\alpha} \sum_{n \in \boldsymbol{N_T}} p_n y_n \tag{8.2a}$$

$$\begin{aligned}
\texttt{s.t.} \quad & h^0_\ell + \sum_{i \in I_\ell} v^0_{i\ell} \pi^0_{i\ell}(1-\delta) + \sum_{c \in C} v^0_{c,e}(1-d) = \sum_{i \in I_\ell} x^0_{i\ell}\pi^0_{i\ell}(1+\delta) + \sum_{c \in C} x^0_{c,e}(1+d) \\
& + \sum_{c \in C} \left\{ \sum_{j \in JC_c} \Big[npc^0_{c,j} * pc^0(e^0_c, K_j) \Big] \right\} \\
& + \sum_{j \in JS_\ell} \Big[ncs^0_{\ell,j} * cs^0(S^0_\ell, K_j) + nps^0_{\ell,j} * ps^0(S^0_\ell, K_j) \Big],
\end{aligned} \tag{8.2b}$$

$$\begin{aligned}
& h^0_c + \sum_{i \in I_c} v^0_{ic}\pi^0_{ic}(1-\delta) + \frac{1}{e^0_c} x^0_{c,e} = \sum_{i \in I_c} x^0_{ic}\pi^0_{ic}(1+\delta) + \frac{1}{e^0_c} v^0_{c,e} \\
& + \sum_{j \in JS_c} \left[ncs^0_{c,j} * cs^0(S^0_c, K_j) + nps^0_{c,j} * ps^0(S^0_c, K_j) \right], \quad \forall\, c \in C
\end{aligned} \tag{8.2c}$$

$$\begin{aligned}
& h^n_\ell + \sum_{i \in I_\ell} v^n_{i\ell}\pi^n_{i\ell}(1-\delta) + \sum_{c \in C} v^n_{c,e}(1-d) + \\
& + \sum_{j \in JS_\ell} \left\{ \Big[ncs^{p(n)}_{\ell,j} * max(S^n_\ell - K_j, 0) + nps^{p(n)}_{\ell,j} * max(K_j - S^n_\ell, 0) \Big] \right\} \\
& + \sum_{c \in C} \left\{ \sum_{j \in JC_c} \Big[npc^{p(n)}_{c,j} * max(K_j - e^n_c, 0) \Big] \right\} \\
& = \sum_{i \in I_\ell} x^n_{i\ell}\pi^n_{i\ell}(1+\delta) + \sum_{c \in C} x^n_{c,e}(1+d) + \sum_{c \in C} \left\{ \sum_{j \in JC_c} \Big[npc^n_{c,j} * pc^n(e^n_c, K_j) \Big] \right\} \\
& + \sum_{j \in JS_\ell} \Big[ncs^n_{\ell,j} * cs^n(S^n_\ell, K_j) + nps^n_{\ell,j} * ps^n(S^n_\ell, K_j) \Big], \\
& \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\}
\end{aligned} \tag{8.2d}$$

$$\begin{aligned}
& h^n_c + \sum_{i \in I_c} v^n_{ic}\pi^n_{ic}(1-\delta) + \frac{1}{e^n_c} x^n_{c,e} + \\
& + \sum_{j \in JS_c} \left\{ \Big[ncs^{p(n)}_{c,j} * max(S^n_c - K_j, 0) + nps^{p(n)}_{c,j} * max(K_j - S^n_c, 0) \Big] \right\} \\
& = \sum_{i \in I_c} x^n_{ic}\pi^n_{ic}(1+\delta) + \frac{1}{e^n_c} v^n_{c,e} \\
& + \sum_{j \in JS_c} \left[ncs^n_{c,j} * cs^n(S^n_c, K_j) + nps^n_{c,j} * ps^n(S^n_c, K_j) \right], \\
& \forall\, c \in C, \quad \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\}
\end{aligned} \tag{8.2e}$$

cont.

$$V_\ell^n = \sum_{i \in I_\ell} w_{i\ell}^{p(n)}\, \pi_{i\ell}^n$$

$$+ \sum_{j \in JS_\ell} \left\{ \left[ncs_{\ell,j}^{p(n)} * max(S_\ell^n - K_j, 0) \,+\, nps_{\ell,j}^{p(n)} * max(K_j - S_\ell^n, 0) \right] \right\}$$

$$+ \sum_{c \in C} \left\{ \sum_{j \in JC_c} \left[npc_{c,j}^{p(n)} * max(K_j - e_c^n, 0) \right] \right\}$$

$$+ \sum_{c \in C} \left\{ e_c^n \left[\sum_{i \in I_c} w_{ic}^{p(n)} \pi_{ic}^n + \sum_{j \in JS_c} \left[ncs_{c,j}^{p(n)} * max(S_c^n - K_j, 0) \right.\right.\right.$$

$$\left.\left.\left. + \, nps_{c,j}^{p(n)} * max(K_j - S_c^n, 0) \right] \right] \right\}, \quad \forall n \in \boldsymbol{N_T} \tag{8.3a}$$

$$\sum_{j \in JS_c} \left[ncs_{c,j}^n * cs^n(S_c^n, K_j) \,+\, nps_{c,j}^n * ps^n(S_c^n, K_j) \right] \le e_c^n w_{k_c}^{p(n)} \pi_{k_c}^n,$$

$$\forall c \in C_0, \;\; \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{8.3b}$$

$$\sum_{j \in JC_c} [npc_j^n] \le \sum_{i \in I_c} w_{i_c}^n \pi_{i_c}^n, \quad \forall c \in C, \;\; \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{8.3c}$$

$$R_n = \frac{V_\ell^n}{V_\ell^0} - 1, \quad \forall n \in \boldsymbol{N_T} \tag{8.3d}$$

$$\bar{R} = \sum_{n \in \boldsymbol{N_T}} p_n R_n, \tag{8.3e}$$

$$\bar{R} \ge \mu, \tag{8.3f}$$

$$y_n \ge L_n - z, \qquad \forall n \in \boldsymbol{N_T} \tag{8.3g}$$

$$y_n \ge 0\,, \quad \forall n \in \boldsymbol{N_T} \tag{8.3h}$$

$$L_n = -R_n, \qquad \forall n \in \boldsymbol{N_T} \tag{8.3i}$$

$$w_{ic}^0 = b_{ic} \,+\, x_{ic}^0 \,-\, v_{ic}^0, \qquad\qquad \forall\, i \in I_c\,, \;\; \forall\, c \in C_0 \tag{8.3j}$$

$$w_{ic}^n = w_{ic}^{p(n)} \,+\, x_{ic}^n \,-\, v_{ic}^n, \qquad\qquad \forall\, i \in I_c\,, \;\; \forall\, c \in C_0, \;\; \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup 0\} \tag{8.3k}$$

$$x_{ic}^n \,\ge\, 0\,, \quad w_{ic}^n \,\ge\, 0\,, \quad \forall i \in I_c\,, \;\; \forall c \in C_0, \;\; \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{8.3l}$$

$$0 \,\le v_{ic}^0 \,\le\, b_{ic}, \qquad \forall i \in I_c\,, \;\; \forall c \in C_0 \tag{8.3m}$$

$$0 \,\le v_{ic}^n \,\le\, w_{ic}^{p(n)}, \qquad \forall i \in I_c\,, \;\; \forall c \in C_0, \;\; \forall\, n \in \boldsymbol{N} \setminus \{\boldsymbol{N_T} \cup \boldsymbol{0}\} \tag{8.3n}$$

This formulation minimizes the CVaR (8.2a) of portfolio losses at the end of the horizon, while constraining the expected portfolio return (8.3e).

Equation (8.2b) impose the cash balance conditions in the base currency ℓ at the first decision stage. We equate the sources and the uses of funds. Total availability of funds stems from initially available reserves, revenues from the sale of initial asset holdings and amounts received through incoming currency exchanges in the spot market. Correspondingly, the uses of funds include the total expenditures for the purchase of assets and stock and currency options and outgoing currency exchanges in the spot market. Note that the entire budget is placed in the available securities that is, we don't have investments in risk-free interest rate (T-Bills), nor do we have borrowing. These could be simple extensions of the model. Note also that linear transaction costs are charged for transactions of assets as well as for currencies. In order to simplify the notation and the formulation, all currency transactions are made through the base currency. Without loss of generality, we do not allow direct transactions between foreign currencies. When reducing holdings in one asset in favor of assets in a different currency, the proceeds from the sale of assets are transferred between respective currencies always through an intermediate conversion to the base currency.

Equation (8.2c) imposes the cash balance conditions in every foreign currency at the first decision stage; again, the availability of funds stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. The uses of funds include the total expenditures for the purchase of assets and stock options in each market and outgoing currency exchanges in the spot market.

Equations (8.2d) and (8.2e) impose the cash balance conditions in every currency at any intermediate decision state (the former for the base currency, the latter for each foreign currency); Now the total availability of funds comes from exogenous reserves, if any, revenues from sale of asset holdings in the portfolio at hand, and the payoffs from the exercise of the options (we consider stock and currency options in the base currency, and only stock options at each foreign currency) decided at the predecessor node. Again the uses of cash include the purchase of assets and options, and outgoing currency exchanges in the spot market.

The final value of the portfolio at leaf node $n \in \boldsymbol{N_T}$ is computed in (8.3a). This equation expresses the total terminal value of the portfolio in units of the base currency. The total terminal value reflects the proceeds from the liquidation of all final asset holdings at the corresponding market prices, the stock option payoffs and the proceeds from exercise of currency options, expiring at the end of the horizon. The proceeds of revenues in foreign currencies (assets and stock option payoffs) are converted to the base currency by applying the respective spot exchange rates at the end of the horizon.

Equation (8.3b) limits the total expenditure for purchases of simple options in each currency. This expenditure is not permitted to exceed the value of the position in the corresponding stock index. This constraint is imposed in order to ensure that options should be purchased so as to cover the exposure in the underlying stock index (i.e., for the intended hedging purpose) and not for speculative purposes.

Equation (8.3c) limits the total number of currency put options that can be purchased in each currency. The total position in put options of each foreign currency is restricted by the total value of assets that are held in the respective currency after the portfolio revision. The idea is that currency puts are used for hedging purposes only, and can cover up to the foreign exchange rate exposure of the portfolio held at the respective decision state.

Equation (8.3d) defines the return of the portfolio during the planning horizon at leaf node $n \in \boldsymbol{N_T}$. Equation (8.3e) defines the expected return of the portfolio at the end of the horizon, while equation (8.3f) imposes a minimum target bound, (μ), on the expected portfolio return over the planning horizon. Constraints (8.3g) and (8.3h) are the definitional constraints for determining

`CVaR`, while equation (8.3i) defines portfolio loss as the negative return. Equations (8.3j) enforce balance constraint for each asset in the first decision stage, while equations (8.3k) similarly impose the balance constraint for each asset at all intermediate decision states. These equations determine the resulting composition of the revised portfolio after the purchase and sale transactions of assets. Short positions in assets are not allowed. So, constraints (8.3l) ensure that the units of assets purchased, as well as the resulting holdings in the rebalanced portfolio are nonegative. Finally, constraints (8.3m) and (8.3n) restrict the sales of each asset by the corresponding holdings in the portfolio at the time of a rebalancing decision.

Portfolio Optimization Model with Quantos and Currency Options

Here, we present separately the model with quanto and currency options.

$$\texttt{min} \quad z + \frac{1}{1-\alpha}\sum_{n\in \boldsymbol{N_T}} p_n y_n \tag{8.4a}$$

$$\begin{aligned}\texttt{s.t.} \quad & h^0_\ell + \sum_{i\in I_\ell} v^0_{i\ell}\pi^0_{i\ell}(1-\delta) + \sum_{c\in C} v^0_{c,e}(1-d) = \sum_{i\in I_\ell} x^0_{i\ell}\pi^0_{i\ell}(1+\delta) + \sum_{c\in C} x^0_{c,e}(1+d) \\ & + \sum_{c\in C}\left\{\sum_{j\in JC_c}\left[npc^0_{c,j} * pc^0(e^0_c, K_j)\right]\right\} \\ & + \sum_{j\in JS_\ell}\left[ncs^0_{\ell,j} * cs^0(S^0_\ell, K_j) + nps^0_{\ell,j} * ps^0(S^0_\ell, K_j)\right] \\ & + \sum_{c\in C}\left[\sum_{j\in JQ_c}\left[ncq^0_{c,j} * cq^0(S^0_c, K_j) + npq^0_{c,j} * pq^0(S^0_c, K_j)\right]\right]\end{aligned} \tag{8.4b}$$

$$h^0_c + \sum_{i\in I_c} v^0_{ic}\pi^0_{ic}(1-\delta) + \frac{1}{e^0_c}x^0_{c,e} = \sum_{i\in I_c} x^0_{ic}\pi_{ic}(1+\delta) + \frac{1}{e^0_c}v^0_{c,e} \quad \forall\, c\in C \tag{8.4c}$$

$$\begin{aligned} & h^n_\ell + \sum_{i\in I_\ell} v^n_{i\ell}\pi^n_{i\ell}(1-\delta) + \sum_{c\in C} v^n_{c,e}(1-d) + \\ & + \sum_{j\in JS_\ell}\left\{\left[ncs^{p(n)}_{\ell,j} * max(S^n_\ell - K_j, 0) + nps^{p(n)}_{\ell,j} * max(K_j - S^n_\ell, 0)\right]\right\} \\ & + \sum_{c\in C}\left[\sum_{j\in JQ_c}\left[ncq^{p(n)}_{c,j} * max(\bar{X}_c S^n_c - K_j, 0) + npq^{p(n)}_{c,j} * max(K_j - \bar{X}_c S^n_c, 0)\right]\right] \\ & + \sum_{c\in C}\left\{\sum_{j\in JC_c}\left[npc^{p(n)}_{c,j} * max(K_j - e^n_c, 0)\right]\right\} \\ & = \sum_{i\in I_\ell} x^n_{i\ell}\pi^n_{i\ell}(1+\delta) + \sum_{c\in C} x^n_{c,e}(1+d) + \sum_{c\in C}\left\{\sum_{j\in JC_c}\left[npc^n_j * pc^n(e^n_c, K_j)\right]\right\} \\ & + \sum_{j\in JS_\ell}\left[ncs^n_j * cs^n(S^n_\ell, K_j) + nps^n_j * ps^n(S^n_\ell, K_j)\right], \\ & + \sum_{c\in C}\left[\sum_{j\in JQ_c}\left[ncq^n_j * cq^n(\bar{X}_c S^n_c, K_j) + npq^n_j * pq^n(\bar{X}_c S^n_c, K_j)\right]\right], \\ & n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\}\end{aligned} \tag{8.4d}$$

$$\begin{aligned} & h^n_c + \sum_{i\in I_c} v^n_{ic}\pi^n_{ic}(1-\delta) + \frac{1}{e^n_c}x^n_{c,e} = \sum_{i\in I_c} x^n_{ic}\pi^n_{ic}(1+\delta) + \frac{1}{e^n_c}v^n_{c,e}, \\ & \forall\, c\in C, \quad n\in \boldsymbol{N}\setminus\{\boldsymbol{N_T}\cup 0\}\end{aligned} \tag{8.4e}$$

cont.
$$V_\ell^n = \sum_{i\in I_\ell} w_{i\ell}\,\pi_{i\ell}^n$$
$$+ \sum_{c\in C}\left\{ \sum_{j\in JC_c} \left[npc_{c,j}^{p(n)} * max(K_j - e_c^n, 0)\right]\right\}$$
$$+ \sum_{j\in JS_\ell} \left[ncs_{\ell,j}^{p(n)} * max(S_\ell^n - K_j, 0) \,+\, nps_{\ell,j}^{p(n)} * max(K_j - S_\ell^n, 0)\right]$$
$$+ \sum_{c\in C}\left[\sum_{j\in JQ_c} \left[ncq_{c,j}^{p(n)} * max(\bar{X}_c S_c^n - K_j, 0) \,+\, npq_{c,j}^{p(n)} * max(K_j - \bar{X}_c S_c^n, 0)\right]\right]$$
$$+ \sum_{c\in C}\left[e_c^n [\sum_{i\in I_c} w_{ic}\pi_{ic}^n] \right] \quad \forall\, n \in \boldsymbol{N_T} \tag{8.5a}$$
$$\sum_{j\in JQ_c} \left[ncq_{c,j}^n * cq^n(S_c^n, K_j) \,+\, npq_{c,j}^n * pq^n(S_c^n, K_j)\right] \le e_c^n w_{k_c}^n \pi_{k_c}^n,$$
$$\forall\, c \in C, \quad \forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{8.5b}$$
$$\sum_{j\in JS_\ell} \left[ncs_{c,j}^n * cs^n(S_\ell^n, K_j) \,+\, nps_{c,j}^n * ps^n(S_\ell^n, K_j)\right] \le w_{k\ell}^n \pi_{k\ell}^n.$$
$$\forall\, n \in \boldsymbol{N} \setminus \boldsymbol{N_T} \tag{8.5c}$$

and also (8.3c), (8.3d), (8.3e), (8.3f), (8.3g), (8.3h), (8.3i), (8.3j), (8.3k), (8.3l), (8.3m), (8.3n).

This optimization model differs from the previous one in the constraints that account for the cashflows, as the transactions of the quantos and their payoffs are now in the base currency. Equation (8.4b) imposes the cash balance condition for the base currency ℓ at the first decision stage. Again, total availability of funds stems from initially available reserves, revenues from the sale of initial asset holdings, and amounts received through incoming currency exchanges in the spot market. The uses of funds include the total expenditures for the purchase of assets and simple options on the base stock index, the purchases of currency options, the purchases of quanto options on foreign stock indices, and outgoing currency exchanges in the spot market. Equation (8.4c) imposes the cash balance constraints in foreign currencies $c \in C$ at the first decision stage. In this case, cash is used only for purchases of assets and outgoing currency exchanges in the spot market; in the case of quanto options there are no purchases of options in foreign currencies.

Equations (8.4d) and (8.4e) impose the cash balance conditions in every currency at any intermediate decision state; Now the total availability of funds comes from exogenous reserves, if any, revenues from the sale of asset holdings in the portfolio at hand, and payoffs from the exercise of stock (on the base index) and quanto options and the exercise of currency options, all decided at the predecessor node. Note that options are now purchased only in the base currency. Again the uses of funds include the purchase of assets and options, and outgoing currency exchanges in the spot market.

The final value of the portfolio at leaf node $n \in \boldsymbol{N_T}$ is computed in (8.5a). The total value of the revised portfolio at the end of the horizon accounts for the following: the proceeds from the liquidation of all assets (domestic and foreign) at the respective asset prices, the payoffs of stock option on the domestic stock index, the payoffs of currency options and the payoffs of quantos on foreign stock indices. Again, the proceeds of foreign revenues are valued in terms of the base currency by employing the applicable spot exchange rate at the end of the horizon.

Finally, constraints (8.5b) and (8.5c) limit the maximum expenditure for quantos on each foreign market, and simple options on the base stock index respectively, to the value of the position in the respective underlying stock index.

The pricing method used to value quanto options is given in the Appendix.

8.3 Experimental Design

Market and currency risk are the main sources of risk of international portfolios in our setting. Thus, controlling these risk factors is an important task for controlling and improving the performance of international investments. The results from previous chapters where clear: Hedging separately the market and the currency risks using options is extremely beneficial compared to totally unhedged positions. The results indicate also the suitability of options in controlling risk compared to other hedging techniques (international diversification, forward contracts).

This study combines the results from all previous studies, showing the effectiveness of options in risk management, and goes two steps further: First, we examine the effectiveness of controlling both risk factors in an integrated manner for international diversified mixed asset portfolios using options, thus capturing any interactions among these factors. Second, we extend the models into a multistage setting. Several hedging strategies, using stock and currency options are evaluated and compared each other. We use the results from the previous studies, and additionally we solve multistage stochastic programming models that consider both kinds of options (stock and currency options). We show the performance improvements as we gradually move to a more integrative risk management framework using options. We investigate and compare the performance of the following hedging strategies:

1. Totally unhedged portfolios. Optimal internationally diversified portfolios are determined in this case, but without any explicit regard to cover against either market or currency risk (i.e., neither options nor currency forward exchange decisions are considered).

2. Control of currency risk only by incorporating in the portfolio optimization models forward exchange contracts.

3. Control of currency risk only by incorporating in the portfolio optimization models currency options.

4. Control of market risk only by incorporating in the portfolio optimization models simple options on the stock indices.

5. Joint protection against all risks with the use of simple options and currency options.

6. Use of the integrative options (quantos) to jointly protect against both market and currency risks.

7. Multi-stage extension of the models with stock and currency options.

8. Alternative trading strategies of options (either stock or currency options).

The first strategy constitutes the basic benchmark against which the other strategies are compared in order to determine the incremental benefits from risk hedging decisions. We compare the performance of portfolios without options, versus portfolios with either stock or currency options separately, and finally the performance of portfolios with both kinds of options. In all tests we use selective hedging strategies.

The following three cases address separately each of the primary risk factors in order to assess the impact of controlling each type of risk in an international portfolio. The second and third strategies address the hedging of currency risk compared to naked positions, the former using forward exchange contracts, the latter using currency options. The fourth case address the hedging of market risk using simple options on stock indices.

The fifth strategy examines the effectiveness of alternative means for addressing both market and currency risks in an integrated manner using options. We investigate whether simultaneously hedging both risk factors using options is beneficial compared to hedging each risk factor separately. By comparing the results in this case against those from the previous three cases, we can determine the incremental benefit from controlling both types of risk instead of just either of them alone. In the sixth case we integrate the market and the currency risks using quanto options, and we study the effectiveness of this integration (we also include currency options to hedge the currency risk associated with the positions in foreign stock and bond indices). This is the more general, integrated risk management framework that is developed in this thesis.

This integrated framework is then extended into a multistage setting, allowing us to investigate the additional performance improvements of multistage models compared to their single-stage counterparts. Finally, we analyze the alternative performance of different trading strategies of multiple options. In the cases that involves consideration of options, we apply the option trading strategies discussed in chapter 6 for stock options and in chapter 7 for currency options. Giving access to such options, and given each time the specific trading strategy using combinations of these options on the respective underlying assets, the model [8.2] decides the optimal exposure to the market and currency risk (the positions in domestic and foreign assets) and the optimal position in hedge (the position in stock and currency options) in order to achieve the minimum shortfall risk at the level of the desirable target expected return.

The multistage models permit rebalancing in each decision state, where existing options may be exercised, and new option contracts could be purchased. We investigate the possible improvements in the performance of international portfolios of multistage models, over their single-stage variants.

To our knowledge, the incorporation of selective hedging decisions consisting of stock and currency options for hedging both the market and currency risks in the context of normative models for optimal international portfolio management is an entirely novel contribution. The introduction of options as risk hedging instruments in multistage portfolio optimization models is also novel. The superiority of multistage stochastic programming model over the myopic one are virtually nonexistent in the literature. In this work we show that the extension of the models in the multistage setting, when options are considered to hedge the various risks, leads to extremely higher returns over their single-stage counterparts.

We use dynamic tests to evaluate and compare the performance of the hedging strategies and models. These tests repeatedly apply the models in backtesting experiments using real market data on a rolling horizon basis during the period 05/1998 to 11/2001 (i.e., 43 months). Starting with an initial cash endowment in April 1998 each model was set up and executed to decide the initial portfolio composition. A set of scenarios is then generated based on the statistical characteristics of monthly observed market prices during the previous ten years. The appropriate model was solved and the optimal portfolios were recorded (first-stage portfolios, for multistage models). The clock was then advanced one month. The realized return of the optimal portfolio was determined on the basis of the revealed market prices of the assets and the exchange rates, plus the payoffs from any options that are considered in the specific model. A new set of scenarios was then generated by matching the statistics of the random variables to their estimates from the monthly observations of their market values during the previous ten years. With these scenarios as input, and using the portfolio composition and cash positions resulting from the previous decisions as a starting point, the new model was solved. The process was repeated for each successive month and the ex post realized returns were recorded and compounded. Thus, the backtesting simulations demonstrate the actual returns that would have been realized had the decisions of the models been implemented during the simulation period 04/1998-11/2001.

8.3.1 Hedging the Market and Currency Risks with Options

First, we analyze the performance of alternative trading strategies of combinations of options. Then, using the appropriate hedging strategies, we investigate the effects of hedging against the various risk factors, either separately or jointly, in an integrated manner. Remember that in chapter 6 we showed that when stock options are considered in international portfolios, the strangle strategy gives the best ex ante and ex post performance. Using the strangle strategy, the downside risk if there is only a small change in the value of the stock index is less with the strangle than with the other trading strategies. This is because the strangle is the cheaper alternative as the prices of its constituent options are the lower. Moreover, in chapter 7, we observed that when stock options are considered in the model to hedge against the currency risk, the BearSpread trading strategy results in the best ex post performance.

We first investigate whether the same trading strategies exhibit the best performance when both currency and stock options are considered simultaneously in the models.

Figure 8.1 compares the ex post performance of portfolios with the alternative trading strategies of stock options that were introduced in chapter 6. Currency options that form the BearSpread strategy are used to hedge the currency risk. It is clear that regardless of the interactions between the two risk factors, the Strangle strategy of call options is the more effective hedging strategy against the market risk. Thus we confirm the results of chapter 6, showing that they are valid even in the case of multiple kinds of options.

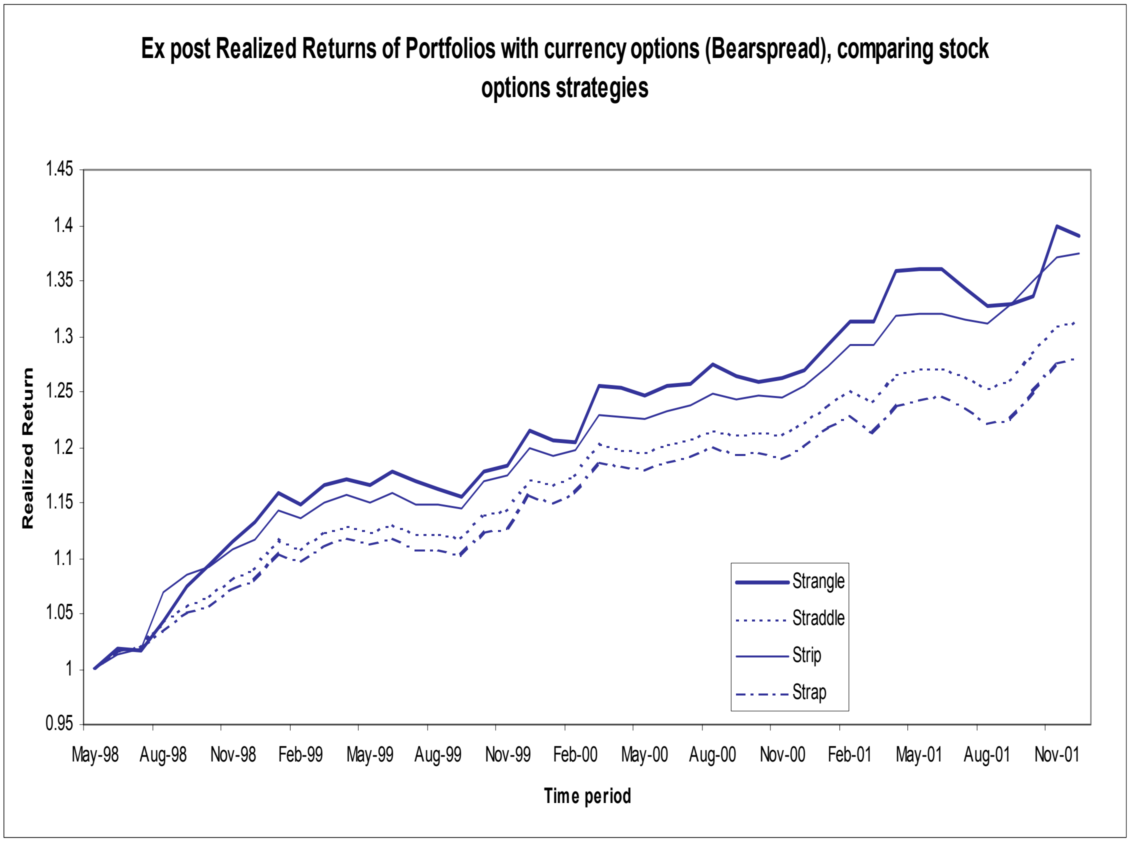


Figure 8.1: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. The strangle trading strategy of stock options is the best hedging scheme against the market risk.

Figure 8.2 compares the ex post performance of portfolios with BearSpread versus put "in-the-money" currency options, in a multistage setting. Stock options that form the strangle strategy are used to hedge the market risk. It is clear that the BearSpread strategy of put currency options is the more effective hedging strategy against the currency risk. Again, the results that have been confirmed in chapter 7 are still valid in this multi-options framework.

Next, we compare the ex post performance of international portfolios with increasingly integrated hedging strategies against the risk factors. We start with a single-stage model for international asset allocation, which is totally exposed to market and currency risks. Then, we investigate the performance improvements when forward rates are used to (partly) hedge the currency risk, while the market risk is not explicitly addressed except via international diversification. We continue by introducing currency options to hedge currency risk. Next, we investigate the performance of portfolios where stock options hedge the exposure to market risk. Finally, we develop an integrated decision

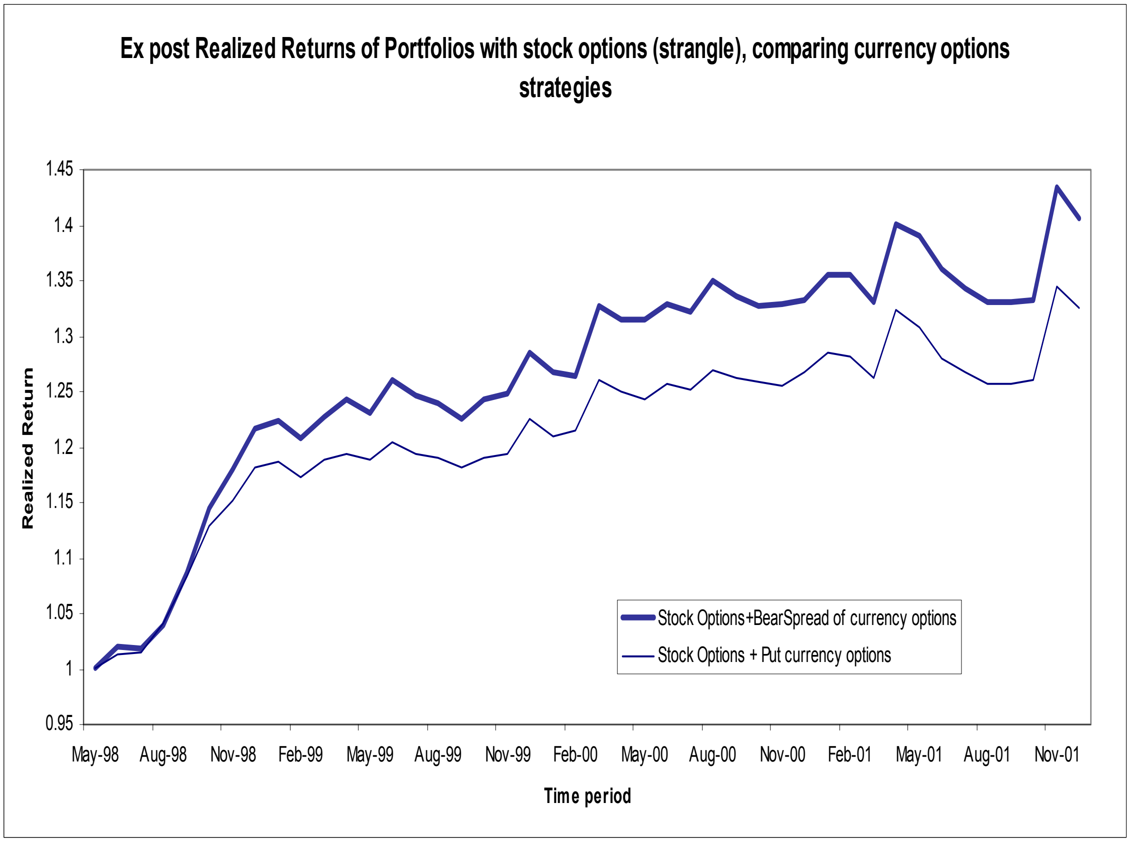


Figure 8.2: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. The BearSpread trading strategy of currency options is the best hedging strategy against the currency risk.

framework where both kinds of options are incorporated in multistage stochastic programming models simultaneously, examining their effectiveness to control the total risk of international portfolios. We investigate the performance improvements that this integrated framework offers in the performance of international portfolios, over the simpler separated hedging schemes against the risk factors.

Figure 8.3 contrasts the ex post performance of totally unhedged international portfolios, versus the performances of portfolios with alternative hedging tactics, moving gradually from the totally naked, to the fully integrated hedging strategy using options against the various risk factors.

First, we observe that the totally unhedged portfolios exhibit the worst performance. The performance of these naked portfolios is characterized by large fluctuations, thus offering very high levels of risk. This is expected as these portfolios are exposed to market and currency risks. Next, we observe that hedging the currency risk using forward contracts improves the performance of the portfolios, as it offers stability, although this performance is kept in very low levels. Instead of using forwards, we add in the model currency options that form the BearSpread strategy to hedge against the currency risk. Remember from chapter 7 that this trading strategy leads to best ex post performance of portfolios, compared to strategies with a single put option. We observe that the inclusion of these options offers additional improvements in the performance of portfolios although this hedging strategy exhibits large losses in the last period between February and November 2001. The losses followed the market crisis of September 11*th* are extremely high, making this hedging scheme unappropriate to manage portfolios when these are exposed to high levels of market risk.

Then we add simple stock options that form the strangle strategy in order to hedge the market risk. We observe a large improvement in the performance of international portfolios when stock options are used, compared to unhedged portfolios. Remember that the strangle strategy limits the downside losses while at the same time improves the upside potential of the portfolios, and the premium paid to buy the options is very low. Moreover, stock options offers stability in the performance, while the losses associated with the market crisis after September 11*th* are now considerably lower compared to the case with currency options only. We observe that the stock options protect the portfolios from unfavorable market movements, while the currency options alone result in great losses during big market crisis, which is expected.

So far, we observe that hedging either risk factor separately using options, offers clear improvements in the performance of portfolios compared to totaly unhedged portfolios. The asymmetric nature of option payoffs makes them the best choice as hedging instruments.

Finally, in order to capture interactions between asset returns and exchange rates, we hedge the market and the currency risk using options, in an integrated manner. Stock (simple or quantos) and currency options are used simultaneously against these risk factors, permitting any interactions among the risk factors to be considered. The result is clear: we observe an outstanding improvement in the performance of international portfolios when both risk factors are considered simultaneously. This framework where options are used to hedge the risks in an integrated manner constitute an effective risk management tool. The last observation is that quantos clearly outperform simple options. Especially when the portfolio with simple options starts to exhibit fluctuations, the performance of portfolios with quantos is much more stable, with lower losses and much higher growth. Even during the period between July and October 2001 where all the other models exhibit losses, the model with quanto options result in almost nonnegative ex post returns.

So far, in this single-stage setting, we show that increasingly integrated views towards total risk management are more effective compared to consideration of constituent risk components in isolation. We implement stochastic programming models that incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively.

8.3.2 The Dynamic Model

The next step is to extend the models into a multistage setting. Figure 8.4 contrasts the ex post performance of totally unhedged portfolios versus the performances of portfolios with different kinds of hedging instruments against both risk factors. The first observation is that the same patterns of performances are observed in this multistage setting.

Again portfolios without options exhibit the worst performance. Adding forwards to hedge against the currency risk offers stability in the performance of international portfolios, while the losses are much lower in this case. The multistage extension of the model with forwards have smoothed the

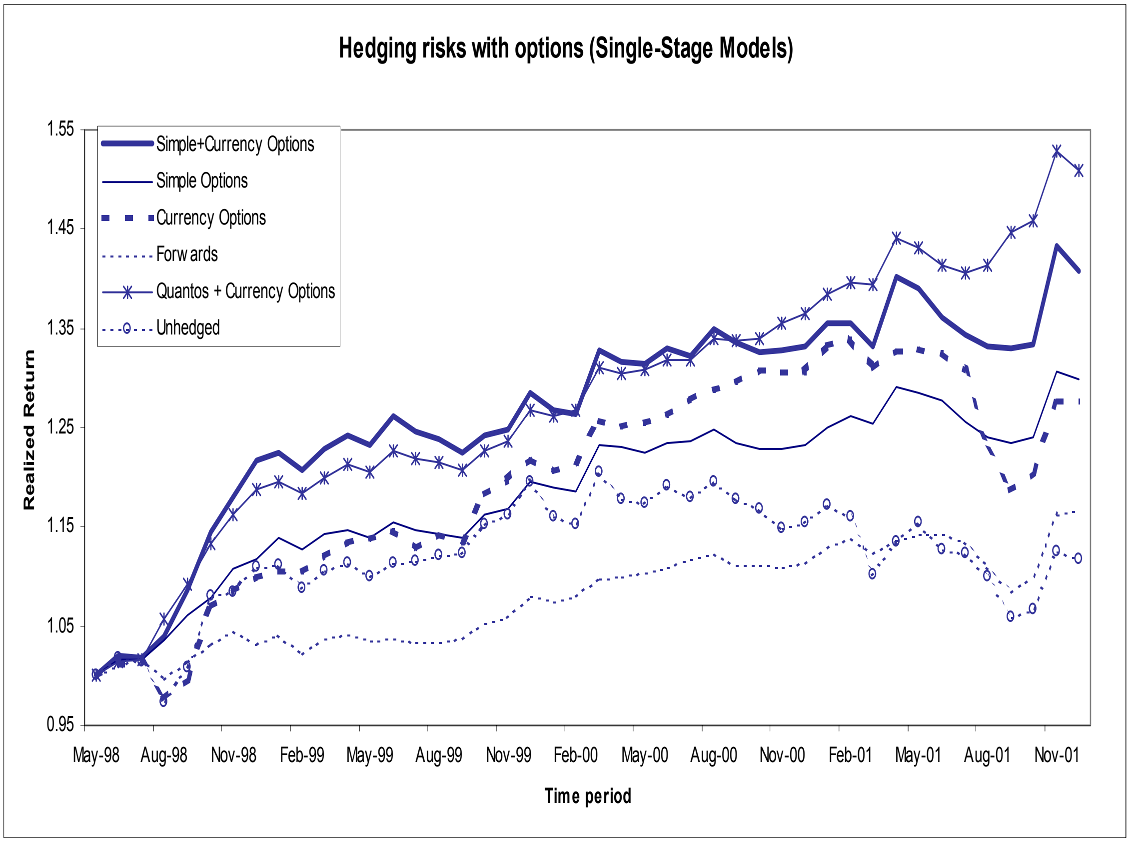


Figure 8.3: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. Increasingly integrated views towards risk (using options to hedge these risks) gives better results in the single-stage setting

performance of these portfolios. Using currency options that form the BearSpread strategy to hedge the currency risk offers additional improvements in the performance of portfolios compared to forwards exchange contracts. But again the currency options could not protect the portfolios from great losses accompanied with the crisis followed September $11th$.

Adding stock options that form the strangle strategy, improves considerably the performance of international portfolios. We observe that hedging using options offsets the market risk, leading in performance improvements. There is a beneficial effect of a hedge versus a diversification-only portfolio. The diversification-only portfolio addresses nonsystematic risks although it is incapable of moderating market risks properly. At the times when protection is needed the most (e.g., September 11th), the simple diversification of nonsystematic risk is partial protection, at best, because major price changes will occur in clusters, given important market events, rather than in isolated instances. Market risk clearly dominates in such circumstances and the protection arrangement using appropriate trading strategies of options must take into consideration.

In any case, hedging either risk factor separately using options, offers clear improvements in the performance of portfolios over that of portfolios without options. Finally, hedging simultaneously the market and the currency risks using options offers performance improvements over the individual hedging against each of the risk factors. This scheme permits cross hedging effects between markets and currencies and produce a portfolio which jointly optimizes market and currency holdings, as it takes into account interactions among movements in the underlying asset returns and exchange rates. The last observation is again that in the multistage setting, quanto options slightly outperform simple stock options. Hence, these integrative instruments constitute even more effective risk management tools. We observe that using quantos and currency options the performance of the portfolios is very smooth, very stable, and no losses have been observed, even after the crisis of September 11*th*. Comparing Figures 8.3 and 8.4 we observe that the two-stage models offers stability in the performance of portfolios, the losses are lower, and the compounded realized return is higher, compared to their single-stage counterparts.

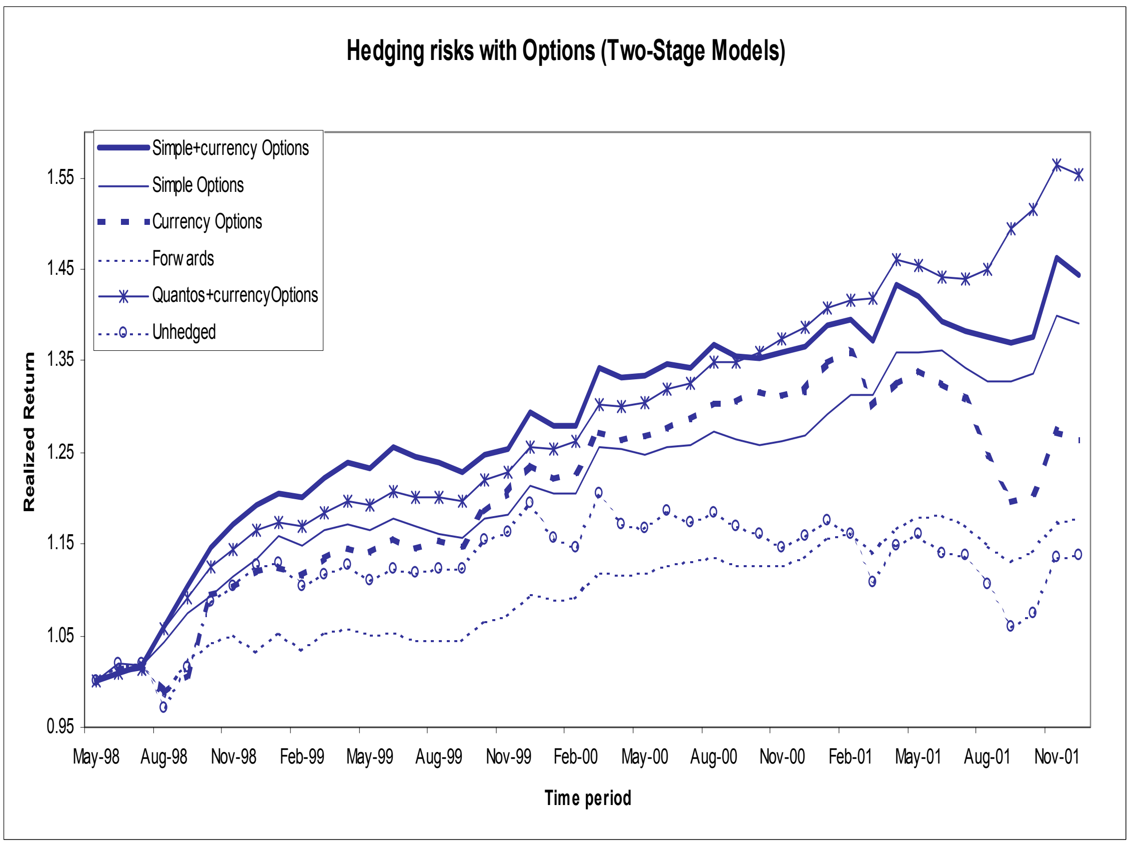


Figure 8.4: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. Increasingly integrated views towards risk (using options to hedge these risks) gives better results in the multistage setting

In order to analyze further the improvements in the performance of portfolios when we move to a multistage setting, we did some additional comparisons. Figure 8.5 contrasts the ex post performance of single versus multistage models with simple stock options that form the strangle strategy. In all cases forward contracts control the currency risk. We observe that when we permit rebalancing of portfolios in intermediate stages, the performance of international portfolios is clearly higher. The Figure also shows the performance of portfolios without options, as the reference line. The same pattern is observed when we use different trading strategies of options, such as straddle, strip and strap. We show that multistage models offer significant performance improvements over their single-stage counterparts, a result that has not been observed in many studies in the literature so far. The same result is observed when we use quantos instead of simple options. From Figure 8.6 we additionally observe that the two-stage model with quantos outperforms the single-stage one, yielding small performance improvements, especially in the second subperiod of our 43 months period. Even after September 11th, portfolios with quantos do really well, without considerable losses.

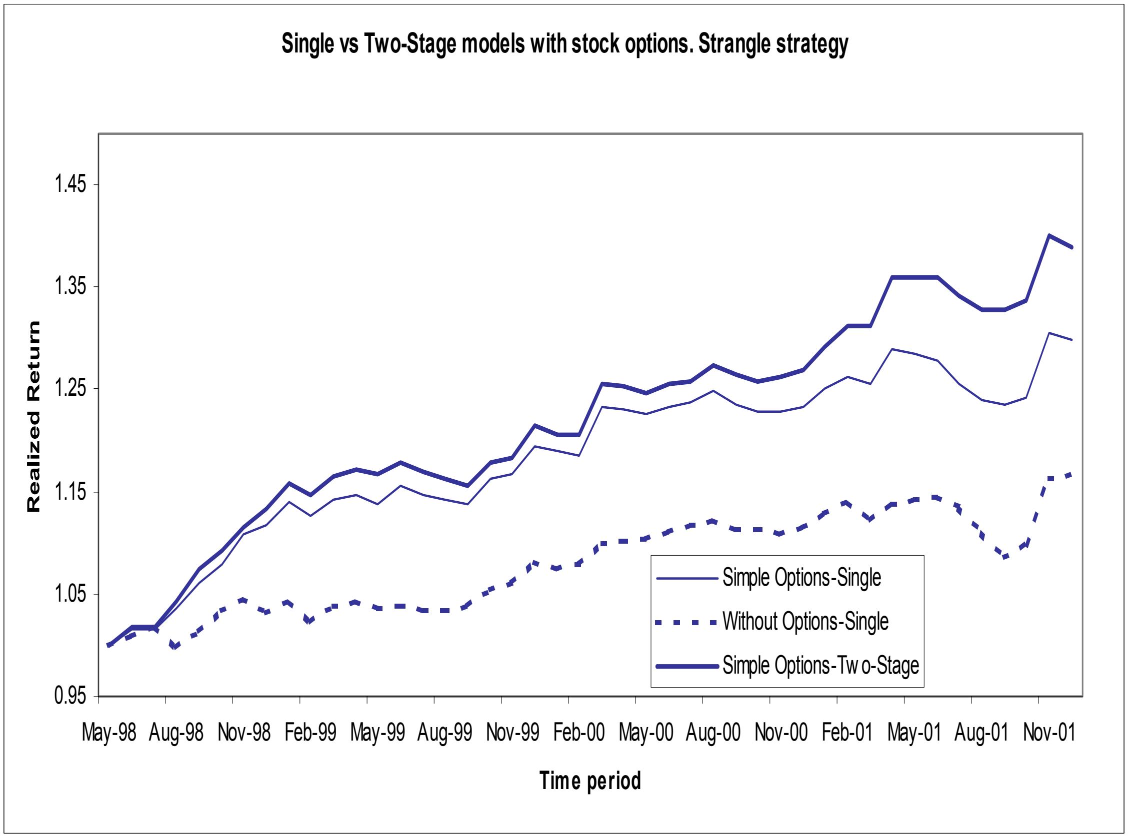


Figure 8.5: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and stock options. Multi-stage models with simple options clearly improve the performance of single-stage models.

Finally, Figure 8.7 contrasts the ex post performance of single versus multistage models with stock and currency options that form the strangle and BearSpread strategies respectively. So far we have observed that both in a single as well as in a multistage setting, hedging both risk factors using options offers the best ex post performance in international portfolios. In this Figure we observe a slightly better performance in multistage models over the single ones. Thus, regardless of the options we incorporate in the portfolios, (stock or currency options, or both) the multistage extension of the models offers additional performance in the portfolios, over their single-stage counterparts.

Overall, the results of this study are very clear. The integrative risk management framework, where options are used simultaneously to hedge against all the risk factors that affect the performance

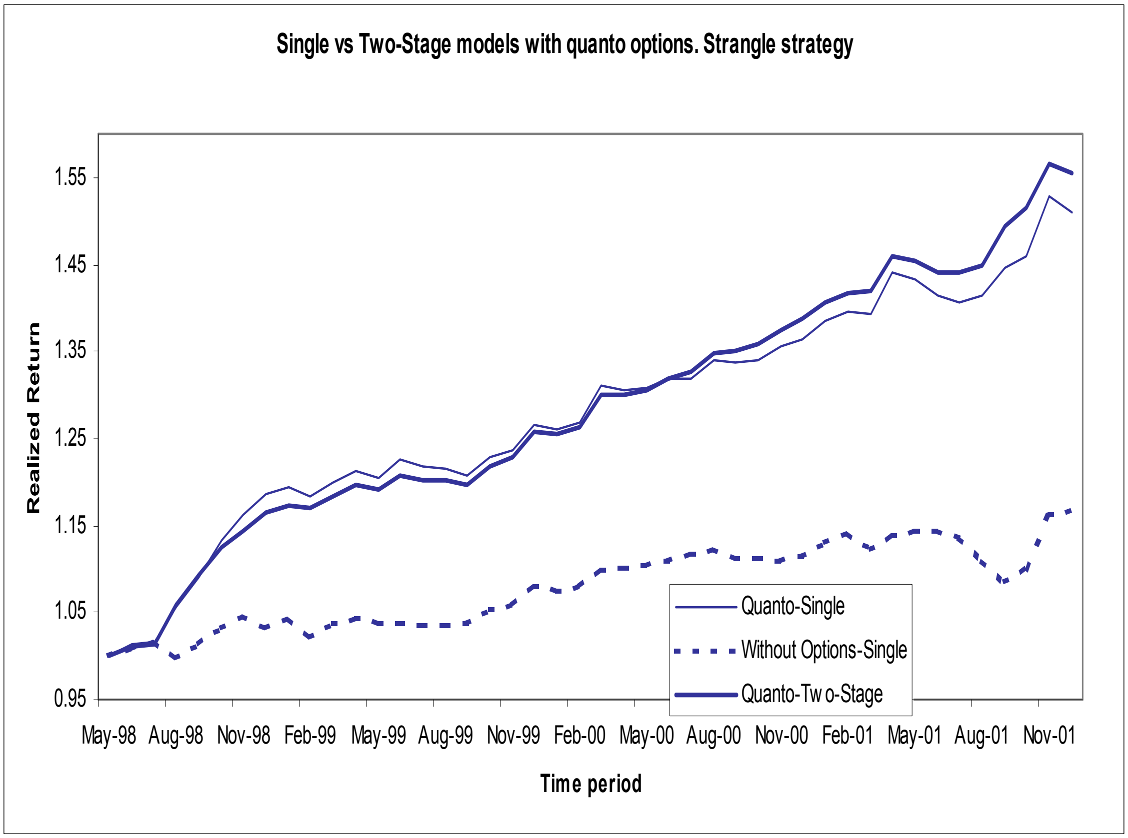


Figure 8.6: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. Multi-stage models with quanto options slightly improve the performance of single-stage models.

of international portfolios, is the more effective one. The fact that multiple risks are considered and hedged simultaneously using options, allows the model to capture any interdependencies among these risk factors, resulting in the most powerful hedging scheme. Finally, and very importantly, the multistage extension of the models yield additionally performance improvements, regardless of the strategy or the hedging instrument that is used. Multi-stage models clearly outperform their single-stage counterparts and in some cases they exhibit exceptional performance improvements.

8.4 Conclusions

In this chapter we combined and extended the developments of all previous chapters in the context of an integrated risk management framework. We showed that increasingly integrated views towards total risk using management are more effective compared to consideration of constituent risk components in isolation. We implemented multistage stochastic programming models that incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively. These models shape the overall portfolio risk profile accounting for both the market and the currency risks, thus contributing directly to the objective of total risk

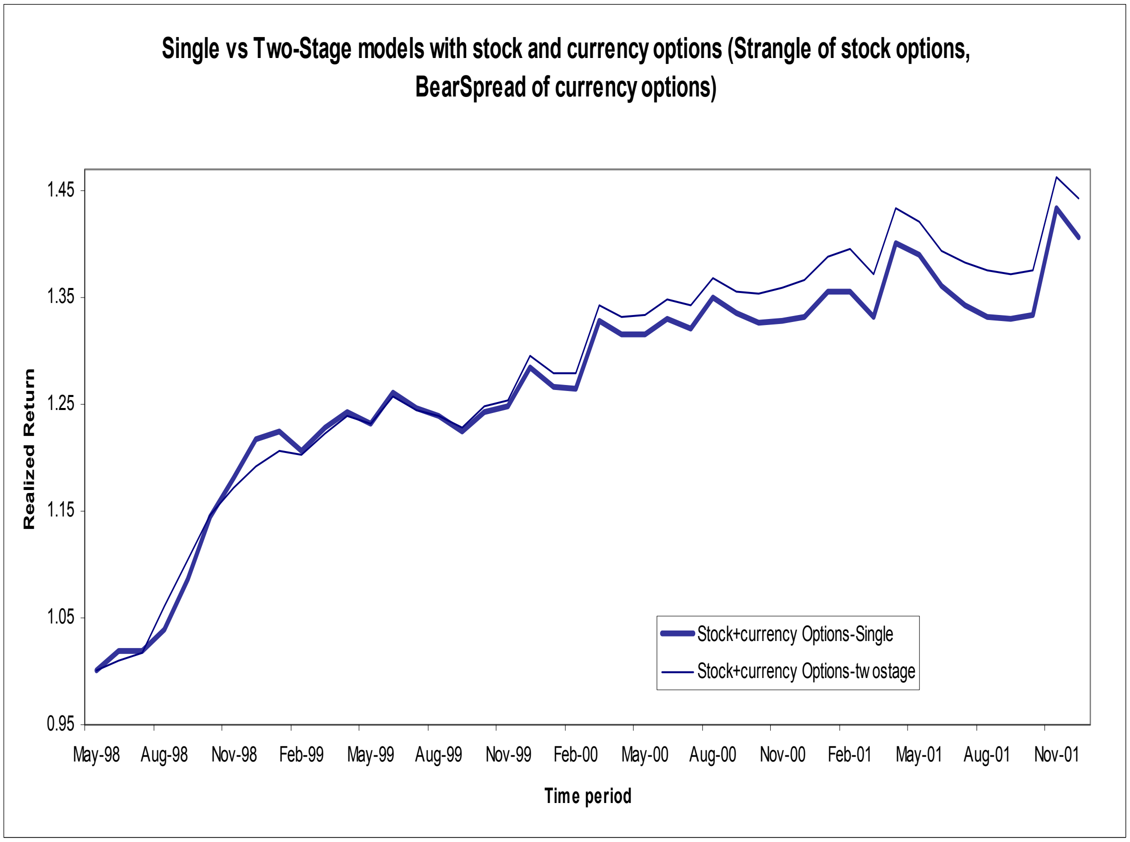


Figure 8.7: Ex-Post realized returns of CVaR optimization models for internationally diversified portfolios of stocks, bonds, and options. Multi-stage models with stock and currency options improve the performance of single-stage models

management.

Empirical results indicate that hedging each risk factor independently using options, offers performance improvements over portfolios without options. But when both risk factors are hedged simultaneously using appropriate trading strategies of options, the additional performance is outstanding. The results indicate the superiority of this integrated risk management scheme. The simultaneous use of stock and currency options yields the best ex post performance of international portfolios. Moreover, the multistage model clearly outperforms the myopic (single-stage) variant of the model, regardless of the trading strategy and the hedging instruments that are used. In every case, independently of the risk factors that are hedged, multistage models offer the best performance. This is a very important result. It was not clear so far, whether the multistage models offer performance improvements over their single-stage counterparts. In this chapter we prove the superiority of multistage models.

The integrated modelling framework of this chapter provides a complete basis to comprehensively address all relevant decisions in international portfolio management in a unified manner. Additionally we observe that using quantos the portfolios perform even better, with higher growth when the time is good, and lower losses when the time is bad.

The next step in this work is to design and incorporate derivative securities in multistage stochastic programming models for hedging against multiple risk factors. This problem has three components. The first one is the designing of individual derivatives, specifying jointly the optimal exercise price and time to maturity. Non-linear optimization algorithms can be used for that. These optimization programs are multimodal, and Tabu search procedures could be developed for their solution. The second component is the pricing of these derivatives. The third one is their incorporation in international portfolios for hedging purposes. Appropriately designed derivatives may be proved superior instruments according to the utility function or risk measure that is optimized.

8.5 Appendix: Pricing Quantos

The underlying asset V of quanto options is the product of the foreign asset price S and the predetermined exchange rate $\bar{X}$, that is $V = S\bar{X}$.

Let $x = lnS_T\bar{X} - lnS_0\bar{X} = lnS_T - lnS_0 \Rightarrow S_T = S_0e^x$, where S_T is the price of the foreign stock at maturity, and S_0 is the current stock price.

The central moments of x are the following:

$$\begin{aligned} k_1 &= E(x) \\ k_2 &= E(x-k_1)^2 \\ k_3 &= E(x-k_1)^3 \\ k_4 &= E(x-k_1)^4 \end{aligned}$$

The skewness and kurtosis are defined as:

$$\begin{aligned} \gamma_1 &= \frac{k_3}{k_2^{3/2}} \\ \gamma_2 &= \frac{k_4}{k_2^2} \end{aligned}$$

In the "risk-neutral" structure, the call quanto price depends on the conditional distribution of x:

$$\begin{aligned} cq &= e^{-rdt}E(\bar{X}S_T - K)^+ \\ &= e^{-rdt}\int_{log(K/(\bar{X}S_0))}^{\infty}(\bar{X}S_0 - K)f(x)dx \end{aligned} \tag{8.6}$$

where f is the conditional density of x, r and r_f are the domestic and the foreign riskless rates respectively. Integrating, we can find the price of the quanto call option.

8.5.1 Under Normality

In the simple case where x is normal, the stochastic process for the underlying asset V is estimated as following:

The exchange rates e and the foreign asset prices S are Lognormally distributed, following Geometric Brownian Motion:

$$\frac{de}{e} = (r - r_f)dt + \sigma_e dZ_e \tag{8.7}$$

and

$$\frac{dS}{S} = r_f dt + \sigma_S dZ_S \tag{8.8}$$

The risk-neutral dynamics of (Se) are of the form:

$$\frac{d(Se)}{(Se)} = rdt + \sigma_{Se} dZ_{Se}, \tag{8.9}$$

where the volatility σ_{Se} is

$$\sigma_{Se} = \sqrt{\sigma_E^2 + \sigma_S^2 + 2\sigma_e \sigma_S \varrho_{Se}}. \tag{8.10}$$

For quantos, we need to specify the process followed by the underlying asset $S\bar{X}$:

$$\frac{d(S\bar{X})}{(S\bar{X})} = \frac{dS}{S} = (r - \delta)dt + \sigma dZ \tag{8.11}$$

Note that we can eliminate $\bar{X}$ from (8.11) since it is a constant.
Next, we need to determine the dividend term δ and the volatility σ. Let $F = Se$. Then

$\frac{\partial F}{\partial e} = S$, $\frac{\partial^2 F}{\partial e^2} = 0$, $\frac{\partial F}{\partial S} = e$, $\frac{\partial^2 F}{\partial S^2} = 0$, $\frac{\partial^2 F}{\partial e \partial S} = 1$.

From Ito's Lemma we have:

$$dF = \frac{\partial F}{\partial e} de + \frac{\partial F}{\partial S} dS + \frac{1}{2}\frac{\partial^2 F}{\partial e^2} de^2 + \frac{1}{2}\frac{\partial^2 F}{\partial S^2} dS^2 + \frac{\partial^2 F}{\partial e \partial S} dedS, \tag{8.12}$$

where $dedS = eS\sigma_e\sigma_S dZ_e dZ_S = eS\sigma_e\sigma_S\varrho_{eS}dt$ (ϱ_{eS} is the correlation between the exchange rate e and the foreign asset S).

Substituting (8.7) and (8.11) into (8.12) gives

$$dF = Se[(r - r_f)dt + \sigma_e dZ_e] + Se[(r - \delta)dt + \sigma dZ] + Se\sigma_e\sigma\varrho_{Se}dt \tag{8.13}$$

or

$$\frac{dF}{F} = (r - rf + r - \delta + \sigma_e\sigma\varrho_{Se})dt + \sigma_e dZ_e + \sigma dZ \tag{8.14}$$

and, finally,

$$\frac{dF}{F} = (2r - r_f - \delta + \sigma_e\sigma\varrho_{Se})dt + \sigma' dZ', \tag{8.15}$$

where $\sigma' = \sqrt{\sigma_e^2 + \sigma^2 + 2\sigma_e\sigma\varrho_{Se}}$ is the volatility of dF/F.
But F=Se, so the processes (8.9) and (8.15) are identical.

Equating the drift and volatility terms of (8.9) and (8.15), we have $r = 2r - r_f - \delta + \sigma_e\sigma_S\varrho_{Se}$ or $\delta = r - r_f + \sigma_e\sigma_S\varrho_{Se}$

and

$$\sigma' = \sqrt{\sigma_e^2 + \sigma^2 + 2\sigma_e\sigma\varrho_{Se}} = \sqrt{\sigma_e^2 + \sigma_S^2 + 2\sigma_e\sigma_S\varrho_{Se}}$$

or

$$\sigma = \sigma_S.$$

Substituting δ and σ in (8.11) we have the stochastic process of the underlying asset $S\bar{X}$ of a quanto option:

$$\frac{d(S\bar{X})}{(S\bar{X})} = (r_f - \sigma_e \sigma_S \varrho_{Se})dt + \sigma_S dZ. \tag{8.18}$$

We define the standardized variable

$$\omega = \frac{x - \mu}{\sigma\sqrt{dt}}$$

where $\mu = (r_f - \sigma_e \sigma_S \varrho_{Se})dt$.

Then the density function is,

$$f(\omega) = (2\pi\sigma)^{-1/2} e^{\frac{-(x-\mu)^2}{2\sigma^2 dt}} \tag{8.20}$$

The integral 8.6 has two terms:

$$\begin{aligned} \int_{log(K/(\bar{X}S_0))}^{\infty} \bar{X}S_0 f(x))dx &= \bar{X}S_0 e^{\mu} N(d_1) \\ \int_{log(K/(\bar{X}S_0))}^{\infty} K f(x))dx &= K N(d_1 - \sigma\sqrt{dt}) \end{aligned}$$

The resulting pricing formula for the quanto call option under the assumption of normality is

$$cq = \bar{X}S_0 e^{(r_f - r - \sigma_e \sigma_S \varrho_{Se})dt} N(d_1) - K e^{-rdt} N(d_1 - \sigma\sqrt{dt}) \tag{8.21}$$

where

$$d_1 = \frac{ln\frac{\bar{X}S}{K} + (r_f - \sigma_e \sigma_S \varrho_{Se} + \sigma^2/2)dt}{\sigma\sqrt{dt}} \tag{8.22}$$

8.5.2 Accounting for Skewness and Excess Kurtosis

If x exhibits nonzero skewness and excess kurtosis we use a Gram-Charlier series expansion which approximates the density function for the standardized random variable $\omega = \frac{x-\mu}{\sigma\sqrt{dt}}$.

The Gram-Charlier density is

$$f(\omega) = \varphi(\omega) - \gamma_1 \frac{1}{3!}\varphi'''(\omega) + \gamma_2 \frac{1}{4!}\varphi''''(\omega) \tag{8.23}$$

where

$$\varphi(\omega) = (2\pi)^{-1/2} e^{-\frac{\omega^2}{2}} \tag{8.24}$$

is the standard normal density, and $\varphi^i(\omega)$, is the $i$the derivative of $\varphi(\omega)$.

We need to solve the integral

$$\begin{aligned} \int_{\omega^*}^{\infty} (\bar{X}S_0 e^{\mu+\sigma\sqrt{dt}\omega} - K) f(\omega)d\omega &= \int_{\omega^*}^{\infty} (\bar{X}S_0 e^{\mu+\sigma\sqrt{dt}\omega} - K)\varphi(\omega)d\omega \\ &- \frac{\gamma_1}{3!}\int_{\omega^*}^{\infty} (\bar{X}S_0 e^{\mu+\sigma\sqrt{dt}\omega} - K)\varphi'''(\omega)d\omega \\ &+ \frac{\gamma_2}{4!}\int_{\omega^*}^{\infty} (\bar{X}S_0 e^{\mu+\sigma\sqrt{dt}\omega} - K)\varphi''''(\omega)d\omega \\ &= I_1 - \frac{\gamma_1}{3!} I_2 + \frac{\gamma_2}{4!} I_3 \end{aligned}$$

with

$$\omega^* = (ln\frac{K}{\bar{X}S_0} - \mu)/\sigma\sqrt{dt} \tag{8.25}$$

The first piece has the same for as 8.21:

$$e^{-rdt}I_1 = \bar{X}S_0e^{(r_f-r-\sigma_e\sigma_S\varrho_{Se})dt}N(d_1) - Ke^{-rdt}N(d_1-\sigma\sqrt{dt})$$

The second piece is evaluated by repeated application of integration by parts:

$$I_2 = -\sigma\sqrt{dt}K\varphi(\omega^*)(\omega^*+\sigma\sqrt{dt}) - (\sigma\sqrt{dt})^3e^{edt} * (\ 8.21) - (\sigma\sqrt{dt})^3KN(-\omega^*)$$

which uses the property of derivatives of the normal density

$$lim_{x\to\infty}e^x\varphi(x) = 0$$

For the third piece

$$I_3 = -\sigma\sqrt{dt}K\varphi(\omega^*)[(\omega^*)^2 - 1 + \omega^*\sigma\sqrt{dt} + \sigma^2dt] + \sigma^4dt^2e^{edt} * (\ 8.21) + \sigma^4dt^2KN(-\omega^*)$$

Substituting we have (eliminating terms which involves σ^3 and σ^4)

$$\begin{aligned} cq &= e^{-rdt}(I_1 - \frac{\gamma_1}{3!}I_2 + \frac{\gamma_2}{4!}I_3) \\ &= \bar{X}S_0e^{(r_f-r-\sigma_e\sigma_S\varrho_{Se})dt}N(d_1) - Ke^{-rdt}N(d_1-\sigma\sqrt{dt}) \\ &+ \frac{\gamma_1}{3!}[e^{-rdt}\sigma\sqrt{dt}K\varphi(\omega^*)(\omega^*+\sigma\sqrt{dt})] \\ &+ \frac{\gamma_4}{4!}\{e^{-rdt}\sigma\sqrt{dt}K\varphi(\omega^*)[(\omega^*)^2 - 1 + \omega^*\sigma\sqrt{dt} + \sigma^2dt]\} \end{aligned} \tag{8.30}$$

Then by substituting $\omega^* = \sigma - d_1$, and by using the identity

$$\bar{X}S_0e^{(\mu-r)dt}\varphi(d_1) = Ke^{-rdt}\varphi(d_1-\sigma\sqrt{dt})$$

we result in the price of the quanto call option

$$\begin{aligned} cq &= \bar{X}S_0e^{(r_f-r-\sigma_e\sigma_S\varrho_{Se})dt}N(d_1) - Ke^{-rdt}N(d_1-\sigma\sqrt{dt}) \\ &+ \bar{X}S_0e^{(\mu-r)dt}\varphi(d_1)\sigma\sqrt{dt}[\frac{\gamma_1}{3!}(2\sigma\sqrt{dt}-d_1) - \frac{\gamma_2}{4!}(1-d_1^2+3\sigma\sqrt{dt}d_1 - 3\sigma^2dt)] \end{aligned} \tag{8.32}$$

where

$$d_1 = \frac{ln\frac{\bar{X}S}{K} + (\mu+\sigma^2/2)dt}{\sigma\sqrt{dt}} \tag{8.33}$$

Chapter 9

Summary and Conclusions

This thesis studies the problem of international portfolio management. Risk management of international portfolios is a major issue, since multiple risk factors are present in this context. It develops suitable modelling frameworks for addressing problems of managing optimally international portfolios and controlling the associated risks. The major aim is to investigate the relative effectiveness of alternative hedging instruments and strategies for controlling the primary risks that affect the performance of international portfolios. We first studied suitable tactics for controlling exposure to risk factor separately, assessing the impact of hedging the particular factor in the performance of the portfolios. Then we analyzed the additional value of controlling against all risk factors using derivatives, in an integrated manner.

The study concerns the development, implementation and validation of scenario-based stochastic programming models for actively managing international investment portfolios of stock and bond indices in multiple currencies. Forward contracts and European options (including quantos) are incorporated in the portfolio optimization models as risk hedging instruments. The models address jointly the volatility of asset returns, the volatility of exchange rates, and their correlations. Integrated simulation and optimization procedures are employed to address these issues in a common framework. Appropriate scenario generation methods are used to generate jointly scenarios for asset returns and exchange rates, so as to capture their interdependencies. When multiple elements of uncertainty, such as asset returns and exchange rate fluctuations are considered simultaneously, capturing correlations among them is very important. The scenarios represent discrete, joint distributions of the random variables and constitute the primary inputs to the stochastic optimization models. The models determine jointly the portfolio positions (not only across the different markets, but also to specific assets within each market) and the levels of hedging with forward contracts, or via "fairly" priced derivative securities. The optimization models incorporate institutional considerations (no short sales, transaction costs) and determine a solution that is optimal, according to specific objectives, with respect to the postulated scenario set. Thus, several interrelated decisions that were previously examined separately, are cast in a common framework with consequent benefits to the investors.

Stochastic programming models take an integrated view of the problem. We can incorporate transaction costs, multiple variables (asset returns and exchange rates), market incompleteness, regulatory and any other market specific restrictions, that can be handled simultaneously in this framework. Moreover, the modelling framework we develop in this thesis is not restricted to the choice of the scenario generation procedure. Alternative methods can be employed.

First, we developed a general dynamic multistage framework for international portfolio management. The stochastic programming model captures decision dynamics, include an operational treatment of hedging decisions by means of implementable forward exchange contracts, and account for the effect of transaction costs. A moment-matching scenario generation procedure is adopted to represent uncertainty in asset returns and exchange rates in accordance with their observed empirical distributions.

We adapted, implemented and empirically validated appropriate methods to price options in accordance with discrete scenario sets that depict the distribution (and dynamic evolution) of asset prices. We confine our attention to European-type options that can be exercised only at the expiration time (maturity). We applied two different methods for pricing the options in accordance with the discrete distributions of a scenario tree. The proposed option pricing methods are motivated from the need to achieve internal consistency in stochastic programming models when options are incorporated, but their use is not confined to these models only. The proposed methods hold their own as promising option pricing methods. We demonstrated through empirical validation tests using market prices for the S&P500 stock index and options on this security that the proposed valuation procedures yield prices that are closer to the market prices of the options than those obtained by the Black-Scholes method, especially for deep out-of-the-money options.

We introduced stock options in the portfolio for hedging against the market risk. The introduction of options broadens the investment opportunity set and provides instruments geared towards risk control due to the asymmetric and nonlinear form of option payoffs. We suitably extended the portfolio optimization model so as to incorporate the options and we empirically investigated the impact that these options have on the performance of international portfolios of financial assets.

We investigated the use of currency options as means of controlling the currency risk of foreign investments in the context of international portfolios. Due to the asymmetric nature of their payoffs, options are particularly suitable instruments for risk management. Currency options can cover against losses from potential unfavorable changes in exchange rates, while preserving the upside potential. We investigated the effectiveness of decision strategies that employ currency options to control currency risk exposures in portfolios of international financial assets. To this end, we empirically compared such strategies against the alternative use of currency forward contracts as means of controlling currency risk.

Finally, we combined and extended all the the developments in the context of an integrated risk management framework. We implemented multistage stochastic programming models that incorporate both stock and currency options to hedge the risk factors in a joint fashion, thus capturing all interacting decisions comprehensively. These models shape the overall portfolio risk profile accounting for both the market and the currency risks, thus contributing directly to the objective of total risk management.

The new finding and contributions of this thesis are numerous.

We integrated a simulation and optimization framework for managing portfolios of international financial assets. Reliable models for managing financial risks, once they have been identified, in an integrated manner are still in their infancy. Our models capture the primary risk factors in international portfolios and jointly determine the selection of particular investments across multiple markets and the appropriate hedging strategies.

Previous studies considered the hedging decisions separately from investment decisions. We demonstrated that our holistic view that incorporates the instruments and investment decisions in a common portfolio optimization framework is an effective approach for managing the total risk exposure of international portfolios. This aim is a central theme of this dissertation. All our developments evolve around this central goal.

We employed optimization models for international portfolio management that minimize the excess shortfall risk of international portfolios as captured by the Conditional-Value-at-Risk (CVaR) measure. Due to the observed asymmetry of asset returns in international portfolios and the asymmetric option payoffs, the CVaR risk measure is more appropriate than alternative metrics such as the Mean-Absolute-Deviation (MAD) and variance.

We developed multistage stochastic programming models for managing international portfolios under uncertainty. Multistage models help decision makers gain useful insights and adopt more effective decisions. They shape decisions based on longer-term benefits and avoid myopic reactions to short-term market movements that may potentially prove risky. They determine appropriate dynamic contingency (recourse) decisions under changing economic conditions that are represented by means of scenario trees. As dynamic models consider longer planning horizons and account for portfolio rebalancing decisions at multiple time periods, they should reasonably be expected to outperform myopic models. Yet, comparative studies to establish the incremental benefits of multistage stochastic programs in comparison to single-period models can scantly be found in the published literature. We showed through extensive numerical experiments that multistage models give performance improvements over their single-stage counterparts. Hence, we established that multistage stochastic programming models are effective decision support tools for international portfolio management.

We adapted suitable methods for pricing and incorporating European type options in scenario-based stochastic programming models. The options are priced in accordance with the postulated scenario sets, thus yielding an internally consistent model framework. The pricing procedures comply with the no-arbitrage conditions. Using these methods, we priced options at any node of the scenario tree, thus allowing transactions with options at any decision stage. The option pricing procedures are not dependent on any explicit assumption regarding the distribution of the underlying security. In principle, they can be applied to any arbitrary discrete distribution (scenario tree) for the prices of the underlying. We validated through extensive empirical tests the proposed option pricing valuation procedures when the scenario sets for the prices of the underlying securities are generated by a moment-matching method. We extended the valuation procedures to price quantos; these are fixed exchange rate options on a foreign equity. They are used to treat jointly the market and currency risks of a position in a foreign equity index.

We incorporated stock options in stochastic programming models for risk management purposes. These tools provide the means to investigate the performance of alternative tactics to mitigate market risk, including popular strategies that enforce specific combinations of stock index options. We found that the inclusion of options in the portfolio can materially reduce the downside risk. Portfolios that include options have return distributions with significantly lower tails and exhibit (more) positive skewness in contrast to the distributions of portfolios without options. Regardless of the trading strategy of options that is selected, the incorporation of options in international portfolios improves significantly the performance of these portfolios. Although, it is not clear enough which of the alternative options (simple of quantos) is indeed more preferable, since their performance is almost the same, in all cases quantos exhibit slightly better results than simple options, and never worse. Thus, integration of market and currency risk into portfolios hedging strategies does pay. Finally, among the trading strategies of options, portfolios with strangle strategy yields the best ex ante and ex post performance.

We analyzed the hedging effectiveness of currency options in international portfolios. We examined the hedging role of currency options and we identified appropriate trading strategies of currency options by comparing empirically their performance against the use of currency forward contracts. Despite the advocacy of currency options for foreign exchange risk management, their incorporation in practical portfolio management models had remained largely unexplored. Empirical results indicate that forward contracts are more effective hedging instruments than single put options. Both static as well as dynamic tests show that the performance of portfolios with forward contacts is better than that of put options, except when "in-the-money" put options are used in multi-stage models. But appropriate combinations of put options, like BearSpread, lead to performance improvements. These strategies of put options can benefit from favorable exchange rate movements, resulting in small periods with high expected return (when they capture the movement of the underlying asset) and large periods with stable performance. Forwards, on the other hand, give more stable performance, but since they lock in a prespecified forward rate, they cannot take advantage of any movements of the underlying.

We developed an integrated risk management framework where the main risk factors are considered simultaneously using appropriate types of instruments (forward contracts and options). Considering both market and currency risks in a unified manner allows us to assess the sensitivity of the portfolio value to each risk factor, and hence assess the significance of each risk component to the overall risk of the international portfolio. The options schemes proposed in this study work best — and most efficiently — when we considered simultaneously both risk factors. Multistage stochastic programming models for international portfolio management generate optimal positions in international assets and optimal hedge levels. We used appropriate trading strategies to hedge against the risk factors either independently or jointly. We found that increasingly integrated views towards risk result in more effective risk management strategies, compared to unhedged positions. The more risk factors we hedge, the more stability in the performance of portfolios we get, and the greater the performance we attaint.

Our empirical results show that quantos provide particularly effective instruments for risk hedging purposes. This is due to the integrative nature of these instruments that cover both the market risk of the underlying security and the associated currency risk.

Overall, empirical results indicate that hedging each risk factor independently using options, offers performance improvements over portfolios without options. But when both risk factors are hedged simultaneously using appropriate trading strategies of options, the additional performance is outstanding. The results indicate the superiority of this integrated risk management scheme. The simultaneous use of stock and currency options yields the best ex post performance of international portfolios. Moreover, the multistage model clearly outperforms the myopic (single-stage) variant of the model, regardless of the trading strategy and the hedging instruments that are used. In every case, independently of the risk factors that are hedged, multistage models offer the best performance. This is a very important result. It was not clear so far, whether the multistage models offer performance improvements over their single-stage counterparts. In this thesis we proved the superiority of multistage models.

The integrated modelling framework of this thesis provides a complete basis to comprehensively address all relevant decisions in international portfolio management in a unified manner.

All the models are tested empirically using market data. We compared the risk-return profiles (efficient frontiers) of alternative decision tactics in static tests. We also ran backtesting experiments on a rolling horizon basis for more substantive comparisons. Observed values of asset prices and exchange rates are used to compute the realized returns of the optimal portfolios. This ex post evaluation of realized returns provides a more reliable performance assessment of alternative hedging strategies and instruments.

We should note here that the results are context dependent, and may not be exactly the same in subsequent periods. What we do provide in this thesis is the machinery, the modelling tools that allow us to solve international portfolio management problems. Essential features of our modelling tools include the representation of uncertainty in a manner that captures jointly the co-movements of the risk factors, the pricing of options consistently with the postulated scenarios, the selection of the appropriate investments in each market, and the specification of appropriate hedging decisions. All these decisions are considered in an integrated framework.

Next step in this direction of research is to investigate the currency risk hedging of international portfolios using instruments such as swaps, swaptions or relative exotic options on exchange rates. Despite the widespread advocacy of swaps or swaptions in risk management, the question concerning their proper application as a currency hedging instrument remains largely unexplored. Models should be developed that: Firstly, price the derivatives by taking in to account that the distribution of exchange rates is not normal but exhibit high skewness and kurtosis. Secondly, incorporate appropriate trading strategies of these derivatives that achieve superior hedge, taking into account the cost of purchasing them. And thirdly, specify jointly the level of currency hedging and the investments in particular assets in each market.

Of particular interest is the pricing of either American or European style options, (stock options, exotic options) on various underlying assets with arbitrary distributional assumptions, and the incorporation of these derivatives in portfolios for hedging purposes. The uncertainty of the underlying assets could be represented by discrete distributions that are depicted in terms of scenario trees. This is a flexible modelling approach as it can accommodate general distributions and arbitrary dynamics in the evolution of the random variables. These models of the price evolution of the risk assets are linked with dynamic programming algorithms to arrive at options prices resulting from optimal exercise prices. Complex exotic options can then be priced consistently with postulated scenario set. These options could be used in mathematical programs to minimize the downside risk of the international portfolios.

Another problem to address is the design of derivatives through integrated simulation and optimization models. This problem has three components. The first one is the designing of individual derivatives, specifying jointly the optimal exercise price and time to maturity. Non-linear optimization algorithms can be used for that. These optimization programs are multimodal, and Tabu search procedures could be developed for their solution. The second component is the pricing of these derivatives. The third one is their incorporation in international portfolios for hedging purposes. Appropriately designed derivatives may be proved superior instruments according to the utility function or risk measure that is optimized.

A particularly interesting problem is to incorporate corporate bonds in international portfolios. To develop a framework for simulating default-free and defaultable bonds consistent with observed term structures of interest rates and credit spreads. The simulations should now include disparate sources of risks such as market and currency risk, interest rates, credit spreads, credit quality migrations, defaults and the corresponding recovery risk. The simulations should capture the correlation between the migration and default events. Capturing this non-independence of extreme events is a current topic of research in both the portfolio simulation and the credit derivatives pricing context. These simulations should be used as inputs to a multi-period stochastic optimization problem. The framework of combined simulation and optimization strategies can be extended to other securities subject to market and credit risk, such as, interest rate swaps.

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